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An Analysis of the Logic of Justification and its Use to Formalize Gettier Problems

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Abstract

Plato's analysis of knowledge states that knowledge is justified true belief. Following this analysis epistemic justification establishes a connection between true belief and knowledge. In this paper we analyze the Logic of Justification developed by Sergei Artemov. The Logic of Justification is a formalization of epistemic justification. Artemov uses the Logic of Justification to formalize the Gettier problems *Gettier's Case I* and *Fake Barn Country* and states that the Logic of Justification is better at dealing with Logical Omniscience. In this paper we will analyze Artemov's basic justification logic J_0, justification logic of belief J and justification logic of knowledge JT to find out whether the Logic of Justification captures all the important aspects of epistemic justification. We will analyze Artemov's formalizations of the Gettier problems to find out whether the Logic of Justification is capable to give an adequate formalization of Gettier problems. Furthermore we will review whether the Logic of Justification's is better at dealing with Logical Omniscience than epistemic modal logic.

Contents

1 Introduction

This paper is about the formalization of *epistemic justification*. Plato's analysis of knowledge states that *knowledge is justified true belief* [2]. Following this analysis epistemic justification establishes a connection between true belief and knowledge. So epistemic justification is very important when we want to *reason about knowledge*.

In the following quote Robert Moore gives several reasons why reasoning about knowledge is an important field of research within Artificial Intelligence:

> [T]he first question that comes up in reasoning about knowledge in AI is how to represent information about what someone knows. The fact that this is not altogether obvious has been a prime motivation for work in AI on reasoning about knowledge. Another motivation comes from AI work on planning. In trying to formulate a plan of action to achieve some goal, an agent may not have enough information. It is often necessary to reason about what knowledge is needed to carry out a plan and how that knowledge can be obtained. A third motivation is found in recent work in natural-language processing that tries to take into account the mental state of the person that the system is communicating with in interpreting and generating utterances. This usually requires reasoning about what that person knows. Finally, there are connections to work on nonmonotonic reasoning - most nonmonotonic reasoning systems have special inference rules that do not apply if their conclusions are known to be false, and in some systems this involves explicit reasoning about what the system itself knows. ([7], p. 81)

As we will see in this paper Moore's first question and his final motivation are important when we want to formalize epistemic justification.

Moore's first question is 'how to represent information about what someone knows.' One approach Artificial Intelligence has to do this is by using logic. Logic can be used to formalize notions and concepts in logics. Using these logics we can formulate formulas and then by applying inference rules we can reason about the notions and concepts. In 1962 Hintikka formalized the notions "knowledge" and "belief" in modal logic [5]. Following Plato's analysis of knowledge Hintikka's epistemic modal logic lacks a formalization of epistemic justification. Sergei Artemov states that this deficiency is displayed most prominently in the *Logical Omniscience* defect of the Modal Logic of Knowledge [1]. And in the next quote Ronald Fagin and Joseph Halpern state why Logical Omniscience is a problem when we want to reason about knowledge in Artificial Intelligence:

> There has long been interest in both philosophy and AI in finding natural semantics for logics of knowledge and belief. The standard approach has been the so-called possible-worlds model. The intuitive idea, which goes back to Hintikka [17], is that besides the true state of affairs, there are a number of other possible states of affairs, or possible worlds. Some of these possible worlds may be indistinguishable from the true world to an agent. An agent is then said to *know* or *believe* fact φ if φ is true in all the worlds he thinks possible.
>
> As has been frequently pointed out in the literature, possible-worlds semantics for knowledge and belief do not seem appropriate for mod-

> elling human reasoning since they suffer from the problem of what Hintikka calls logical omniscience. In particular, this means that agents are assumed to be so intelligent that they must know all valid formulas, and that their knowledge is closed under implication, so that if an agent knows p, and knows that p implies q, then the agent must also know q ([3], pp. 39-40)

A formalization of epistemic justification was finally introduced in 2008 by Sergei Artemov in his article "The Logic of Justification". In this article Artemov describes a general logical framework, Justification Logic, for reasoning about epistemic justification. As a case study Artemov also uses the Logic of Justification to formalize the Gettier problems *Gettier's Case I* and *Fake Barn Country* [1]. Artemov states that by using the Logic of Justification we can keep track of justification terms and escape the problem of Logical Omniscience.

Gettier problems are cases in epistemology introduced by Edmund Gettier that constitute an attack Plato's knowledge is justified true belief analysis [2]. In these cases, we describe situation in which a person has a justified true belief but no knowledge, usually because there is a further truth, a defeater, that would defeat the person's justification if the person knew it.

Defeaters are also very important, when we want to reason about knowledge in Artificial Intelligence. Note that Moore's final motivation states that 'most non-monotonic reasoning systems have special inference rules that do not apply if their conclusions are known to be false'. These conclusions can be known to be false if it is known that there is a defeater that defeats the premises.

In this paper we will analyze the Logic of Justification and its uses to formalize Gettier problems. For the Logic of Justification to be an adequate formalization of epistemic justification, it must capture all the important aspects of epistemic justification and therefore must also say something about defeaters, and it must be able to give an adequate formalization of Gettier problems. We will also review whether the Logic of Justification is better at dealing with Logical Omniscience then epistemic modal logic. This paper will answer the following research questions:

(1) Does the Logic of Justification capture all the important aspects of epistemic justification?
(2) Does the Logic of Justification yield an adequate formalization of Gettier problems?
(3) Is the Logic of Justification better at dealing with Logical Omniscience then epistemic modal logic?

Section 2 will describe the philosophical theories of knowledge that Artemov uses as building blocks for his formalization of epistemic justification. It will also analyze the reasoning behind Gettier problems and give a description of *Gettier's Case I* and *The Fake Barn Country.* Section 3 will give a description of Artemov's basic justification logic J_0, the justification logic of belief J, and the justification logic of knowledge JT. Section 4 will show how Artemov uses J and JT to formalize *Gettier's Case I* and *The Fake Barn Country.* Section 5 will be a reflection on Artemov's formalization of epistemic justification and will review whether the Logic of Justification is better at dealing with Logical Omniscience then epistemic modal logic. Section 6 will conclude with the answer to the research questions of this paper.

2 Gettier problems

In this section we will look at the following theories of knowledge, which Artemov has used as building blocks for his Logic of Justification: Plato's analysis of knowledge, Lehrer-Paxson's *Undefeated Justified True Belief* theory, and Goldman's *The Causal Theory of Knowing.* We will analyze the reasoning behind Gettier problems and give a description of *Gettier's Case I* and *The Fake Barn Country*, which Artemov has formalized using the Logic of Justification.

2.1 Plato's analysis of knowledge

Plato's analysis of knowledge states that a person S knows a proposition p if and only if (i) p is true, (ii) S believes p, and (iii) S is justified in believing p. In this analysis of knowledge we see that for true belief to become knowledge, justification is needed. But when are you justified in believing a proposition. The sentence "is justified in believing that" can also be read as "has adequate evidence for" or "has the right to be sure that" [2]. In this paper we will use the reading "has adequate evidence for", so you are justified in believing a proposition P, for example "it is raining", when you have adequate evidence for this proposition. In this example a couple of things that count as adequate evidence are: You observe that it is raining, you remember that it is raining, or someone you trust tells you that it is raining.

Note that if we follow Plato's analysis of knowledge, epistemic justification establishes a connection between true belief and knowledge. So to be able to establish a connection between true belief and knowledge is an important aspect of epistemic justification.

2.2 Gettier problems and defeaters

Edmund Gettier states that the three conditions "justification", "truth" and "belief" are not sufficient for knowing a proposition [2]. In his article "Is Justified True Belief Knowledge?" Gettier introduces a category of thought experiment, now known as Gettier problems, in which we describe cases where a person has a justified true belief but no knowledge. Gettier problems conflict with Plato's analysis of knowledge, because Plato's analysis states that all cases of justified true belief are cases of knowledge, so it should not be possible to create Gettier problems. Gettier problems are possible because Plato's analysis of knowledge does not say that for a justified true belief to become knowledge, the justification should be indefeasible.

A justification is indefeasible if

> there is no further truth which, had the subject known it, would have defeated [subjects] present justification for the belief ([1], p. 481)

A truth P defeats the justification j of Q, if whenever S is justified in believing Q then S has to retract the status of a justification from j. If P defeats j, then P is called the defeater of j.

Now that we now what defeaters are, we can look at the reasoning behind Gettier problems. Gettier states two things about epistemic justification:

(1) You can be justified in believing a false proposition.

(2) Justification is closed under entailment: If you are justified in believing P and P entails Q and you deduce Q from P and accept Q as the result of the deduction, then you are justified in believing Q [2].

In the rest of the paper these two statements will be called the first and the second principle of Gettier. Gettier problems usually start with a person forming a justified false belief, which is possible because of Gettier's first principle. This is usually because this person does not know that there is a defeater which would defeat the justification of his belief. During the Gettier problem the person usually infers a new justified belief from his justified false belief. This is possible because of Gettier's second principle. And at the end of the Gettier problem this new belief happens to be true, so the person has a justified true belief. But as we will see in *Gettier's Case I* and *Fake Barn Country* this justified true belief is intuitively no knowledge.

One important thing to note is that Gettier's second principle and the definition of indefeasible both contain a temporal factor. In Gettier's second principle you can only accept Q as the result of the deduction *after* you deduced Q from P. And *present* denotes a temporal aspect in the definition of indefeasible justifications.

So temporal factors are also an important aspect of epistemic justification.

2.3 Gettier's Case I

Let's take a look at the first Gettier problem, *Gettier's Case I*:

> Suppose that Smith and Jones have applied for a certain job. And suppose that Smith has strong evidence for the following conjunctive proposition: (a) Jones is the man who will get the job, and Jones has ten coins in his pocket. Smith's evidence for (a) might be that the president of the company assured him that Jones would in the end be selected, and that he, Smith, had counted the coins in Jones's pocket ten minutes ago. Proposition (a) entails: (b) The man who will get the job has ten coins in his pocket. Let us suppose that Smith sees the entailment from (a) to (b) and accepts (b) on the grounds of (a), for which he has strong evidence. In this case, Smith is clearly justified in believing that (b) is true.
>
> But imagine, further, that unknown to Smith, he himself, not Jones, will get the job. And, also, unknown to Smith, he himself has ten coins in his pocket. Proposition (b) is then true, though proposition (a), from which Smith inferred (b), is false. In our example, then, all of the following are true: (i) (b) is true, (ii) Smith believes that (b) is true, and (iii) Smith is justified in believing that (b) is true. But it is equally clear that Smith does not know that (b) is true; for (b) is true in virtue of the number of coins in Smith's pocket, while Smith does not know how many coins are in Smith's pocket, and bases his belief in (b) on a count of the coins in Jones's pocket, whom he falsely believes to be the man who will get the job. ([2], pp. 1-2)

In the following analysis of *Gettier's Case I* we will look at how the two principles of Gettier make it possible to form a justified true belief which is no knowledge and which defeater is used.

The first principle states that you can be justified in believing a false proposition. In *Gettier's Case I* Smith is justified in believing the false proposition (a). Smith believes (a), for which he has strong evidence, but (a) is false. The second principle states that justification is closed under entailment. In *Gettier's Case I* Smith is justified in believing proposition (b), because (a) entails (b), Smith sees this entailment and accepts (b) as the result of this entailment. In *Gettier's Case I* the defeater is the fact that Smith will get the job and that he has ten coins in his pocket. If Smith had justified in believing this further truth, then he would have not been justified in believing (a) and could not have inferred (b). So Smith is justified in believing (b), because of the two principles of Gettier and not knowing about the defeater. It is also the case that (b) is true. So it is clear that Smith is justified in believing the proposition (b), but it is also clear that Smith does not know (b).

2.4 Fake Barn Country

Fake Barn Country is a Gettier problem that is commonly attributed to Kripke and Goldman. This Gettier problem describes a case where we deduce something we do not know from something we know. This problem conflicts with the *Epistemic Closure Principle* which states that knowledge is closed under entailment. From this principle it follows that we know everything that we can deduce from our knowledge, so it should not be possible to create this Gettier problem. Note that the Epistemic Closure Principle follows from Logical Omniscience that the Epistemic Closure Principle is similar to Gettier's second principle, which states that justification is closed under entailment. The following quote is a description of *Fake Barn Country*:

> Suppose I am driving through a neighborhood in which, unbeknownst to me, papier-mâché barns are scattered, and I see that the object in front of me is a barn. Because I have barn-before-me percepts, I believe that the object in front of me is a barn. Our intuitions suggest that I fail to know barn [sic]. But now suppose that the neighborhood has no fake red barns, and I also notice that the object in front of me is red, so I know a red barn is there. This juxtaposition, being a red barn, which I know, entails there being a barn, which I do not [1].

In this example the defeater is the fact that only the red barns are real barns. If I do not know this defeater, I can form the false belief that the non-red object in front of me is a barn. I am justified in this belief because of the barn-before-me perception.

In *Gettier's Case I* we deduce a justified belief that also happens to be true. In *Fake Barn Country* we work the other way around. We start with the justified true belief that the object in front of me is a red barn. From this justified true belief we deduce the justified false belief that the object in front of me is a barn. If we follow Plato's analysis of knowledge and knowledge is justified true belief, then this case violates the Epistemic Closure Principle, because we deduce something we do not know from something we do know.

2.5 Undefeated Justified True Belief

In the previous paragraphs we have seen that Gettier problems should be cases of knowledge if we follow Plato's analysis of knowledge. This is because in Plato's analysis of knowledge justifications can be defeated. To get from Plato's analysis of knowledge to an analysis of knowledge that is capable of dealing with Gettier problems, Lehrer and Paxson suggested to add a fourth condition to Plato's analysis stating that the justification should be indefeasible [6]. This adjustment of Plato's analysis of knowledge is now known as the *Undefeated Justified True Belief* theory. This theory deals well with Gettier problems, because by making a justification indefeasible, there may not be a unknown defeater that defeats the justification. And we have seen that in a Gettier problem we can only be justified in believing a false propositions if we do not know the defeater. In the section 3 we will see that Artemov uses this theory of knowledge as a building block for the Logic of Justification.

2.6 The Causal Theory of Knowing

Another adjustment to Plato's analysis of knowledge is *The Causal Theory of Knowing* developed by Alvin Goldman [4]. Artemov gives the following interpretation of Goldman's principle:

> [A] subject's belief is justified only if the truth of a belief has caused the subject to have that belief (in the appropriate way), and for a justified true belief to count as knowledge, the subject must also be able to correctly reconstruct (mentally) that causal chain. ([1], p. 481)

Note that this theory denies Gettier's first principle which states that you can be justified in believing a false proposition. If we use this theory of knowledge, *Gettier's Case I* is not a case of justified true belief and therefore no knowledge. In *Gettier's Case I* Smith should not be justified in believing that Jones will get the job, because there is no causal chain between his justification and the truth that Jones will get the job, because it is not true that Jones will get the job. Also by using this theory of knowledge in *Fake Barn Country* I should not be justified in believing that the object in front of me is a barn, because it is not true that the object in front of me is a barn. In section 3 we will see that next to the Undefeated Justified True Belief theory also this theory is used by Artemov in his formalization of epistemic justification.

In this section we have seen that epistemic justification establishes a connection between a true belief and knowledge and that next to defeaters and temporal factors, this is also an important aspect of epistemic justification. We have reviewed the theories of knowledge that Artemov uses for his formalization of epistemic justification. We have seen the reasonings behind Gettier problems and reviewed the Gettier problems *Gettier's Case I* and *Fake Barn Country*.

3 Justification Logic

In this section we are going to take a look at how Artemov has formalized epistemic justification. The result of Artemov's formalization of epistemic justification is the Logic of Justification. In this section we look at the basic justification logic J_0 and two of its extensions: The justification logic of belief J and the justification logic of knowledge JT. In section 4 we will see how Artemov uses J and JT to formalize the Gettier problems *Gettier's Case I* and *Fake Barn Country*.

3.1 Language

The language of J_0 is built on the usual propositional language and contains the following set of justification terms t and formulas φ:

$$t ::= c \mid x \mid (t \cdot t) \mid (t + t)$$
$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \mid \varphi \leftrightarrow \varphi \mid t : \varphi$$

- Where 'c' are constants,
- 'x' are variables,
- '$\cdot$' is the Application-operator,
- '$+$' is the Sum-operator,
- 'p' is an atomic proposition,
- '$t{:}\varphi$' is the justification assertion of justification term t to formula φ.

In the next subsection we will see the informal reading of the various symbols. The Logic of Justification also uses the meta-variables $=$ s, u and v to range over justification terms.

Artemov gives the following reason why he has built the Logic of Justification on the language of propositional logic:

> At this stage, we are concerned first with justifications, which provide a sufficiently serious challenge on even the simplest Boolean base. Once this case is sorted out in a satisfactory way, we can move on to incorporating justifications into other logics. ([1], p. 479)

By restricting the expressiveness of the Logic of Justification, Artemov buys more control and this is important for Artificial Intelligence applications. In section 5 we will see whether the Logic of Justification is still expressive enough to give an adequate formalization of Gettier problems.

3.2 Informal reading of justification terms

In general $t{:}F$ denotes 't is a justification for F'. In the justification logic of belief J, $t{:}F$ denotes 't is the reason to believe F' and in the justification logic of knowledge JT, $t : F$ denotes 't is the reason to know F'. In the Logic of Justification we can build justification terms with the binary operations: Application '$\cdot$' and Sum '$+$'. In general the formula $(s \cdot t) : F$ denotes 'The product of s and t is a justification for F', and the formula $(s + t) : F$ denotes 's is a justification for F and the joint justification of s and t is also a justification for F or vice versa'.

The formula $t{:}F$ and the use of binary operations '$\cdot$' and '$+$' to build justification terms come from Artemov's Logic of Proofs [1]. In the Logic of Proofs the formula $t : F$ denotes 't is a proof for F'. It is not strange that Artemov also used this notation for the Logic of Justification, because you can say that proofs are a type of justification. In this paper I will not elaborate more on the Logic of Proofs, because it is very similar to the Logic of Justification.

3.3 Syntax

Now that we have seen the language of J_0 we will take a look at its axioms and rules. Using these axioms and rules we can reason about epistemic justification in the Logic of Justification. Artemov has given J_0 the following axioms:

A1. Classical propositional axioms and rule Modus Ponens,
A2. Application Axiom $s:(F \to G) \to (t:F \to (s \cdot t):G)$,
A3. Monotonicity Axiom $s:F \to (s+t):F, s:F \to (t+s):F$. ([1], p. 483)

Axiom A1 is used because the Logic of Justification is built on propositional logic.

In section 2 we have seen that Gettier's second principle states that justification is closed under entailment. This is very similar to the Epistemic Closure Principle, which states that knowledge is closed under entailment. In modal logic the closure principle is formalized using the Distribution Axiom:

$$\Box(P \to Q) \to (\Box P \to \Box Q)$$

In the Modal Logic of Knowledge we interpret the $\Box$-operator as knowledge operator and the Distribution Axiom will be a formalization of the Epistemic Closure Principle. Artemov also assumes that justification is closed under entailment and formalizes this property with axiom A2. This looks like a good formalization because A2 is very similar to the Distribution Axiom, but does A2 capture the temporal factor in Gettier's second principle?

In section 2 we have reviewed Lehrer-Paxson's *Undefeated Justified True Belief* theory. Artemov states that the full reading of this theory is beyond the scope of the Logic of Justification. To be able to formalize Lehrer-Paxson's condition Artemov reformulates this condition in the following way:

> Denoting present justification for the belief as the assertion $s:F$, we reformulate Lehrer-Paxsons condition as
>
> > *given $s{:}F$, for any evidence t, it is not the case that t would have defeated $s:F$.*
>
> The next step is to formalize t does not defeat $s:F$. This informal statement seems to suggest an implication
>
> > *if $s:F$ holds, then the joint evidence of s and t, which we denote here as $s+t$, is also an evidence for F, that is, $(s+t):F$ holds.*
>
> ([1], pp. 481-482)

After these steps Artemov formalizes the Undefeated Justified True Belief theory with the axiom A3. Note that in this theory 'would have' says something about modality. Because modality is beyond the scope of the Logic of Justification Artemov changes *would have* into 'does'. Also note that in this reformulation the defeaters change from *further truths* to *present justifications.* This reformulation will be further discussed in section 5.

If we look at the axioms of J_0 we will see that J_0 does not have an axiom or rule that states that our axioms are justified. Artemov states that J_0 is a justification logic for skeptical persons, because skeptical persons do not have justifications for their axioms.

In the justification logic of belief J Artemov wants the users to have justifications for their axioms and that these justifications are also justified. The following rule states that if we have an axiom, this axiom is justified by any justification. And the justifications of axioms are also justified by any justification:

> R4: Axiom Internalization Rule: For each axiom A and any constants $e_1, e_2, \ldots, e_n$, infer $e_n : e_{n1} : \ldots : e_1 : A$.

From this Axiom Internalization Rule it follows that $\vdash A \Rightarrow \vdash t : A$. Artemov defines the justification logic of belief J as:

$\mathsf{J} = \mathsf{J}_0 + \mathrm{R4}$.

The justification logic of knowledge JT is an extension of the justification logic belief J. The difference between these two logics is that JT has the Factivity Principle:

$t : F \to F$.

The Factivity Principle states that you can only be justified in knowing a formula if that formula is true. Artemov defines JT as:

$\mathsf{JT} = \mathsf{J} +$ Factivity Principle.

Artemov uses Goldman's *The Causal Theory of Knowing* as reason for using the Factivity Principle in his Logic of Justification:

> Goldman's principle makes it clear that a justified belief (in our language, a situation t justifies F for some t) for an agent occurs only if F is true, which provides the Factivity Axiom for 'knowledge-producing' justifications ([1], p. 481)

And Artemov states the following about the Axiom Internalization Rule R4:

> With a certain amount of good will, we can assume that *the 'causal chain' leading from the truth of F to a justified belief that F* manifests itself in the Principle of Internalization ([1], p. 481)

So the difference between the justification of belief J and the justification logic of knowledge JT is the Factivity Principle and Artemov states that this Factivity Principle provides 'knowledge-producing' justifications. In section 5 we will discuss the role of the Factivity Principle in the Logic of Justification.

3.4 Semantics

Now that we have seen the syntax, axioms and rules of J_0, J and JT, there is still an important thing left to talk about. And that is the semantics of $t : F$. Next to an informal reading of justification terms, we also need a formal interpretation.

Artemov has enriched Kripke models $(W, R, \Vdash)$, where W is the set of possible worlds and R the accessibility relation, with an admissible evidence function $\mathcal{E}$ to get a J-model $M = (W, R, \mathcal{E}, \Vdash)$ such that $\mathcal{E}(t, F) \subseteq W$ for any justification t and formula F. Artemov states that this informally means that $\mathcal{E}(t, F)$ specifies the set of possible worlds where t is considered admissible evidence for F.

With this admissible evidence function $\mathcal{E}$ Artemov gives the following semantics of $t : F$:

> $u \Vdash t : F$ if and only if
>
> (1) F holds for all possible situations [sic], that is, $v \Vdash F$ for all v such that uRv;
> (2) t is an admissible evidence for F at u, that is, $u \in \mathcal{E}(t, F)$.
>
> ([1], p. 489)

Let us start with saying that the semantics of $t : F$ are not very explanatory. The justification term t and formula F can both be complex, so the semantics is not compositional. Artemov states that in J we use J-models and in JT we use JT-models [1]. The only difference between these models lies in the accessibility relation R. The accessibility relation is reflexive in the justification logic of knowledge JT and not in the justification logic of belief J. But now the question rises if the admissible evidence function $\mathcal{E}$ establish a connection between true belief and knowledge.

In this section we have seen how Artemov has formalize epistemic justification. But is this formalization expressive enough to give an adequate formalization of Gettier problems? And do the axioms and the semantics capture all the important aspects of epistemic justification? We will come back to these questions in section 5.

In the next section we will see how Artemov has used J and JT to formalize the Gettier problems *Gettier's Case I* and *Fake Barn Country*.

4 Gettier problems formalized in Justification Logic

In section 2 we looked at how Gettier problems work and analyzed the Gettier problems *Gettier's Case I* and *Fake Barn Country*. In section 3 we looked at how Artemov has formalized epistemic justification. In this section we will look at how Artemov uses the Logic of Justification to formalize the Gettier problems *Gettier's Case I* and *Fake Barn Country*.

4.1 Gettier's Case I

Gettier's Case I contains a definite description, namely "the man who", and references to coins and pockets. Artemov states that *Gettier's Case I* could have been made simpler without losing its power. Artemov developed a version of *Gettier's Case I* that is based on the same material, but does not make use of definite descriptions and references to coins and pockets. Artemov calls this version *Streamlined Case I* [1]. In the next paragraph I will present and formalize a slightly revised version of *Streamlined Case I*. Artemov presents and formalizes *Streamlined Case I*, which does not make use of variables, using a quantifier-free first-order logic version of the Logic of Justification. Because the first-order logic version of the Logic of Justification is beyond the scope of this paper, our revised version of *Streamlined Case I* will be a propositionalized version. It is possible to propositionalize Artemov's first-order version, because it is quantifier-free and does not make use of variables.

In *Streamlined Case I* Smith has a false belief P 'Jones will get the job'. From P we can deduce $P \vee Q$, where Q denotes 'Smith will get the job'. Smith sees the entailment of $P \vee Q$ from P and accepts $P \vee Q$ as the result of the entailment. So following Gettier's second principle if Smith were justified in believing 'Jones will get the job', then he would also be justified in believing 'Jones will get the job or Smith will get the job'. And if we follow Gettier's first principle it is possible for Smith to be justified in the believing the false proposition P. In *Streamlined Case I* Smith is justified in believing that Jones will get the job, but unknown to him he will get the job, so it is true that $v : P$ for some v and $P \vee Q$. So if Smith were to be justified in believing $P \vee Q$, so that it were the case that $u : (P \vee Q)$ for some u, then Smith would have the justified true belief $P \vee Q$. But if we follow the same intuition as in *Gettier's Case I* we do not say that Smith knows $P \vee Q$. The following derivation in J will show that $u : (P \vee Q)$ can be deduced from $v : P$ as formalization of *Streamlined Case I*:

1. $v : P$, *assumption, (This is a justified false belief)*;
2. $P \to P \vee Q$, *propositional axiom*;
3. $c : (P \to P \vee Q)$, *from 2, by Axiom Internalization R4*;
4. $c : (P \to P \vee Q) \to [v : P \to (c \cdot v) : (P \vee Q)]$, *Application Axiom A2*;
5. $v : P \to (c \cdot v) : (P \vee Q)$, *from 4 and 3, by rule Modus Ponens;*
6. $(c \cdot v) : (P \vee Q)$, *from 5 and 1, by rule Modus Ponens. (This is a justified true belief)*

We now have deduced $u : (P \vee Q)$ for some u from $v : P$, because a justification term u can be an application of two justification terms $(c \cdot v)$, as we have seen in the syntax of the Logic of Justification in section 3.

We now have formalized *Streamlined Case I*, which is a simplified version of *Gettier's Case I*, in J similar to Artemov's formalization. But is this an adequate formalization of *Streamlined Case I*? We derived a justified true belief from a justified false belief, but the qualifications had to be added at a meta level.

Now that we have seen how Artemov formalizes *Streamlined Case I* in J we might want to know if it is also possible to formalize *Streamlined Case I* in JT. Artemov states that this is not possible, because Gettier problems cannot be formalized in JT [1]. This is, as Artemov states, because the Factivity Principle in JT follows from the *The Causal Theory of Knowing* and in section 2 we have seen that if we follow the *The Causal Theory of Knowing Gettier's Case I* is not a case of justified true belief.

The next derivation is a formalization of the revised version of *Streamlined Case I* in JT, which is similar to Artemov's formalization of *Streamlined Case I* in JT. The next derivation will show that *Streamlined Case I*, and therefore Gettier problems, cannot be formalized in JT, because the derivation will end in a contradiction:

1. $v : P$, *assumption*;
2. $\neg P$, *assumption, (Jones will not get the job, because Smith will get the job)*;
3. $v : P \to P$, *Factivity Principle*;
4. P, *from 3 and 1, by rule Modus Ponens*;
5. $\bot$, *from 2 and 4.*

We now have formalized *Streamlined Case I* in JT similar to Artemov formalization. But is this an adequate formalization of *Streamlined Case I*? In section 5 we will discuss if it is correct that we use $v : P$ as assumption in this derivation.

4.2 Fake Barn Country

Now that we have looked at Artemov's formalization of *Gettier's Case I* in the Logic of Justification we are going to take a look at his formalization of *Fake Barn Country*.

In section 2 we have seen that in *Fake Barn Country* we derive something we do not know from something we know and that this violates the Epistemic Closure Principle, which states that knowledge is closed under entailment. Artemov states that because *Fake Barn Country* violates the Epistemic Closure Principle, *Fake Barn Country* cannot be formalized in Epistemic Modal Logic. This is true, because in Modal Logic we have the Distribution Axiom:

$\Box(P \to Q) \to (\Box P \to \Box Q)$

In the Modal Logic of Knowledge we interpret the $\Box$-operator as knowledge operator and the Distribution Axiom will state the same as the Epistemic Closure Principle [5]. Using this Distribution Axiom the following derivation, which we will use as formalization of *Fake Barn Country*, will end in a contradiction. In the following derivation we will use B to denote 'The object in front of me is a barn' and we will use R to denote 'The object in front of me is red'. So $\neg\Box B$ will denote 'I do not know that the object in front of me is a barn' and $\Box(B \wedge R)$ will denote that 'I know that the object in front of me is a red barn'. In the following derivation Artemov shows that *Fake Barn Country* cannot be formalized in the Modal Logic of Knowledge:

1. $\neg\Box B$, *assumption*;
2. $\Box(B \wedge R)$, *assumption*;
3. $(B \wedge R) \to B$, *logical axiom*;
4. $\Box((B \wedge R) \to B)$, *from 3, by Necessitation*;
5. $\Box((B \wedge R) \to B) \to (\Box(B \wedge R) \to \Box B)$, *Distribution Axiom*;
6. $\Box(B \wedge R) \to \Box B$, *from 5 and 4, by rule Modus Ponens*;
7. $\Box B$, *from 6 and 2, by rule Modus Ponens*;
8. $\bot$, *from 7 and 1.*

This derivation is a good example why the Modal Logic of Knowledge suffer from the problem of *Logical Omniscience*. Let us again take a look at what Fagin and Halpern say about *Logical Omniscience*:

> [T]his means that agents are assumed to be so intelligent that they must know all valid formulas, and that their knowledge is closed under implication, so that if an agent knows p, and knows that p implies q, then the agent must also know q [3],p....

Note that the Distribution Axiom follows from the *Logical Omniscience* problem and that by using the Distribution Axiom it is not possible to formalize *Fake Barn Country* in the Modal Logic of Knowledge.

Artemov states the following about this incapability of epistemic modal logic:

> [E]pistemic modal logic is capable only of telling us that there is a problem, whereas Justification Logic helps to analyze what has gone wrong. We see that closure holds as it is supposed to, and we see that if we keep track of justifications we can analyze why we had a problem [1],p....

Artemov has formalized *Fake Barn Country* using both J and JT. Artemov states that when using J and JT to formalize *Fake Barn Country* the Epistemic Closure Principle will hold, because we will not end in a contradiction at the end of the derivation. We will only look at Artemov's formalization of *Fake Barn Country* in JT, because the formalization in J is very much alike to the formalization in JT.

In the next derivation we will use $\neg u : B$ to denote 'u is not a good reason to know that the object in front of me is a barn' and we will use $v : (B \wedge R)$ to denote 'v is a good reason to know that the object in front of me is a red barn'. The next derivation will show that if we follow Artemov's formalization of *Fake Barn Country* in JT, we will can deduce $s : B$ for some s from $v : (B \wedge R)$ and that $s : B$ and $\neg u : B$ will not contradict each other:

1. $\neg u : B$, *assumption*;
2. $v : (B \wedge R)$, *assumption*;
3. $(B \wedge R) \rightarrow P$, *logical axiom*;
4. $s : [(B \wedge R) \rightarrow B]$, *from 3, by Axiom Internalization*;
5. $s : [(B \wedge R) \rightarrow B] \rightarrow [v : (B \wedge R) \rightarrow (s \cdot v) : B]$, *from 4, by Application;*
6. $v : (B \wedge R) \rightarrow (s \cdot v) : B$, *from 5 and 4, by rule Modus Ponens*;
7. $(s \cdot v) : B$, *from 6 and 2, by Modus Ponens.*

[1],pp 487-488.

Artemov states that $\neg u{:}B$ and $(s{\cdot}v){:}B$ do not contradict each other, because they denote that in the first case you do not have a good reason to know B and in the second case that you do have a good reason to know B. Note that this problem was not visible when using the Modal Logic of Knowledge. Also Note that in this case we could not match the justifications u and $(s \cdot v)$. In this derivation we deduced $s : B$ for some s. This s could not be u, because it was already stated that u is not a good reason to know B.

We now have seen how Artemov has used the Logic of Justification to formalize Gettier problems. But do we get adequate formalizations of Gettier problems when using the Logic of Justification? And does keeping track of justification really help us when dealing with the problem of *Logical Omniscience*?

5 Reflection

In this section we will answer the research questions of this paper:

(1) Does the Logic of Justification capture all the important aspects of epistemic justification?
(2) Does the Logic of Justification yield an adequate formalization of Gettier problems?
(3) Is the Logic of Justification's formulation of dealing with Logical Omniscience a useful extension of epistemic modal logic?

In section 2 we have seen the following important aspects of epistemic justification:

- Defeaters,
- Temporal factors,
- Establishment of connection between true belief and knowledge.

In section 3 we have seen how Artemov has formalized epistemic justification. And in section 4 we have seen how Artemov uses the Logic of Justification to formalize the Gettier problems *Streamlined Case I* and *Fake Barn Country* and we have seen how the Logic of Justification deals with Logical Omniscience.

In the following subsections I will argue why the answers to each of the research questions is: No.

5.1 Establishment of connection between true belief and knowledge

Does the Logic of Justification establish a connection between true belief and knowledge? If we look at the justification logic of belief J and the justification logic of knowledge JT, we see two different interpretations of the formula $t:F$. In the justification logic of belief $t:F$ is interpreted as 't is the reason to believe F.' In the justification logic of knowledge $t:F$ is interpreted as 't is the reason to know F.' In both logics $t:F$ does not give a connection between believing a true proposition F and knowing F. In J you do not have the capabilities to talk about truth and knowledge, and in JT you do not have the capabilities to talk about belief.

But might the semantics of $t:F$ perhaps establish a connection between true belief and knowledge? In section 3 we have seen that Artemov gives the following semantics of $t:F$:

> $u \Vdash t:F$ if and only if
>
> (1) F holds for all possible situations [sic], that is, $v \Vdash F$ for all v such that uRv;
> (2) t is an admissible evidence for F at u, that is, $u \in \mathcal{E}(t, F)$.
>
> ([1], p. 489)

The semantics of $t:F$ is the same for J and JT, but as I stated earlier, there is a difference between the interpretation of $t:F$ in J and $t:F$ in JT. The difference here between J and JT is that J uses J-models $M = (W, R, \mathcal{E}, \Vdash)$ and JT uses JT-models $M = (W, R, \mathcal{E}, \Vdash)$. The difference between these two models lies in the property of the accessibility relation R. The accessibility relation is reflexive in the justification logic of knowledge JT and not in the justification logic of belief J. In epistemic modal logic it is also the case that knowledge is reflexive and belief is not.

So the difference between belief-models and knowledge-models depends on the property of the accessibility relation. In section 3 we have seen that Artemov has enriched Kripke models with the admissible evidence function $\mathcal{E}$. But if we look at the semantics of $t:F$ we do not see that $\mathcal{E}$ affects R. So the only thing that the Logic of Justification adds to epistemic modal logic is the admissible evidence function $\mathcal{E}$, which does not effect the accessibility relation R, which establishes a connection between belief-models and knowledge-models.

So the Logic of Justification does not establish a connection between true belief and knowledge.

5.2 Defeaters

Does the Logic of Justification capture the defeaters aspect of epistemic justification? In section 2 we have seen that defeaters are further truths that defeat justifications. A truth P defeats the justification j of Q, if whenever S is justified in believing Q then S has to retract the status of a justification from j. One of the conditions from the Lehrer and Paxson's *Undefeated Justified True Belief* theory is that for justified true belief to count as knowledge, its justification must be indefeasible. Artemov formulates this conditions in the following way:

> there is no further truth which, had the subject known it, would have defeated [subject's] present justification for the belief ([1], p. 481)

Note that *further* and *would have* denotes that this condition contains a modality factor.

In section 3 we have seen that Artemov formalizes this conditions with the Monotonicity Axiom:

$s : F \to (s + t) : F$,
$s : F \to (t + s) : F$. ([1], p. 483)

This axiom only states that *actual justifications* may not defeat each other and note that *actual* does not denote a modality factor.

The Logic of Justification does not capture the defeaters aspect of epistemic justification, because defeaters are *further truths* that defeat justifications and not *actual justifications* that defeat justifications.

5.3 Temporal factors

Does the Logic of Justification contain temporal factors? In section 2 we have seen that the second principle of Gettier states that justification is closed under entailment. This means that if you are justified in believing P and P entails Q and you deduce Q from P and accept Q as the result of the deduction, then you are justified in believing Q [2]. Note that this formulation contains a temporal factor. First, you must believe P, and P must entail Q, after that you must deduce Q from P, and after that you must accept Q as the result of the deduction.

If we look at *Gettier's Case I*, we see that *Gettier's Case I* also contains a temporal factor, because it is based on Gettier's second principle:

> Let us suppose that Smith sees the entailment from (a) to (b) and accepts (b) on the grounds of (a), for which he has strong evidence ([2], p. 2)

Artemov formalizes Gettier's second principle with the Application Axiom:

$s : (F \to G) \to (t : F \to (s \cdot t) : G)$ ([1], p. 483)

This axiom does not contain a temporal factor. You can argue that there is an order in $(s \cdot t)$, but Artemov does not make it explicit that this is a temporal order.

So the Logic of Justification does not contain temporal factors.

5.4 Adequate formalization of Gettier problems

Does the Logic of Justification yield an adequate formalization of Gettier problems? In section 4 we have seen that Artemov states that Gettier problems can only be formalized in the justification logic of belief J and not in the justification logic of knowledge JT. And in section 3 we have seen that the difference between J and JT is the Factivity Principle:

$t : F \to F$. ([1], p. 488)

In section 2 we have seen that *Gettier's Case I* is a case of knowledge if we follow Plato's analysis of knowledge and that *Gettier's Case I* is not case of knowledge if we follow the *The Causal Theory of Knowing*. So it might look acceptable when Artemove states that the Factivity Principle follows from the *The Causal Theory of Knowing*.

The Facitivity Principle states that you can only know a proposition if that proposition is true. But this also follows from Plato's analysis of knowledge which allowed Gettier problems to work. So it is not true that Gettier problems do not work when using the Factivity Principle. Gettier problems cannot be formalized in JT, because JT cannot talk about belief. If we look at the derivation of *Streamlined*

Case I in JT in section 4 we see that, if we follow Artemov's derivation, we start with the assumption $v : P$. In JT this means that Smith has a reason to *know* that Jones will get the job. But that is not true in *Streamlined Case I*, because Smith only has a reason to *believe* that Jones will get the job.

But now we can ask ourselves whether Gettier poblems can be formalized in J. If J should be able to formalize Gettier problems, then J must have some way of saying about the truth of the proposition you are believing. But the justification logic of belief does not have that capability. For example in the formalization of *Gettier's Case I* in J in section 4, if we follow Artemov's formalization, we had to assume on a meta level that $v : P$ is a justified false belief and that $(c \cdot v) : (P \vee Q)$ is a justified true belief.

As we have seen in section 2 Gettier problems contain defeaters and temporal aspects. In the previous subsection we have argued that the Logic of Justification does not capture the defeaters aspect of epistemic justification and does not contain temporal factors.

So the Logic of Justification does not yield an adequate formalization of Gettier problems, because of the incapability of J to talk about truth and knowledge, the incapability of JT to talk about belief and because the Logic of Justification does not capture all the important aspects of epistemic justification.

5.5 Dealing with Logical Omniscience

In section 4 we have seen that we could not formalize *Fake Barn Country* in the Modal Logic of Knowlege, because epistemic modal logic suffers from the problem of Logical Omniscience. In section 4 we have also reviewed the following quote of Artemov:

> Justification Logic provides natural means of escaping logical omniscience by keeping track of the size of justification terms ([1], p. 482)

But do we escape Logical Omniscience when we keep track of the size of justification terms? If we follow Artemov's formalization of *Fake Barn Country* in *JT* in section 4 we did not end in a contradiction. Artemov states that because $\neg u : B$ and $(s \cdot v) : B$ do not contradict each other, the Epistemic Closure Principle still holds. But note that this result is made possible by using the Application Axiom. But we have already seen that Gettier's second principle, which state that justification is closed under entailment, contains a temporal factor and that the Application Axiom, which is intended to be a formalization of this principle does not contain temporal aspects. So the question might rise if we would get the same results if the Application Axiom does contain temporal factors.

But following the quote of Fagin and Halpern from the introduction of this paper, the Epistemic Closure Principle is only the second point of Logical Omniscience. We still have the problem that:

> agents are assumed to be so intelligent that they must know all valid formulas ([3], pp. 39-40)

The Logic of Justification does not escape this problem because this can also be achieved in JT by using the Axiom Internalization Rule:

> R4: Axiom Internalization Rule: For each axiom A and any constants $e_1, e_2, \ldots, e_n$, infer $e_n : e_{n1} : \ldots : e_1 : A$. ([1], p. 484)

By using the Internalization Rule in JT we are justified in knowing all valid formulas. And by using the Internalization Rule we are also justified in knowing every justification for knowing a valid formula. It is true that by using the Logic of Justification we can keep track of the size of justification terms. But it is still a problem that the size of justification terms is infinite.

So the Logic of Justification might be better at dealing with Logical Omniscience than epistemic modal logic, because it does not contain temporal aspects. But the Logic of Justification does not escape Logical Omniscience.

6 Conclusion

In this paper I have analyzed the Logic of Justification developed by Sergei Artemov. The Logic of Justification does not capture all important aspects of epistemic justification, because it does not establish a connection between true belief and knowledge, it does not capture the defeaters aspect and it does not contain temporal factors. Because of this and the incapability of the justification logic of belief J to talk about truth and knowledge, the incapability of the justification logic of knowledge JT to talk about belief, the Logic of Justification does not yield adequate formalizations of Gettier problems. Because that the Logic of Justification does not contain temporal factors it might be better at dealing with Logical Omniscience than epistemic modal logic, but it does not escape the problem of Logical Omniscience. The Logic of Justification is not an adequate formalization of epistemic justification and therefore should not be used when we want use logic to reason about knowledge in Artificial Intelligence. Further research can be done in developing a formalization of epistemic justification that does establish a connection between true belief and knowledge and does capture the defeaters aspect and contains temporal factors. This formalization of epistemic justification should then be able an adequate formalization of Gettier problems. Whether this formalization of epistemic justification is able to escape Logical Omniscience remains to be seen.

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