# Increasing the Chemical potential By using a Local Optical Ultra-tight Dipole-Trap (I.C.L.O.U.D) 

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#### Abstract

Although the experimental set-ups to get in the temperature regime for Bose-Einstein Condensation have been available for many years, there is an even colder regime where we are eager to do research in. It is in this regime that the temperature times the Boltzmann constant becomes of the order of the chemical potential. In this regime exciting new things can be measured, for example, so-called first and second sound switch their behaviour. In the experiment described in this report we reach this regime not by decreasing temperature, but by increasing the chemical potential by making our trap tighter. Usually, this also means that the temperature goes up, but we suppress this effect by creating a thermal bath in the trap. We succeeded in this by using an existing harmonically shaped magnetic trap with a tight dimple shaped optical dipole trap added in the centre. The fact that this trap is not harmonic changes the physics involved. This report features simulations and an experimental part. It this report it is shown that we have reached the regime where the chemical potential and the temperature become of the same order.


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## Chapter 1

## Introduction and Concepts

### 1.1 Introduction

This research aims to get to a new regime in Bose-Einstein Condensation, where we want to increase the ratio between the chemical potential and the temperature. The process of BoseEinstein Condensation has been available for almost a century with liquid helium [1] and for low interacting gases for almost two decades. [2] [3] 4] 6] [7. Bose-Einstein Condensation is a state of matter that occurs at low temperatures or high particle densities. In this research however we aim to reach another regime where the chemical potential becomes larger then the Boltzmann constant times the temperature. In this regime new physics can be measured for example first and second sound, which are discussed in the outlook (chapter 6.3) of this report. The current set-ups used for Bose-Einstein condensation, which use harmonic traps, have difficulty reaching this regime. In this research another method is used, where we increase the chemical potential by using a trap that is tight in the lower energy regions of the potential, but wide in the high energy regions. By allowing a thermal bad in the wide part of the trap, and increasing the tightness in the centre of the trap, particle densities and thus the chemical potential is increased, while suppressing the temperature increase.

A broader explanation to Bose-Einstein Condensation in the context of this report and a explanation of this change in trap shape is presented in section 1.2. Next, in chapter 2 the necessary theoretical description to describe this process is presented. We then simulate a number of experiments in chapter 3 A part of these experiments are then performed in a real set-up (chapter (4) of which the results are presented in chapter 5

This master research actually consisted of two projects. A second small project concerning Particle Flux was done before the large project involving non-harmonic dimple traps. This report primarily discusses the large project. Since the discussion of the large project has a clear structure, the part about the small project would be out of place at any position in the report. Therefore the small project is discussed in Appendix G. The fact that it is in an appendix does not mean that it is not important. The large project is discussed in the other parts of this report. The small project about Particle Flux was done in collaboration with fellow master student Martijn van 't Woud BSc, and therefore Appendix G is not completely written by me, but has a second author, namely him.The small project has an extra supervisor, namely Ph.D. student Pieter Bons MSc. This report continues discussing non-harmonic dipole traps.

### 1.2 Concepts

A Bose-Einstein condensate can be seen as a macroscopic occupation of an energy state, and also as a state where the wave functions of the individual particles overlap so much that it can be described by one single quantum mechanical wave function. 8 However, in this section we describe the phenomena in a way more fit to this research, by looking at the chemical potential.

### 1.2.1 Concept of a cloud in a harmonic trap

Consider a trapping potential of any shape 1 , with a cloud of trapped (bosonic) particles in it. Now quantum mechanics learns us that there are only quantized levels of energy allowed. So if our particles were fermions, we would know that there could be only one particle per energy state, but since we have bosons, multiple particles can be in one state 9 . Since our cloud has a temperature, and that way also a thermal energy (kinetic energy), not all particles occupy the lowest energy state, but in fact spread over an entire spectrum of energy levels. Of such a trapped cloud of atoms the chemical potential ( $\mu$ ) compared to the minimum of the trap is negative for normal temperatures. For those using the definition of chemical potential as the amount of energy needed to add a particle to the system this might be counter-intuitive as it suggests that one would gain energy from putting a particle in a trap. But let it be noted that this definition applies only for constant entropy, so that is forgetting about the $T d S$ in the equation for internal energy given by:

$$
\begin{equation*}
\mathrm{d} U=T \mathrm{~d} S-p \mathrm{~d} V+\mu \mathrm{d} N \tag{1.1}
\end{equation*}
$$

Of course if someone would simply add a particle to the system the entropy wouldn't be constant at all. However at zero temperature the $-T d S$-term vanishes and therefore drops out of the equations, making (at zero temperature) the chemical potential with or without entropy change exactly the same. So at zero temperature this definition becomes useful. So what is the chemical potential at zero temperature? First of all at zero temperature there is no longer thermal energy to be overcome and because of that thermal energy is no longer in the equations. Second of all, one must note that since there is no longer any thermal energy to let the particles spread over the energy levels, the particles will fall into a macroscopically occupied ground state. If this ground state has a non-zero energy (compared to the minimum of the trap) then of course an added particle would need to overcome this energy. But, in fact this energy is very small compared to another source of energy that must be overcome, interaction energy. Since we have a cloud of particles, particles interact with each other meaning that adding a particle means adding interaction energy. In general the interaction energy with all the other particles scales with the density. So this means that, since the interaction energy is positive, that the chemical potential at zero temperature is non-zero and positive. Now we are able to conclude that we have two regimes, high temperature with a negative chemical potential, and zero temperature with a positive one. The conclusion must be that there must be a temperature in between where the chemical potential is zero. At this temperature, the system switches over from this regime, where the ground state is macroscopically occupied, to the regime where all particles spread over all states. Basically this means that when the chemical potential becomes in the order of zero there is a transition from the regime where the interaction energy dominates, to the one where the thermal energy does. So if we start at zero-temperature and we increase it, more and more particles start to occupy higher states until the macroscopically occupied state disappears at a certain temperature $T_{c}$, where the chemical potential is almost zero 2 One may recognise the

[^0]particles occupying this ground state as the (Bose-Einstein) condensate, and the other particle the thermal cloud.

Of course if we have a higher density of particles, we have a higher chemical potential, so we would need to move to higher temperatures to force it zero. This shows that the condensation temperature is in fact not a steady value, and it therefore depends on the density. If one would for example make the trap tighter, the density and thus the chemical potential would increase, and therefore the condensation temperature $T_{c}$ would also increase. This makes clear that for the properties of a condensate it is the proportion between the chemical potential and the temperature that basically defines its properties. So increasing the chemical potential at constant temperature is very similar to decreasing temperature at a constant chemical potential. Let us define this proportion as

$$
\begin{equation*}
\chi=\frac{\mu}{\mathrm{k} T} \tag{1.2}
\end{equation*}
$$

with the Boltzmann constant there to make it dimensionless.

### 1.2.2 Concept of a cloud in a "dimple" trap

So we now know that we have condensation as soon as this $\chi$ becomes positive. Reaching this regime has been possible for almost a century with liquid helium [1] and for low interacting gases for almost two decades. [2] [3 [4 [5] (6) 7] In this research however we aim to reach another regime where $\chi$ is not only positive, but where it becomes close or larger than one. In this regime it is for example predicted that first and second sound switch behaviour.[10, page 69] (See more in section 6.3.2 However as it turns out reaching low enough temperatures to get $\chi$ in the order of one is very hard to achieve experimentally. So we decided to go the other way, we decided to increase the chemical potential instead, effectively doing the same thing. As said before this can easily be done by making our trap tighter. Unfortunately, if one would just take a harmonic trap, and would just make it tighter, the temperature would also increase undoing the effect. So what we will use is the fact that the energy used for temperature is mostly in the higher energy regions of our potential, while the condensate is of course in the lower regions. By introducing a dimple in our trap, as shown in Figure 1.1 we can increase the density in the centre, and thus the chemical potential, but because we do not compress in the higher energy regions of our trap, the higher energy particles work as a large thermal bath, suppressing the heating effect of that one would have in a normal harmonic trap. Basically this method has been used before by Ketterle to go from a negative to a positive $\chi$, to make a condensate without throwing away any particles. [11, Section 3] This research considers a large simulation part (Chapter 3) where it is shown using a Hartree-Fock approach that this way it is indeed possible to increase $\chi$. It also shows that this gives a large spatial spread to the thermal particles compared to the condensate, thus confirming our theory of a thermal bath. It then considers that we might be able to use this spatial separation to throw away our thermal bath after it has done its job, and therefore leaving us with a high $\chi$ cloud which we can then insert back in a harmonic trap, basically using the spatial separation to make spilling more effective. After the simulation part there is an experimental part (Chapters 4 and 5 where this method is actually applied in experiments and is shown to be working, that means we where able to get $\chi$ bigger than one.


Figure 1.1: Visual impression of including a dimple in the trap. The dimple will increase density for the low energy particles but will leave the high energy particles unharmed

## Chapter 2

## Theoretical Description

To describe the experiment described in chapter 1 we need to know how a cloud of atoms behaves at low temperatures. The lowest temperature is of course zero temperature and since this is the most simple case we start off using a Hartree approximation for zero temperature (section 2.1). Next, we make corrections to this model for non-zero temperature in section 2.2 since we are not at zero-temperature and we are interested in the temperature behaviour (see section 1.2).

### 2.1 Zero Temperature

At zero-temperature all particles are in the same state so normally we are allowed to write for the N-particle wave function:[8] page 160]

$$
\begin{equation*}
\Psi\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \ldots, \mathbf{r}_{N}\right)=\prod_{i=1}^{N} \phi\left(\mathbf{r}_{i}\right) \text { with } \int d \mathbf{r}\left|\phi\left(\mathbf{r}_{i}\right)\right|^{2}=1 \tag{2.1}
\end{equation*}
$$

where $\Psi$ is the many-particle wave function and $\psi$ is the single-particle wave function. However this does not take into account the interactions between the atoms, and those cannot be neglected, so we take those into account simply by using the effective interaction given by $U_{\text {eff }}\left(\mathbf{r}, \mathbf{r}^{\prime}\right)=$ $U_{0} \delta\left(\mathbf{r}-\mathbf{r}^{\prime}\right)$, with $U_{0}$ given by [8, page 120]

$$
\begin{equation*}
U_{0}=\frac{4 \pi a \hbar^{2}}{m} \tag{2.2}
\end{equation*}
$$

which is the contact interaction. Here $a$ is the scattering length, $\hbar$ is the Planck constant and $m$ is the mass. This effective interaction changes the many-particle wave function. In other words, the regular single-particle Hamiltonian $\left(H=\sum_{i=1}^{N}\left(\frac{\mathbf{p}_{i}^{2}}{2 m}+V\left(\mathbf{r}_{i}\right)\right)\right)$ has not yet the interactions in it, since there is only one particle, and that is why we can't just use 2.1. Via appendix A we now obtain:

$$
\begin{equation*}
V(\mathbf{r})+U_{0} n(\mathbf{r})=\mu, \text { or } n(\mathbf{r})=0 \tag{2.3}
\end{equation*}
$$

the solution to the time independent Gross-Pitaevskii equation in the Thomas-Fermi approximation. So this give us the formula for our density distribution 2.3 and our total energy A. 12 .

### 2.2 Non-zero temperature

For non-zero temperature we can't say that all particles are in the ground state and we need to include the particles that are not in the condensed state, the excited particles. The density of condensed particles we now no longer represent by $n$ as in section 2.1, but by $n_{0}$. The density of particles in excited states is given by $n_{\mathrm{ex}}$, also known as thermal cloud.
Let us start off by making the correction to the equations in section 2.1 by looking at what changes for the condensate because of the presence of thermal particles. For this we look to the energy of a condensed particle. Looking at A.12 one sees that the average energy per particle is given by $V+\frac{1}{2} U_{0} n_{0}$. However the energy for a condensed particle is given by the chemical potential in (2.3). The factor two is explained because interaction is always shared between two particles. Now according to the Fock approximation the change in this because of the presence of excited particles is $2 n_{\text {ex }} U_{0}$ (see [8, eq 8.92]) so let us add this to (2.3) yielding:

$$
\begin{equation*}
\mu=V+U_{0} n_{0}(\mathbf{r})+2 U_{0} n_{\mathrm{ex}}(\mathbf{r}) \tag{2.4}
\end{equation*}
$$

for the chemical potential. In equilibrium this $\mu$ is independent of (r). This means that we know our density distribution of condensed particles as soon as when we know our distribution of excited particles.

Now let us look at the excited particles. First of all, we know that they are distributed over the different energy states by the Bose-Einstein distribution:

$$
\begin{equation*}
f_{\mathbf{p}}=\frac{1}{\exp \left(\beta\left(\epsilon_{\mathbf{p}}-\mu\right)-1\right)} \tag{2.5}
\end{equation*}
$$

with $\beta=\frac{1}{\mathrm{k} T}, \epsilon_{\mathbf{p}}$ the energy of an excited particle and $\mathbf{p}$ simply the momentum. So we have to find out what the energy of an excited particle is. Classically this is simply the kinetic energy $\frac{p^{2}}{2 m}+V$, but of course we again need to include the effect of interactions. According to the Fock approximation for a state with momentum $p \equiv|\mathbf{p}|$ this is simply:

$$
\begin{equation*}
\epsilon_{\mathbf{p}}=\frac{p^{2}}{2 m}+V+2 n(\mathbf{r}) U_{0} \tag{2.6}
\end{equation*}
$$

Here it is important that the state it self isn't macroscopically occupied, so that we can neglect those particles. ${ }^{1}$.
So our $\epsilon_{\mathbf{p}}-\mu$ in 2.5 becomes simply

$$
\begin{equation*}
\epsilon_{\mathbf{p}}-\mu=\frac{p^{2}}{2 m}+U_{0} n_{0} \tag{2.7}
\end{equation*}
$$

We now know the distribution over the excited states, so we can now integrate over the momentum to obtain the total density, using:

$$
\begin{equation*}
\int_{0}^{p_{\max }} \frac{d p}{(2 \pi \hbar)^{3}} \frac{4 \pi p^{2}}{e^{\beta\left(\frac{p^{2}}{2 m}+n_{0} U_{0}\right)}-1}=\frac{\sqrt{m^{3}}}{\sqrt{2} \pi^{2} \beta^{3 / 2} \hbar^{3}} \int_{0}^{\beta\left(E_{\max }-2 n U_{0}-V\right)} d x \frac{\sqrt{x}}{e^{x} z^{-1}-1} \tag{2.8}
\end{equation*}
$$

[^1]with z equal to:
\[

$$
\begin{equation*}
z=e^{-\beta n_{0} U_{0}} \tag{2.9}
\end{equation*}
$$

\]

with $E_{\max }$ the maximally allowed value for 2.6 and $p_{m} a x$ the corresponding value for $p$. If let $E_{\text {max }}$ be infinite ${ }^{2}$ the integral becomes equal to:

$$
\begin{equation*}
n_{\mathrm{ex}}=\frac{\operatorname{Li}_{3 / 2}(z)}{\Lambda_{\mathrm{deB}}^{3}} \tag{2.10}
\end{equation*}
$$

with $\Lambda_{\mathrm{deB}}=\frac{h}{\sqrt{2 \pi m \mathrm{k} T}}$ the De Broglie wave length. This makes $n_{\mathrm{ex}}$ dependent of $n_{0}$. We now have a full circle and we can insert our expression for $n_{\text {ex }}$ back into 2.4 yielding:

$$
\begin{equation*}
\mu=V(\mathbf{r})+U_{0} n_{0}(\mathbf{r})+2 U_{0} \frac{\mathrm{Li}_{3 / 2}(z(\mathbf{r}))}{\Lambda_{\mathrm{deB}}^{3}} \tag{2.11}
\end{equation*}
$$

From this equation $n_{0}$ can be obtained, that can then be inserted back into 2.10 to obtain $n_{\text {ex }}$. Unfortunately (2.11) cannot be solved analytically, so in section 3.1 we numerically solve this equation, using an iterative method.

Now that we have obtained the density distributions, we would also like to know what the changes are to the total energy of the system. Without the effects of excited particles the total energy of the condensate is given by

$$
\begin{equation*}
E\left(n_{0}(\mathbf{r})\right)=\int d \mathbf{r} H_{0} \tag{2.12}
\end{equation*}
$$

with $H_{0}$ the energy density given by

$$
\begin{equation*}
H_{0}(\mathbf{r})=V(\mathbf{r}) n_{0}(\mathbf{r})+\frac{1}{2} U_{0} n_{0}(\mathbf{r})^{2} \tag{2.13}
\end{equation*}
$$

which is a modified version of A.12. To include the excited particles we need to know the energy density $H$ at position mathbfr. This is simply, realising that they are Bose-distributed,

$$
\begin{equation*}
H_{e x}(\mathbf{r})=\int d \mathbf{p} \frac{\epsilon_{\mathbf{p}}}{\exp \left(\beta\left(\epsilon_{\mathbf{p}}-\mu\right)-1\right)}=n_{\mathrm{ex}}(\mathbf{r})\left(2 n(\mathbf{r}) U_{0}+V(\mathbf{r})\right)+\frac{3 \sqrt{2}(m \pi)^{\frac{3}{2}}(\mathrm{k} T)^{\frac{5}{2}} \operatorname{Li}_{5 / 2}(z(\mathbf{r}))}{h^{3}} \tag{2.14}
\end{equation*}
$$

so integrating this over all possible $p$ gives us the energy density of the excited particles. We have recognised 2.8 in the constant terms of $\epsilon_{\mathbf{p}}$ to simplify our equation. Next we realise that we need a term to account for the interaction energy between excited and condensed particles, which was not accounted for in 2.13 , which is then given by:

$$
\begin{equation*}
H_{e x i n t}(\mathbf{r})=\frac{n_{0}(\mathbf{r}) n_{e x}(\mathbf{r}) U_{0}}{2} \tag{2.15}
\end{equation*}
$$

[^2]Now using that $n=n_{e x}+n_{0}$ we can obtain the energy density $H=H_{e x}+H_{0}+H_{\text {ex int }}$ :

$$
\begin{gather*}
H(\mathbf{r})=2 n(\mathbf{r}) n_{\mathrm{ex}}(\mathbf{r}) U_{0}+\frac{n_{0}(\mathbf{r}) n(\mathbf{r}) U_{0}}{2}+n(\mathbf{r}) V(\mathbf{r})+\frac{3{\mathrm{k} T \mathrm{Li}_{5 / 2}(z(\mathbf{r}))}_{2 \Lambda_{\mathrm{deB}}}}{E=\int d \mathbf{r}\left(2 n(\mathbf{r}) n_{\mathrm{ex}}(\mathbf{r}) U_{0}+\frac{n_{0}(\mathbf{r}) n(\mathbf{r}) U_{0}}{2}+n(\mathbf{r}) V(\mathbf{r})+\frac{3{\mathrm{k} T \mathrm{Li}_{5 / 2}(z(\mathbf{r}))}_{2 \Lambda_{\mathrm{deB}}}}{E} .\right.} . \tag{2.16}
\end{gather*}
$$

So we are now able to calculate the total energy.
In (2.3) we also obtained the case where $n_{0}=0$. That is missing in 2.11. Actually there is a whole regime where $V$ is larger then $\mu$ where (2.11) does not have a solution. Because of the third term in (2.11) actually starts already at a lower $V$. This occurs when the temperature is above the condensation temperature, and at large distances. Entering $n_{0}=0$ into 2.10 would give a constant density of excited particles over all space, which is obviously physically impossible. What is happening is that we cannot keep on using the same equations with $n_{0}=0$ when there is no solution, because the whole set of equations was derived under the assumption that there was a solution. So if the ground state is no longer macroscopically occupied, every particle is an excited particle, reducing the energy expression for excited particles (2.6) to:

$$
\begin{equation*}
\frac{p^{2}}{2 m}+V+2 n_{e x} U_{0} \tag{2.18}
\end{equation*}
$$

or when subtracting $\mu$ :

$$
\begin{equation*}
\epsilon_{\mathbf{p}}-\mu=\frac{p^{2}}{2 m}+V+2 n_{\mathrm{ex}} U_{0}-\mu \tag{2.19}
\end{equation*}
$$

We can then again insert this in our Bose distribution and we now find a different integral with as result:

$$
\begin{equation*}
n_{\mathrm{ex}}=\frac{\mathrm{Li}_{3 / 2}(z)}{\Lambda_{\mathrm{deB}}^{3}} \tag{2.20}
\end{equation*}
$$

which seems to have the same form as 2.10 but now $z=\exp \left(-\beta\left(V+2 n_{\mathrm{ex}} U_{0}-\mu\right)\right)$. This means that as soon as we no longer find a solution for $n_{0}$ from (2.11) we must switch to another expression for $n_{\mathrm{ex}}$. In the high temperature limit or in the large $V$ limit this returns to approximate the classical Maxwell-Boltzmann distribution given by:

$$
\begin{equation*}
n_{\mathrm{ex}} \propto \exp (-\beta(V-\mu)) \tag{2.21}
\end{equation*}
$$

Finally we need an expression for the entropy. The total entropy is given by:

$$
\begin{equation*}
S=\int\left(\frac{5}{2} \frac{\operatorname{Li}_{5 / 2}(z)}{\operatorname{Li}_{3 / 2}(z)} n_{\mathrm{ex}} \mathrm{k}+\frac{n_{\mathrm{ex}} n_{0} U_{0}}{T}\right) \tag{2.22}
\end{equation*}
$$

We now have expressions to describe the system in and outside the condensate, as long as we know the temperature, chemical potential and a trap shape. These expressions will be used in chapter 3 to describe our experiment.

## Chapter 3

## Simulations

In this chapter a variety of simulations with the trap shape described in section 1.2 so called dimple traps are presented. We start off in section 3.1 with simply simulating the experiment of switching on the dimple in Figure 1.1 Here we confirm the thermal bath hypothesis from section 1.2 Next we take everything one step further to look if we can use this thermal bath to make an effective cooling step ( in section 3.2).

### 3.1 Hartree-Fock Simulations

In Chapter 2 equation 2.11) cannot be solved analytically. If we subtract $n_{0} U_{0}$ on both sides we see we must find the solution of equation 3.1 for $n_{0}$.

$$
\begin{equation*}
\mu-V(\mathbf{r})-U_{0} n_{0}(\mathbf{r})-U_{0} \frac{\operatorname{Li}_{3 / 2}\left(z\left(n_{0}(\mathbf{r})\right)\right)}{\Lambda_{\mathrm{deB}}^{3}}=0 \tag{3.1}
\end{equation*}
$$

Solving this yields $n_{0}$ as a function of potential and chemical potential and thus our density distribution as a function of position, since the potential is dependent of position. In Figure 3.1 the left-hand side of equation (3.1) is plotted for three different values of $(\mu-V)$, basically $(\mu-V)$ moves the graph up and downward. The root of this graph is located at the solution of (3.1). In appendix $B$ this is discussed further. In the simulation we can now build a function that solves (3.1) when one enters temperature, chemical potential and the value of the potential in it. It will then result in a value for the condensate density at that potential temperature and chemical potential value. We then use an iterative method to solve 3.1, described in appendix B We can now enter the result of the function in 2.10 and that way we directly obtain the thermal density $n_{\text {ex }}$. In the regions where we said the condensate is absent and $n_{0}$ equal to 0 , we will now switch over to a different model, the one described in section 2.2 Once again we can write a function that gives $n_{\text {ex }}$ as a function of the local potential, temperature and chemical potential, again by using an iterative method. This time we solve 2.20 iteratively. We can now combine these functions to get a function that gives the total density. If we scan in space over a given potential we can now get a density picture as a function of position. In Figure 3.2 this is given for a harmonic trap. Note the use of the $\mathrm{Li}_{3 / 2}$ function in the formulas. A closer look on how this is calculated is given in Appendix B So now we are able to calculate the density if we know the chemical potential. But in general we don't know the chemical potential. So what we now do is integrate the density function over space to obtain the number of particles. A closer look on this integration is also given in Appendix B To save some simulation time cylindrical
(a)


Figure 3.1: Plot of the left-hand side of (3.1) at different $(\mu-V)$. The root of this plot corresponds to $n_{0}$. We see that increasing and decreasing $(\mu-V)$ moves the graph up and downwards, resulting in a one solution, two solution and no solution regime. The function is not defined for $n_{0}<0$ and has a finite value at $n_{0}=0$ marked by the dot.
symmetry is assumed, so we can work in a two-dimensional space. Once integrated we obtain the number of particles as a function of chemical potential, temperature and potential shape (see Figure 3.3). Because in our simulation we would like to see the effects that changing the trap shape has on the chemical potential, we now assume that our number of particles remains constant. So we now have a function in which we have the chemical potential as an input which gives the number of particles as an output, but we would like it the other way around. This can be solved using a binary search method. In appendix B we have successfully inverted our function, and we now have a function that gives $\mu$ as a function of temperature, trap shape, and particle number. In Figure 3.4 a simulation that gives $\mu$ as a function of temperature is shown. The trap shape is something we control as an external input, the temperature however is not. Since our function requires temperature as an input we need to calculate the temperature. For this we need another assumption. We assume that, if we change our trap slowly enough that the process is isentropic, meaning that we assume no change in entropy ${ }^{1}$ By using and integrating equation 2.22 over space, we obtain the total entropy as a function of temperature and density distributions. We already have a function that gives the density distributions as a function of

[^3]

Figure 3.2: Density image in a harmonic trap in an Hartree-Fock simulation, relatively close to the condensation temperature $T_{c}$. The green line depicts the total density, the red the density of condensate and the blue the density of thermal cloud. The light-blue line is where the second solution (the purple line) starts to appear and the yellow line depicts the point where $\mu=V$. As one goes to lower temperatures the light-blue and the yellow line move closer together.


Figure 3.3: Number of particles as a function of the chemical potential at set temperature and trap frequency parameters. It can be seen that lower chemical potential means a lower number of particles


Figure 3.4: Chemical potential as a function of temperature in a harmonic trap. We see that the Hartree-Fock model fails to give an accurate description at low temperatures. It can also be seen that $\mu$ drops through zero at a certain temperature, this temperature actually matches the $\mathrm{k} T_{c}=0.94 \hbar \bar{\omega} N^{1 / 3}$ formula in [8, eq 2.20].
temperature, trap shape and chemical potential. And combining this with our function that gives the chemical potential as a function of temperature, trap shape and particle number, we now obtain a function that gives the entropy as a function of number of particles temperature and trap shape. Now we of course know the number of particles and the trap shape, so temperature is the only unknown variable. The only thing we do give as an input to the simulation, is the starting temperature, before we start changing the trap shape. Using this we can also calculate the starting entropy. Since we assume entropy to be constant throughout the simulation we now know the value of entropy. Using a similar binary search method as used before with chemical potential and number of particles, we can now determine the temperature, as a function of trap shape, number of particles, and the starting entropy. Please note that in this binary search, in every step a new temperature is tried, so in every step the binary search for the chemical potential must be done, and in the binary search for the chemical potential in every step the integration must be done, and in every step of the integration the iteration must be done. So this means our simulation time is growing exponentially with the more different types of parameters we want to obtain.

### 3.1.1 Running the simulation

### 3.1.1.1 First results: a crossing

So let us now assume a potential shaped according to 3.2 :

$$
\begin{equation*}
V(\rho, z)=V_{\text {harm }}(\rho, z)+V_{\mathrm{dip}}(\rho, z), \tag{3.2}
\end{equation*}
$$

with:

$$
\begin{align*}
& V_{\text {harm }}(\rho, z)=\frac{1}{2} m\left(\omega_{\rho}^{2} \rho^{2}+\omega_{z}^{2} z^{2}\right)  \tag{3.3}\\
& V_{\mathrm{dip}}(\rho, z)=-\frac{\mathrm{vf}_{V_{0}} P e^{-\frac{2 \rho^{2}}{w_{0}^{2}\left(\frac{z^{2}}{\mathrm{zR}}+1\right)}}}{w_{0}^{2}\left(\frac{z^{2}}{z \mathrm{R}^{2}}+1\right)} . \tag{3.4}
\end{align*}
$$

Here the first part is a regular harmonic potential with magnetic trap frequencies $\omega_{\rho}$ and $\omega_{z}$ in respectively the $\rho$ - and $z$ - direction of a cylindrical coordinate system. The right-hand part comes from the optical dipole trap in section 4.2 which we call our dimple, with depth $V_{0}=\frac{\mathrm{vf}_{V_{0}} P}{w_{0}^{2}}$ where $w_{0}$ is the width (in chapter 5 the focus) of the dimple in the $\rho$ and $z_{R}$ in the $z$ direction (in chapter 5 the Rayleigh length). $P$ is a power of the dimple in watts and $\mathrm{vf}_{V_{0}}=4.5566610^{-37} \mathrm{sm}^{2}$ is a constant for Sodium and a wavelength of 1070 nm as is shown in section 4.2 This constant comes from the detuning of this 1070 nm from the resonance frequency of sodium. This potential is of course not arbitrary, in chapter 5 it can be seen that this is also the one which we use in the experiment. Clearly putting $P$ to zero gives the regular harmonic potential. So let's see what happens when we start to increase the power. The results of such a simulation are shown in Figure 3.5 As can be see there is a point where $\mu$ becomes equal to $\mathrm{k} T$ so this is the point


Figure 3.5: Image depicting the effects on chemical potential and temperature when increasing the dimple depth. Indeed a crossing is observed.
where $\chi$ is one. So the simulation shows that we can quite easily get $\chi=\frac{\mu}{\mathrm{k} T}$ equal to one in this
method. The results in the example in Figure 3.5 where obtained with magnetic trap frequencies of $\omega_{\rho}=2 \pi 100 \mathrm{~Hz}$ and $\omega_{z}=2 \pi 1.5 \mathrm{~Hz}, 200$ million particles, a width of $w_{0}=20 \mu \mathrm{~m}$ and a starting temperature of 400 nK .

### 3.1.1.2 Simulating at the crossing

As said in section 1.2 the regime we want to measure in is the regime where $\mu$ is equal to $\mathrm{k} T, \chi$ is one. So let us determine this crossing point, where $\mu$ becomes equal to $\mathrm{k} T$, as a function of starting temperature. We can again determine this point using the binary search method. This way we obtain a power at which $\chi$ becomes one, as a function of temperature. Now it becomes clear that $\chi$ equal to one cannot be achieved under all circumstances, for example, if the width of our dimple much larger than the size of our cloud, it can be approximated as an harmonic, and we will effectively not even leave the harmonic trap. Also if one starts of at a too high temperature it turns out that it is no longer possible to achieve the crossing, because one needs to go to such a deep dimple, that the entire cloud will be in the dimple, removing the thermal bath concept. An example of that is shown in Figure 3.6. It turns out that there is a maximum


Figure 3.6: Image where a crossing is just missed. This happens if $w_{0}$ gets too large, or $T$ too high.
temperature at which one is still able to find a crossing. Once again this maximum can be found for different dimple widths by using a binary search method. Because this binary search has to do all the previous searching steps for all the other parameters, in every searching step of this search, this is quite time intensive. That is why we choose to give up some accuracy to reduce


Figure 3.7: Image where the simulation was done under circumstances where a crossing is barely achieved, and continued afterwards. We see that at a certain dimple depth there is a second crossing, depicting a flat line for the chemical potential afterwards. A second crossing is expected, because at a certain point the dimple is so deep, that the particles are all in there, making it possible to approximate it with a harmonic potential again.
simulation time, which will be illustrated by the error bars in Figure 3.9

### 3.1.1.3 Results

So once we know this crossing point we can calculate the other parameters at this crossing point. This is done in appendix C. 1 in Figure C. 1 for $w_{0}=10 \mu \mathrm{~m}$ and Figure for $w_{0}=20 \mu \mathrm{~m}$ width of our dimple. A selection of the results at for $w_{0}=20 \mu \mathrm{~m}$ width of our dimple is shown in Figure 3.8. All graphs are as a function of the temperature that we started with. One must note the appearance of a $\mu$ with respect to the zero point of the harmonic trap in one of the graphs. The relevant $\mu$ is of course that one with respect to the potential minimum, which is in the dimple. But in the simulation we always work with a $\mu$ with respect to the defined zero point in our simulation, which is the zero point of the harmonic trap without the dimple. The $\mu$ with respect to the potential minimum can easily be obtained by adding the trap depth. Because at the crossing this is by definition equal to the temperature they are both shown in the same graph. What we also see in the results is that the conceptual image of the process gets confirmed, because if we look at the spatial position of the thermal particles, we see that only a small fraction of them is spatially in the dimple while of the condensed particles a big fraction is, confirming our view of


Figure 3.8: Results at the crossing between $\mu$ and $\mathrm{k} T$. The magnetic trap frequencies are $\omega_{\rho}=2 \pi 100 \mathrm{~Hz}$ and $\omega_{z}=2 \pi 1.5 \mathrm{~Hz}$, we have 300 million particles, and the width is $w 0=20 \mu \mathrm{~m}$. In (a) the power of the crossing is shown, in (b) (c) the thermal bath hypothesis is confirmed, and in (d) the dominance of the condensate is illustrated. All graphs are as a function of the temperature that we started the simulation with.
a thermal bath. The density in the centre is plotted to see if three-particle losses are important in the time scales in our experiment, because this simulation does not include a description of this process.

The results of varying the dimple width are shown in Figure 3.9 . One can see that increasing the dimple width decreases the temperature at which a crossing can be achieved, as one would expect, because if the dimple becomes wide enough it starts to approximate a harmonic trap. Now if we use experimental data from previous experiments to obtain the temperature at which it is possible to start this experiment, we can use this figure to make an estimate of the width we need for our dimple to achieve the crossing between $\mu$ and $\mathrm{k} T$.


Figure 3.9: Maximal temperature at which there is still a crossing as a function of the dimple width $w_{0}$. Since we always start with condensate, the condensation temperature is shown if there is a crossing for all temperatures below the condensation temperature.

### 3.1.2 Discussion and Conclusion

### 3.1.2.1 Discussion

In this type of simulation time and accuracy are inseparable. If one wants a more accurate result, one needs more time. Although in the first simulations the simulation uncertainty is still negligible compared to the value, in Figure 3.9 we see that it becomes significant, simply to keep simulation time reasonable. This is all a consequence that this simulation only uses one dimensional searches. This means that for every trial value for a parameter B , the equations must be solved completely for parameter A, even if that parameter B is still far off. Simulation time could be greatly reduced, or accuracy greatly increased if one would make the requested accuracy in a parameter A dependent of how close B is to the actual result or using a method in which A and B are varied at the same time.

### 3.1.2.2 Conclusion

The simulation confirms our hypothesis. It is possible to achieve a crossing between $\mu$ and $\mathrm{k} T$ using this technique, and we see that this effect coincides with the appearance of a thermal bath. We also see that the experiment only works in certain parameter ranges, and we can use the obtained simulation data to choose the correct parameters in the actual experiment described in section 5.4.2

### 3.2 Positional Separation

In Figures C.1 and C. 2 it is shown that there is a strong positional separation between the thermal bath and the condensate. This gives rise to a new idea, is it possible to use this separation to get rid of the high energy particles? This process, known as spilling, can of course also be done in a regular harmonic trap by "ending" the potential at a certain height. Actually a similar but even more efficient process, called evaporative cooling, in which two particles interact and create a high, and a low energy particle, with the high energy particle being kicked out of the trap is used in most experiments, and is actually also used in our set-up (see section 4.1) to create the starting conditions for this particular experiment.

But, as said, in a regular harmonic trap one does not have the spatial separation between the high energy thermal particles, and the low energy condensate particles. The hypothesis researched in this section is that this spatial separation can be used to make this spilling much more efficient. The first idea is illustrated in Figure 3.10 and considers introducing a barrier after the previous


Figure 3.10: Concept where we use the dimple method to make a positional separation between the thermal cloud and the condensate and then cut off the high energy particles.
experiment. Since the condensate is in the dimple, the idea is that it does not experience anything of this abrupt change in trap shape, while the thermal particles, spatially mostly outside the barrier (as illustrated by Figure C.1) are immediately cut of from the condensate. The result would be having an extreme occupancy of the ground state, or better said, being at an extremely low $T$. One would then be able to turn off the harmonic trap, giving the following potential:

$$
\begin{equation*}
V(\rho, z)=-\frac{\mathrm{vf}_{V_{0}} P e^{\left.-\frac{2 \rho^{2}}{w_{0}^{2}\left(\frac{z^{2}}{\mathrm{zR}}+1\right.}\right)}}{w_{0}^{2}\left(\frac{z^{2}}{\mathrm{zR}^{2}}+1\right)} \tag{3.5}
\end{equation*}
$$

Now let gravity remove the high energy particles, turn the harmonic trap on again and then adiabatically return to a normal harmonic trap, yielding an enormous cooling step. The problem with this approach is the barrier, which as it turns out is difficult to implement in our set-up. We now consider a different simplified approach of the same concept. Basically this concerns doing all the same steps, but without the barrier. The idea is that the condensate is being kept trapped in the dimple and the high energy thermal particles fall out because of gravity.

It was later found out experimentally that it is not possible to keep a dimple trap energetically filled that much ${ }^{2}$ The following subsections however neglect the evaporative cooling effect, and

[^4]only take into account the spilling effects, so in the following sections keeping a trap filled (see footnote!) is not a problem. Please keep this in mind while reading these sections.

Switching off the harmonic trap can be done in two ways, continuous or instantaneous. In the continuous approach the magnetic trap frequency is gradually decreased, making the trap wider until the magnetic trap frequency is zero which is equivalent to it being turned off. This immediately results in a major challenge, since our simulations did not include gravity or the walls of a chamber around it (see chapter 4). In the experiment gravity is in a radial direction, which is challenging since our simulation was 2D in a cylindrical coordinate system, and gravity would require a third coordinate. This is further discussed in section 3.2.1 In the instantaneous approach the trap is turned off instantaneously. This is also challenging since when a change is made instantaneously the adiabatic assumption is no longer valid, meaning that one ends up with a dynamic process. An approach with the formulas from chapter 2 is described in section 3.2.2 An approach using the occupation of the different energy states is described in section 3.2.3.

Gravity was just presented as a challenge in the first approach but as it turns out is a challenge in both approaches. The question is of course why one needs one, because a much simpler approach is apparent. One could simply say that the dimple is overfilled, that is (see footnote) filled to a higher energy than the depth of the dimple, when you start, and all the particles with an energy higher than the depth of the dimple are lost. This would however imply that all the thermal particles are lost, because they all have an energy higher than the depth of the dimple. Further, already in a simple Thomas-Fermi approximation 2.3 of the potential in 3.2 a completely filled dimple is impossible, because the potential integrated over space does not converge and results in a infinite number of particles, basically meaning that everything fits in and nothing falls out. This is exactly why gravity and walls are a problem, in a completely Gaussian potential for example one would be able to say that the result of the method is just a completely filled trap. Since in this potential that means infinite number of particles this is not possible. This is why we turn to the explanations in the following sections.

### 3.2.1 Continuous Approach

In this approach we decrease the magnetic trap frequency adiabatically, so keeping the entropy constant. This requires no adaptations to the simulation described in section 3.1 at all. However that way we wouldn't be accounting for the loss of particles, meaning that this way particles will remain in the trap, even if the potential remains flat (why would they leave?). So we need to include a source of loosing particles. To avoid dealing with the challenge of gravity we use a more simple approach. We introduce a wall. We simply define that at a certain distance from the centre, the particles hit a wall and are lost. We then assume that they also take their entropy with them. We use the following three step model; in the first step a small adjustment to the magnetic trap frequency is made while keeping the entropy constant. In the second step it is calculated how many particles and how much entropy is outside the selected zone, by integrating over the regions outside the wall. Finally we adjust the entropy and particle number with these losses and we repeat the procedure.

### 3.2.1.1 Results

The results of such a simulation are shown in Figure 3.11 and Figure 3.12 . A strange bump is appearing in the temperature, around 0.001 Hz . Basically it seems that decreasing the magnetic trap frequency means a small increase in temperature, at that point. So can decompression


Figure 3.11: Result of a simulation where the magnetic trap frequency is slowly decreased, and particles that hit a wall are removed from the simulation. This image shows the number of particles. The cut-out shows the last part.


Figure 3.12: Result of a simulation where the magnetic trap frequency is slowly decreased. This image shows the effect on the chemical potential (blue) and the temperature (red) The cut-out shows the last part. The strange bump appears at the moment the chemical potential falls below the dimple depth. This effect is independent from the fact if we do or do not remove particles.
mean a temperature increase? It seems so. This effect seems to be there regardless if we do or do not include the wall to account for particle and entropy losses (as was confirmed by another simulation). But if one looks at the chemical potential we can see what is happening. The chemical potential is exactly at this point falling below the dimple depth. This means that the width of the condensate is drastically reduced. So it's switching over from the regime where the very wide harmonic trap still influences the condensate to the regime where the much tighter dimple influences the condensate, effectively meaning that the trap is made tighter, resulting in a increase in temperature. This can also be seen in 2.9 and 2.20 where the $z$ makes a change in formula when the condensate leaves the flat parts of the potential. This happens in a very big area (the entire flat part of the potential), going from almost one regardless of the temperature in (2.9) to having a positive $(V-\mu)$ in the exponent and a positive $n_{\mathrm{ex}}$ in 2.20 . This means that to keep $z$ in the same order $\beta$ has got to decrease so $T$ has got to increase. One wants to keep $z$ the same because the entropy $(2.22)$ must stay the same. In other words since the chemical potential becomes so low, the condensate "falls" into the dimple, meaning that decompression is experienced as a compression by the condensate, since the dimple is much tighter. Further we see both the temperature and the chemical potential vanish in the limit of zero magnetic trap frequency. This is probably an effect of trying to keep thermal equilibrium with the flat parts of this potential. That is, of course, physically unrealistic because the amount of interactions available to keep this equilibrium is getting lower and lower.

### 3.2.1.2 Discussion and Conclusion

This method keeps assuming thermal equilibrium with a region where the density is negligible, which is physically unrealistic. The description by walls is not accurate, because a particle is lost by gravity before it hits a wall, and placing the wall closer to the dimple to simulate gravity doesn't solve this because it's all about the transition from parts that weren't flat to parts that become flat. One would have to bring the wall inward during the simulation, as an unknown function of magnetic trap frequency.

The results of this simulation are not realistic for a real world experiment, because it uses a thermal equilibrium, while the whole concept involves breaking that equilibrium. It shows a chemical potential that goes to zero, and a temperature that does the same, while of course if the chemical potential goes to zero we should return to a harmonic approximation of the dimple. It did however provide us with the insight that in some strange potentials decompression can lead to an effective compression and temperature increase.

### 3.2.2 Deep dimple approach

Next we consider an approach in which we don't look at the crossing but at a much deeper dimple. Actually we continue until the maximal condensate density times $U_{0}$ becomes equal to the dimple depth. Now we assume that all the particles in the condensate are conserved. One can use 2.8 to calculate what part of the energy of the thermal particles is below the dimple depth and the Bose-Distribution from (2.5) to calculate how many excited particles are below the dimple depth. This can then be put back into our original harmonic trap. The results of this are shown in Figure 3.13

### 3.2.2.1 Discussion and Conclusion

The reason for trying this method is because it can be simulated much better then the situation at the crossing, even though we are actually interested in the crossing. The fact that the entire


Figure 3.13: Image depicting the results of the deep dimple approach
condensate is energetically in the dimple means we only have to consider thermal particles, which means that we have to make much less approximations and assumptions. However, we see that the effect isn't that large, and this can be explained because the second term in 2.22 starts to play an important role at this conditions.

### 3.2.3 Energy levels and degrees of occupation

This section considers an instantaneous turn-off of the harmonic trap. Describing the actual behaviour would require to look at the dynamic response of such an abrupt change of trap shape. To avoid this difficult approach, we use a different method. We say that particles that have an energy below the dimple depth are kept trapped, and particles with an energy above that are not. Just like an entropy, a trap shape and a number of particles defines our system (see section 3.1) an energy, trap shape and number of particles defines our system too. So if we can find the new total energy after turning off the harmonic part of the trap, we have our new system. So the question is how to determine this energy. Of course one would like to use the energy given in formula (2.16) . So a first consideration is integrating the Bose-distribution for the energy to a certain maximum energy, in this case the dimple depth. Problem is that the thermal particles have also the chemical potential as energy, meaning that if the chemical potential is higher than the dimple depth, all particles have a too high energy. So the question comes to mind if that would that mean that all particles are lost. The answer to that question is negative, because as soon as we start to loose particles the chemical potential gets lower, and that way the particles do fit in the dimple. The concept discussed in this section is that we go back to the quantum mechanics and look at the energy levels of this potential. We can then look at the degree of occupation of the different energy levels and then calculate the energy and number of particles below the dimple depth, and thus determine the resulting energy again. Section 3.2.3.1 and 3.2.3.2 discusses the energy levels, section 3.2.3.3 looks at the degrees of occupation.

### 3.2.3.1 The Schrödinger equation

To find the energy levels we are required to solve the Schrödinger equation:

$$
\begin{equation*}
H \phi(\mathbf{r})=E \phi(\mathbf{r})=\left[\frac{-\hbar^{2}}{2 m} \nabla^{2}+V(\mathbf{r}, t)\right] \phi(\mathbf{r}) . \tag{3.6}
\end{equation*}
$$

For a harmonic trap this immediately results in a discrete energy spectrum:

$$
\begin{equation*}
\epsilon\left(n_{x}, n_{y}, n_{z}\right)=\left(n_{x}+\frac{1}{2}\right) \hbar \omega_{\rho}+\left(n_{y}+\frac{1}{2}\right) \hbar \omega_{\rho}+\left(n_{z}+\frac{1}{2}\right) \hbar \omega_{z} . \tag{3.7}
\end{equation*}
$$

However for this non-harmonic potential things are more complicated. It turns out that for this complex potential the Schrödinger equation cannot be solved analytically. So we have to turn to numerical methods. Solving the equation in three dimensions turns out to be impossible with the computing power available. So we make the assumption about the wave function that it can be uncoupled:

$$
\begin{equation*}
\phi(x, y, z)=\phi_{x}(x, 0,0) \phi_{y}(0, y, 0) \phi_{z}(0,0, z) \tag{3.8}
\end{equation*}
$$

In the harmonic case this is valid, because there (3.6) splits into three decoupled equations. However in our complex potential this does not happen. It is an approximation that we still
make however, because of the calculation time. Now we can solve the one-dimensional case and we obtain the following one-dimensional equations:

$$
\begin{align*}
& E \phi(\rho, 0)=\frac{-\hbar^{2}}{2 m} \phi^{\prime \prime}(\rho, 0)+V(\mathbf{r}, t) \phi(\rho, 0)  \tag{3.9}\\
& E \phi(0, z)=\frac{-\hbar^{2}}{2 m} \phi^{\prime \prime}(0, z)+V(\mathbf{r}, t) \phi(0, z) \tag{3.10}
\end{align*}
$$

where we have realised that in this potential the equations in the $x$ - and $y$-directions become the same as the one stated for $\rho$ in (3.9). So we need to solve this equation to find the energy levels. Solving the equation is not the main challenge, numerical solving a differential equation is a well known concept. However we want to find a discrete energy spectrum. So the reader might starts to wonder why the energy spectrum is discrete, since the equation can be solved for every energy? That is a crucial point indeed, the Schrödinger equation can be solved for every energy and yields an solution for every energy. The discrete spectrum actually arises because of the extra condition:

$$
\begin{equation*}
\int d \mathbf{r}\left|\phi\left(\mathbf{r}_{i}\right)\right|^{2}=1 \tag{3.11}
\end{equation*}
$$

which means that it is required that the found solution can be normalized. But for nearly all energies the solution of the Schrödinger equation diverges. Only for a discrete spectrum of energies it doesn't. So the question is how to find these energies, because the solution diverges for every energy except for exactly the right one, and a numerical energy is never exact, so always diverges. We need to know if a trial energy is close to a correct energy or not, because it always diverges for a numerical energy. The trick is to implement something we know about a correct solution, that it does not diverges. So a correct solution is zero at large distances, both on the right-side and the left-side. We start off by saying that our solution is zero on the right-hand and the left-hand, and now integrate back from the right to the left, and from the left to the right. Now only for the correct energy our solutions will meet at exactly the same point. But if our trial energy is off we have a mismatch in the solution coming from the right and the one coming from the left, by definition, because if they would meet at the same point, it would be a solution that does not diverge. So we have a mismatch at the meeting point. The trick is to now minimize the mismatch, using the binary search algorithm described in section 3.1. That way the discrete energy value can be obtained. All right so now we know how to find one. But we want to find all of them. Let us first notice that it turns out that there is exactly one solution that never passes through zero, one that does it once, one that does it twice and so on. So if we find the number of zero-passes we know at what energy level we are. We can then again use a binary search algorithm that yields success if a predefined number of zero-passes is achieved, to find the energy range. Next we use the binary search algorithm described above to fine-tune the energy.

In our program we choose to use a slightly adapted version of the method described above and solve the logarithmic derivative of the wave function instead of the wave function itself. In this method the zero-passes turn into poles which are remarkably easy to count because after a pole the solution changes sign at a high value, making it impossible to count an extra pole because of a numerical variation. The method we used is described excellently in [12] and therefore the reader is kindly requested to read that article if he or she wants to know more about it. In Figure 3.14 this method is illustrated.


Figure 3.14: Illustration of the numerical solving of the logarithmic derivative of the wave function. Solving goes from both the right-hand and the left-hand, and the energy gap is what needs to be minimized. The number counting of the number of poles makes sure that we won't miss a solution.

### 3.2.3.2 Energy Levels: Results

The energy levels for our potential are depicted in Figure 3.15 and in Appendix C. 2 in Figures C. 3 to C. 9 We see in Figure C. 4 that it starts off like an harmonic that approximates the dimple for low energy, and it ends up like the harmonic that is still in the potential. In these images the one-dimensional case is considered, in both the $z$-direction and the $x$-direction which is equivalent to the $y$ and $\rho$ direction in formulas when $z$ is zero. By comparing Figure C. 6 to Figure 3.15 it can be seen that the number of states in the $z$ direction is much larger than in $x$ which isn't surprising because the trap is much wider in that direction Now to find the real states, we must look at all the combinations of states in the $x, y$ and $z$-direction. This results in an even bigger number of states, because every combination of three values is a lot more. Then all these combination states must be sorted according to their energy using a sort algorithm to finally obtain the energy levels with their order. To give an impression of the amount of data this gives, for just one dimple depth the energy levels up to about 3 times the dimple depth gives a datafile containing 60 megabyte of data. And we want to scan over an entire spectrum of dimple depths. If one looks at which level one is in height, or to the sorted list just discussed this level number is directly the cumulative density of states. An example of this data is plotted in Figure 3.16 for a certain dimple depth.


Figure 3.15: Energy levels for a radial direction as a function of dimple depth. The energy axes is put to zero at the original zero point of the original harmonic. The dimple width is $w_{0}=10 \mu \mathrm{~m}$.


Figure 3.16: Cumulative density of states as a function of energy. The energy axis is put to zero at the original zero point of the original harmonic. The dimple width is $w_{0}=10 \mu \mathrm{~m}$.

### 3.2.3.3 Degrees of occupation

Now we know what the states are, we need to know what their occupation is, to determine the energy that is in the dimple. For this we realise that the occupation of a certain state is given by the following formula [9, eq. 2.34]:

$$
\begin{equation*}
c=\int d \mathbf{r} \Psi\left(\mathbf{r}_{i}\right) \phi\left(\mathbf{r}_{i}\right) \tag{3.12}
\end{equation*}
$$

since we made the assumption of 3.8 this simplifies to:

$$
\begin{equation*}
c=c_{\rho}^{2} c_{z} \text { with } c_{z}=\int d z \Psi(0, z) \phi(0, z) \text { and } c_{\rho}=\int d z \Psi(\rho, 0) \phi(\rho, 0) \tag{3.13}
\end{equation*}
$$

where since the second equation is just a mere functional form of the one with $x$ and $z$ the second integral must be evaluated Cartesian and not cylindrical. Since we have a method (described in section 3.2.3.1 to find the wave function of a single state, and we know the wave function of the total cloud, namely the square-root of the density distribution in section 3.1 we can now also calculate the integrals above. In this way, we are able to obtain a one-dimensional degree of occupation, for each direction, which we can then multiply to obtain the occupation for each state in the entire space. We now normalize using:

$$
\begin{equation*}
1=\sum_{n} c_{n}^{2}[9, \text { eq. } 2.38] . \tag{3.14}
\end{equation*}
$$

We now see that because in one dimension all the odd states have an odd number of poles, meaning that those functions are odd, and multiplied with the even function for the total number of particles this yields an occupation of zero for half of the states. If we now look to the three-dimensional case this means that only one in eight states seems to be occupied. Now if one looks at the occupation as a function of energy, we obtain for a certain dimple depth the figure presented in 3.17 . For tidiness all the states that have an occupation of zero are left out. This figure is made at the crossing of section 3.1.1.3, so where $\mu=\mathrm{k} T$. One can clearly see that there are more states in the $x$ and $y$-direction combined than in the $z$-direction. We see by looking at the shape of the graph that a high $x$ and $y$ state combined with a low $z$ state might give the same energy as the next $z$ state, but is way less likely than the next $z$ state with a low $x$ and $y$ state.

Now the goal was to calculate the energy of the particles that are in the dimple, so the energy that is kept trapped when turning off the harmonic. To do this one can simply find out the number of particles in an energy state by multiplying the occupational degree with the number of particles. To then find out the number of conserved particles, one could simply add up all the numbers of particles in all states below the dimple depth. For the energy we have to multiply the number of particles in a state with the energy of that state and then sum them over the conserved states. But let us first check if the total energy is the same compared what we obtain by using formula 2.16 from section 2.2 . To obtain the total energy we must sum over all the states. Comparing both methods yields the picture in Figure 3.18.

Clearly they do not match. If the energies do not match, the whole method of recalculating the state of the system according to the methods of section 3.2.3.1 with the new energy and new number of particles would be useless, because the energies already do not match in a case that should yield the same value.


Figure 3.17: Occupational degrees as a function of Energy at a crossing in $\mu$ and $\mathrm{k} T$.

### 3.2.3.4 Discussion

The big question that appears now is why they do not match. An answer will now be presented. In section 2.1 formula 2.1 it is suggested, and then discussed why that formula isn't valid, namely because it doesn't take care of particle interactions, which are absent in the single-particle Hamiltonian. Combining single-particle wave functions with single-particle energies would yield the wrong many-particle wave function. It might seems like we did take into account the particle interactions in this section, because the many-particle wave function used in this section is the one with interactions. But we in fact did not: we expand our many-particle wave function with interactions on a basis that just consists of our single-particle solutions of the Schrödingerequation of the potential. We then use the energies associated with those single-particle solutions, resulting from a Hamiltonian that did not include interactions, because for a single particle there are no interactions to find our total energy. But that would mean we could also do the opposite, build up the many-particle wave function from the basis of the single-particle wave functions, with energies that are the solution of (3.6). But if we would do it that way we wouldn't be taking into account the interactions, because otherwise we would just be able to use (2.1) from section 2.1. We can't however because it doesn't take care of correlations caused by particle interactions. So one can't build up the many-particle wave function out of single-particle wave functions with the single-particle energies. But if one can't build up, one can't expand, at least, one can always do this mathematically, but since we use the single-particle Hamiltonian the energies will not match. And if we are not allowed to expand our many-particle wave function on the singleparticle wave functions from the single-particle Schrödinger-equation with the energies given by the single-particle Schrödinger-equation, we no longer have to deal with the issue of only one in eight states that would be occupied. The wrong value for the energy is now also easily explained.


Figure 3.18: Total energy as a function of dimple depth. The red points depict the line determined using $\sqrt{2.16}$, the blue points the ones determined with the energy level approach. These simulations were done at magnetic trap frequencies of $\omega_{\rho}=2 \pi 100 \mathrm{~Hz}$ and $\omega_{z}=2 \pi 1.5 \mathrm{~Hz}, 300$ million particles, and a dimple width of $w 0=10 \mu \mathrm{~m}$

Furthermore, this method is not valid because it does not take into account evaporative cooling as an extra effect next to spilling. Finally this method assumes the system can be decoupled into the one-dimensional equations which it can't for this potential, but the reasoning above could also be used were the potential completely harmonic.

### 3.2.3.5 Conclusion

The method used in this section is only valid in the non-interacting case, because it assumes that the many-particle wave function is built up out of single-particle wave functions, without taking into account the changes in energy because of interactions. Therefore expanding the manyparticle wave function on the single-particle wave functions to find the energies is not correct and therefore the entire method is not valid. It did however provide us with some insight of how single-particle wave functions result in energy levels in our system, but only that and not more then that. So we have learned some things, but we can't use the results of this section will not be used as experimental input.

## Chapter 4

## Set-up

The set-up of our experiment consist of two parts: an excising set-up that is used for many other experiments that provides us with the starting situation of the experiment discussed in this report. It delivers a cold atomic cloud that is already an Bose-Einstein condensate. Next there is the specific set-up (section 4.2 and 4.3 ) that is used for the experiment described in this report. The part of the set-up that was already present before this research is described in a short summary in section 4.1 because the set-up in our group is discussed in so many reports like the excellent description given by Robert Meppelink in his Ph.D. thesis, which can be found in the references of this report [13].

### 4.1 Present set-up

In this section the set-up to lower the temperature to a condensate is discussed briefly. This set-up provides the starting conditions for the experiments described in this report. The set-up starts with loading particles in a vacuum chamber, in our case sodium atoms. The process of doing this is described quite thoroughly in the particle flux report about the small project done in collaboration with Martijn van 't Woud BSc, that is included in this text as Appendix G. So for the loading of the trap the reader is referred to Appendix $G$. Now once the particles are loaded into the chamber (look in Appendix G to see how they are loaded) where the experiment is performed, they are trapped in a magneto-optical trap (MOT). This trap has the great advantage that it both cools and traps the atoms. This trap works by shining lasers from six directions on the cloud, which are slightly detuned from the resonance frequency of sodium. Now if a particle moves into the direction of the laser, it will acquire a Doppler shift and because of this Doppler shift it will will experience a frequency closer to resonance. When it is in resonance, it absorbs a photon, thus acquiring momentum in the opposite direction. It will then emit a photon in a random direction, meaning that it acquires again a net momentum in a random direction, but if we repeat this process many times, then the net momentum will be in the direction opposite to the laser. By applying lasers in six directions the process eventually slows the atoms in all directions, making them lose speed, and thus temperature. So this cools the atoms, but it does not trap them, so with only lasers the cloud would fall in the direction of gravity. So here is where the coils come in, by applying a gradient in the magnetic field, zero in the centre of the trap, the particles will experience a Zeeman shift when they are not in the centre of the trap, shifting their resonance frequency and making it more likely to get a kick from the laser coming from the direction in which they were displaced, and thus trapping them. With this magneto-optical trap already quite low temperatures can be reached, but still there is a limit in the temperature
that can be reached because particles are still absorbing energy from the lasers (because of the Doppler cooling limit). So now the particles are loaded into a Magnetic trap (MT) (not optical) and which uses the property of particles in a negative spin state (the available spin states are $+1,0$ and -1 ) to have a preference for the locating at the minimum of the magnetic field, because their magnetic moment shifts their energy if the magnetic field changes. By creating a minimum in a magnetic field produced we can make a trap. Since our particles are evenly distributed over the three spin states we would loose two third of our particles, but because of a technique called spin polarization we loose less. For an excellent description of how the coils are configured to produce this magnetic field, the reader is kindly referred to the Master thesis of Martijn van 't Woud BSc ${ }_{-1}^{1}$ The remaining particles are now cooled using the method of evaporative cooling. For this an antenna produces a radio signal, which we call the RF-knife. This kicks out all the particles above a certain energy in the potential. Starting off at a high enough level, no particles are kicked out directly, but sometimes a particle collides with another particle giving it a very high energy and the other particle a very low one, at least for a brief moment. When it reaches the energy the RF-knife is set, it is kicked out, taking away way more energy than the average energy per particle. The RF-knife is then slowly lowered while more and more energy is removed from the cloud, and because only particles that have incidentally a very high energy are taken away from the cloud, the cloud is cooled even further. This way we are able to reach temperatures so low that the cloud will condensate. By setting the end point of our RF-knife we can determine the temperature, although it must be noted that also this method has its limits, it is very hard to get the $\chi$ described in the introduction above one. It is experienced that when you go too low with your RF-knife you start cutting your condensate away, in other words, losing it. We do however achieve a condensate and this means that we have a harmonic trap made by the MT with a condensate and a thermal cloud in it.

### 4.2 The optical dipole trap

So in section 4.1 we had a harmonic trap, but the goal of the experiment is to make a tight dimple into it (Figure 1.1). This is done by using a optical dipole trap, also known as Far Off Resonance Trap (FORT). The trap works by using far of resonance light ( 1070 nm ) so the exciting of particles does not have a significant role, so the light just passes through the atoms. The light does however have an electric field, and this electric field induces a dipole on the atom and start to drive it at that frequency ${ }^{2}$ This is known as the dipole effect. This dipole can then interact with the field, and so the atoms experience a force. Now if one then does the mathematics it turns out that for red-detuned light this results in a potential in which particles have a preference to be at the highest intensity point of the trap. It turns out that the beam's potential shape is identical to the intensity profile. So if we now focus a laser beam on the cloud, we have the highest intensity point in the focus, so the particles will have a preference there. Now the intensity profile of a focused Gaussian beam has the following shape, in a cylindrical coordinate system where the laser beam is pointed along the $z$-direction:

$$
\begin{equation*}
I(\rho, z)=I_{0} \frac{e^{-\frac{2 \rho^{2}}{w_{0}^{2}\left(\frac{z^{2}}{z \mathrm{R}^{2}}+1\right)}}}{\left(\frac{z^{2}}{\left.\mathrm{zR}^{2}+1\right)}\right.} \tag{4.1}
\end{equation*}
$$

[^5]where $w_{0}$ is the focus, and $z_{R}$ the Rayleigh length $3^{3}$ Now as said the resulting potential scales with this so it has the similar shape:
\[

$$
\begin{equation*}
V_{\mathrm{dip}}(\rho, z)=-V_{0} \frac{e^{-\frac{2 \rho^{2}}{\left(\frac{z^{2}}{z \mathrm{R}^{2}+1}\right)}}}{\left(\frac{z^{2}}{\left.\mathrm{zR}^{2}+1\right)}\right.} \tag{4.2}
\end{equation*}
$$

\]

The constant $V_{0}$ can also be derived and it turns out that for sodium atoms it is given by the following formula:

$$
\begin{equation*}
V_{0}=\operatorname{vf}_{V_{0}} \frac{P}{w_{0}^{2}} \tag{4.3}
\end{equation*}
$$

where the constant $\mathrm{vf}_{V_{0}}$ is given by $4.5566610^{-37} \mathrm{sm}^{2}$ for sodium atoms, and $P$ is the power in watts. For a derivation the reader is referred to [15]. In section 5.1 measurements of this profile are presented.

The set-up for achieving this focussed beam is shown in Figure 4.1. By simple Gaussian optics it can be shown that if one shines a collimated beam on a lens, the light is focused and the size of the focus is proportional to the focal length and inversely proportional to the size of the incoming beam.

$$
\begin{equation*}
w_{0}=\frac{2 \lambda f}{\pi S} \text { with } S \text { the size of the incoming beam and } f \text { the focal length. } \tag{4.4}
\end{equation*}
$$

For the purpose of our experiment the tightness dimple is important, because if the dimple is not tight enough we would just return to a magnetic trap (see Figure 3.9). How tight it should be depends on the starting temperature, as explained in section 3.1.1.2 The focal length is limited by the fact that that the lens making the focus should be outside the vacuum chamber that was mentioned in section 4.1 The centre of the magnetic trap and the focus should overlap, so this gives a minimal focal length. This means that the way to get a tight focus is by starting off with a wide beam. The beam size is increased with two telescopes, placed as shown in Figure 4.1. The lenses of the second telescope can be removed, because it is a shared set-up, and the other experiment uses a much wider focus, requiring a smaller beam. The size of the beam with the last telescope is five times as large as the one without it, so that would mean according to Gaussian beam optics that the focus would be 5 times as small as the other beam. In section 5.1 both beams are measured, and it turns out that this is not the case. Since we can't measure the beam size in the vacuum chamber (that would destroy the vacuum), there are two more mirrors, one right after the last lens that can be folded up and down, when it is folded up it interrupts the path into the chamber and shines the light onto the second mirror which then shines it on to a camera. The camera is set at a translation stage which can be accurately shifted backand forwards. We can then image the beam at different distances, by moving the camera. It is important that the beam passes straight through the centre of the condensate, and that when measuring the profile it hits the camera perpendicular to make sure that the beam isn't imaged slanted. The camera is calibrated by moving in the direction perpendicular to the beam and that way it is determined it has a resolution of $5.6 \mu \mathrm{~m} / \mathrm{px}$.

[^6]

Figure 4.1: A part of the set-up. At the right the laser beam leaves the fibre, moves through the two telescopes, and on the top-left it passes through the lens that makes our tight focus.

In the set-up light is seen coming from the right, it actually comes out of a fibre that is located there. The light is coupled into the fibre coming from a infra-red fibre with a wavelength of 1070 nm, but before it goes into the laser it passes through an AOM, which can be used to modulate the intensity of the light.

The light doesn't only pass into the chamber, but it passes through it. At the other end of the chamber the intensity of the light is measured and the measured intensity is then passed back to the AOM, which then modulates an intensity fluctuation in the laser power directly back to a pre-set intensity. This way the power of the beam remains constant. The AOM can be set at a chosen voltage, which corresponds to a certain intensity, of course dependent of the power of the laser itself. To measure this intensity, we have a basic power meter which we can place into the beam to calibrate the AOM. The intensity is measured right after it comes out of the fibre, because there the beam is small and fits on the power meter. Of course the mirrors, lenses and the windows of the vacuum chamber absorb some of that power. To calibrate the losses of the lenses mirrors and windows, we use a relatively large lens with a very short focal length, which we then place in front of our power meter to be able to measure the intensity of the larger beams. By also doing this measurement with the large lens at the point where the beam is still small we can also determine how much of the power this large lens takes away. The measurements of this
are presented in section 5.2

### 4.3 Imaging

To image the resulting cloud we have actually three methods of performing this called top imaging with absorption, side imaging with absorption, and top imaging with phase contrast imaging. Phase contrast imaging is not discussed in this report. For more about this type of imaging see [10, Chapter 3]. The two methods that use absorption imaging are actually the same method from a different direction. In absorption imaging a large laser beam with a wavelength at the resonance frequency of sodium atoms ( 589.16 nm ) is sent through the cloud thereby letting the particles absorb the light. By comparing the resulting pattern with the one in a reference picture without the cloud, we can obtain the amount of light that is absorbed by a column of particles. That way we can determine the density at that point. The difference is that top imaging comes from the top, so if a cloud would fall because of gravity, one would not observe that. The picture coming from this image is magnified three times before it shines onto a camera with a resolution of $8 \mu \mathrm{~m} / \mathrm{px}$, meaning that in the image a distance of 1 px corresponds to $8 / 3 \mu \mathrm{~m}$ in the experiment. In side imaging the cloud is imaged at an angle. So if a cloud would fall it moves in position on the camera. In side imaging the image is first de-magnified before it falls onto the camera. By letting a cloud of atoms fall compared to gravity, this direction is calibrated in section 5.3.1 The angle used in side imaging was determined from an average of three measurements and turned out to be $0.72 \pm 0.02$ radians. An important thing about absorption imaging is that it excites the atoms to a higher electron state, thus losing all the trapping systems and effectively destroying the cloud. This means that it can only be done once with the same sample, so after making an image the experiment has to be repeated completely, which can be quite time intensive.

## Chapter 5

## Measurements and results

In this chapter the Measurements and results are discussed. First the shape of the trap is measured in section 5.1 Next we calibrate our power measurement in section 5.2 and our imaging in section 5.3. Finally the results of the actual experiment are presented in section 5.4 .

### 5.1 The Rayleigh Profile

As already discussed in section 4.2 we measure the Rayleigh profile for our experiment, and for comparison also that of the other experiment done in our group that is done without the second telescope so with a five times smaller beam, that has according to theory a five times bigger focus as in the experiment described in this report. We measure the profile, which has according to theory a Gaussian profile in the direction perpendicular to the beam, cylindrically symmetric, and a Lorentzian intensity profile in the direction of the beam. The width of the Gaussian varies with the square-root of the inverse of that same Lorentzian profile, meaning it becomes wider as one moves further away from the centre. This results in the following model:

$$
\begin{equation*}
I(\rho, z)=I_{0} \frac{e^{-\frac{2 \rho^{2}}{w_{0}^{2}\left(\frac{z^{2}}{z \mathrm{R}^{2}}+1\right)}}}{\left(\frac{z^{2}}{z \mathrm{R}^{2}}+1\right)} \tag{5.1}
\end{equation*}
$$

Now, if the beam is completely Gaussian, the Rayleigh length is given by $\frac{\pi w_{0}^{2}}{\lambda}$. For a derivation of this see [16].

Now measurements are done with the camera as said before by moving it slowly across the profile. At every distance, the camera takes twenty pictures in twenty seconds, and these are then averaged, to take care of fluctuations in the measurement. An image of the camera is a cross section of the profile in the $\rho$ direction, meaning it should have a Gaussian profile. Now of course there is a possibility that the beam doesn't have the same size in the $x$ and the $y$ direction (the $z$ being the beam is moving) so we describe the data with the following formula:

$$
\begin{equation*}
A \exp \left(-2\left(\frac{\left(\left(x-x_{0}\right) \sin (\theta)+\left(y-y_{0}\right) \cos (\theta)\right)^{2}}{w_{0, y}^{2}}+\frac{\left(\left(x-x_{0}\right) \cos (\theta)-\left(y-y_{0}\right) \sin (\theta)\right)^{2}}{w_{0, x}^{2}}\right)\right)+\mathrm{os} \tag{5.2}
\end{equation*}
$$

Now we do a least square fit using this model, and so we get results as a function of distance. For the wide focus the results are shown in Figure 5.1, for the tight focus used in our experiment


Figure 5.1: Width as a function of position for the wide focus, where the data around the focus is shown. Fits to the part of the data around the focus are also shown. The different colours represent the long and short axes for the ellipse. Ideally the green and red points should have the same value. The purple line is a fit to both the " $X$ " and " $Y$ " data.
in Figures 5.2 and 5.3 . The camera measures actually three colours, and since we use infra-red light that passes right through the filters, this just means we get three measurements in the same image, which we then average. Now we notice a few things, first the Gaussian seems to be a little elliptic, noticing the difference in values for one and the other axis of the ellipse. Secondly one sees that far away from the focus, the angle of the main axis is starting to become inaccurately determined. The presented uncertainties are the fit uncertainties of the Gaussian, and in Figure 5.1 where the results of the three colours are presented in a weighted average they are the propagated uncertainties. Taking only the values close to the focus (Figure 5.3 and 5.1) we make a weighted least square fit of the root of the inverse of the Lorentzian profile and determine the width of the focus, and the Rayleigh length. The results are presented in Table 5.1.

We see that the Rayleigh length deviates from the predicted value, giving us the opportunity to calculate the so called $m^{2}$, defined as the ratio between the measured Rayleigh length and the formula that should give this length for a Gaussian $\left(\frac{\pi w_{0}^{2}}{\lambda}\right)$. This $m^{2}$ accounts for the observed deviation being both smaller than one for the tight and for the wide focus. The most remarkable result is perhaps that the focus of the tight dimple is not five times smaller as the one used in the other experiment, while the incoming beam is five times larger. However the result turned out to be reproducible with this experiment. Clearly the reader of chapter 3 realises that the original plan was to use a 10 um focus, which just isn't possible with this experiment at the


Figure 5.2: Angle of the long elliptic axis as a function of distance for the tight focus. It becomes clear that the data around the focus (where the intensity of course is bigger) is the most reliable. The different colours correspond to the different colours that are reported by the camera, although all are used equally because we use infra-red light
moment of writing this report.

### 5.1.1 Discussion

When looking at the estimated variance we see that the fit where we just take all the $w_{x}$ and $w_{y}$ data, we see that doing that is actually not valid for determining the width, since the estimated variance is to large. Clearly the potential isn't cylindrically symmetric meaning that we actually should use a three-dimensional potential in chapter 3 and section 5.4 but for a simulation this turns out to be imposable to keep within the calculation time, and because of the method of imaging described in section 4.3 , cylindrical symmetry must be assumed, since it only measures in two dimensions. That's why it must be assumed that the potential is approximately cylindrically symmetric, giving raise to a larger uncertainty in both the Rayleigh length and the focus. In section 5.4 it turns out that the Rayleigh length actually has the largest partial uncertainty contribution in the final result. Finally it was later observed that the beam seemed to have a ring pattern appearing, suggesting a diffraction pattern of a piece of dirt on one of the lenses or mirrors that it passes. However countlessly cleaning the lenses did not remove the effect. It could be a defect on one of the lenses or mirrors, however we did not verify this yet by replacing the lenses. If a beam is not Gaussian the behaviour is different and this could explain the fact that the expected factor of five does not appear. Again, we did not verify this yet. Replacement of the lenses would be something that should be done in future experiments to find out.


Figure 5.3: Width as a function of position for the tight focus, where the data around the focus is shown. Fits to the part of the data around the focus are also shown. The different colours represent the long and short axes for the ellipse. Ideally the green and red points should have the same value. The purple line is a fit to both the " $X$ " and " $Y$ " data.

### 5.1.2 Conclusion

We determined the focus of our trap at $18.74 \pm 0.36 \mu \mathrm{~m}$, and the Rayleigh length at $1.012 \pm 0.057$ mm . This data is used in section 5.4.

### 5.2 Calibration of the power

Sending a certain signal into the AOM corresponds to a certain light intensity that comes out. Basically the case is a little more complex because of the fact that the power is stabilized by the technique discussed in section 4.2 This actually means that we have to set a certain intensity value that is measured by a photo diode at the end of the set-up. We need to calibrate the values it sends back to the AOM to the light intensity, to know what setting gives what light intensity. Basically the whole system has an input value we can set, and it then delivers a certain power (if this power is provided by the laser of course). The obtained powers as a function of the input voltage are shown in Figure 5.4.

So now we know the power of our laser, but, as explained in section 4.2 the windows might take something away. To calculate what the losses are of the lenses we measure at three points, right after the fibre (on the right in Figure 4.1), right before we enter the vacuum chamber (around the folding mirror in Figure 4.1) and right after the beam leaves the vacuum chamber (not visible in Figure 4.1). We do that with and without the last telescope, (see Figure 4.1) and

|  | tight focus | wide focus |
| :---: | :---: | :---: |
| $w_{0, x}(\mu \mathrm{~m})$ | $20.51 \pm 0.21$ | $43.73 \pm 0.10$ |
| $w_{0, y}(\mu \mathrm{~m})$ | $17.02 \pm 0.40$ | $43.349 \pm 0.076$ |
| $w_{0, a v}(\mu \mathrm{~m})$ | $19.8 \pm 1.4$ | $43.49 \pm 0.18$ |
| $w_{0, g f}(\mu \mathrm{~m})$ | $18.74 \pm 0.36$ | $44.44 \pm 0.192$ |
| $z_{R, x}(\mathrm{~mm})$ | $1.164 \pm 0.045$ | $5.392 \pm 0.047$ |
| $z_{R, y}(\mathrm{~mm})$ | $0.891 \pm 0.052$ | $5.528 \pm 0.071$ |
| $z_{R, a v}(\mathrm{~mm})$ | $1.05 \pm 0.14$ | $5.433 \pm 0.062$ |
| $z_{R, g f}(\mathrm{~mm})$ | $1.012 \pm 0.057$ | $5.52 \pm 0.14$ |
| $\chi_{\text {red }, x}^{2}$ | 0.36 | 0.58 |
| $\chi_{\text {red, }, y}^{2}$ | 1.20 | 1.47 |
| $\chi_{\text {red,gf }}^{2}$ | 1.85 | 16.1 |
| $m_{R, x}^{2}(\mathrm{~mm})$ | $0.942 \pm 0.024$ | $0.9534 \pm 0.0044$ |
| $m^{2}{ }_{R, y}(\mathrm{~mm})$ | $1.046 \pm 0.035$ | $0.9934 \pm 0.0090$ |
| $m^{2} R, a v(\mathrm{~mm})$ | $0.976 \pm 0.049$ | $0.961 \pm 0.016$ |
| $m^{2}{ }_{R, g f}(\mathrm{~mm})$ | $0.981 \pm 0.034$ | $0.967 \pm 0.016$ |

Table 5.1: In this table $w_{0, a v}$ is an weighted average of $w_{0, x}$ and $w_{0, y}$ and $w_{0, g f}$ is a fit to all the data. The same goes for $z_{R}$. $\chi_{\text {red }}^{2}$ is the estimated variance.


Figure 5.4: AOM values versus the measured Power.

|  | $P(\mathrm{~mW})$ with the big lens | $P(\mathrm{~mW})$ without the big lens |
| :---: | :---: | :---: |
| Straight out of the fibre | 196 | 220 |
| Right before entering the chamber |  |  |
| -With the extra telescope | 185 | - |
| -Without the extra telescope | 192 | 174 |
| Right after leaving the chamber |  | - |
| -With the extra telescope | 149 | - |

Table 5.2: Powers measured at different locations in Figure 4.1.
with and without the big lens. The big lens (see section 4.2) makes it possible to measure a big beam on the power meter, but it also takes away some of the power. When the telescope is present measuring without the big lens right before the chamber is not possible, but without the telescope, the focussing makes the beam small enough to be measured on the power meter. The data of all this is given in Table 5.2 .
We can now calibrate the big lens and see that it lets a fraction $0.898 \pm 0.008$ of the light pass through. We can now process these results with the assumption that the window to get into the chamber takes away just as much as the window to get out. If $\alpha$ is the portion of the light allowed through by one window, $\alpha^{2}$ the portion of two. Using the two measurements right before and after we see that a fraction of $0.884 \pm 0.013$ of the light is allowed to pass a single window. We then arrive at the conclusion that $85 \pm 2$ percent of the light coming out of the fibre comes into the chamber. (we added the same uncertainty to the first part of the set-up as we determined to the second part, because there we only have one measurement.)

### 5.2.1 Using the trap frequency

There is actually a second way of determining the power, one could fill the dimple with a small number of particles, and then allow it to do a small oscillation. If we now approximate (4.2) as an harmonic (valid for a small number of particles and a small oscillation) the oscillation has the trap frequency. Approximating 4.2 as an harmonic gives the following equations for the so-called dimple trap frequency:

$$
\begin{equation*}
\omega_{\text {dimple }, \rho}=\frac{2 \sqrt{\frac{\mathrm{vf}_{V_{0}} P}{m}}}{w_{0}^{2}} \text { and } \omega_{\text {dimple }, z}=\frac{\sqrt{2 \frac{\mathrm{vf}_{\mathrm{V}_{0}} P}{m}}}{w_{0} z_{R}} \tag{5.3}
\end{equation*}
$$

This means that we can reverse this and determine the power from here. This procedure is done in the master thesis of Martijn van 't Woud [17] for the wide focus of section 5.1 Combining his measurements for the trap frequency with the focus and Rayleigh length from table 5.1 a power of $P=0.883 \pm 0.020 \mathrm{~W}$ for the $\omega_{\rho}$ measurement is obtained, and a power of $P=0.839 \pm 0.046$ W for the $\omega_{z}$ measurement. In the power determined from the $\omega_{\rho}$ measurement the partial contributions to the uncertainty from the measurement of the trap frequency and the width of the focus are of similar size, in the power determined from $\omega_{z}$ the Rayleigh length dominates in the uncertainty. He then determined the power under the same circumstances but now with the method described in section 5.2 This yielded (after he included the factor of 0.85 for the window) a power of $P=1.36 \mathrm{~W}$. Of course these are different circumstances then the one that are used in this report, but the mismatch must be noted.

### 5.2.2 Discussion and conclusion

Looking at the partial contributions of the different parts of the trajectory, we see that the windows take the most of the light away. Which we can explain with the fact that it turns out they were coated for 589 nm light and we send through 1070 nm .

The light beam looses 15 percent of its power. One must include this result when processing the results in section 5.4 We have also seen that when determining the power from the trap frequencies a mismatch can occur with the measurements with the power meter.

### 5.3 Calibration of side imaging

For side imaging there are actually two things to be calibrated. First of all there is to calibrate what distance corresponds to a certain number of pixels. Of course this is in the specifications of the camera, but we reduce the image, so we calibrate it using a gravity experiment in section 5.3.1 Next we need to know which cross section to use. This is actually highly dependent on magnetic fields and the light, so we calibrate this in section 5.3.2

### 5.3.1 Gravity

As is said in section 4.3 the way to determine the distance per pixel is done by doing a gravity experiment. We turn off the magnetic trap and let the cloud fall by gravity. By varying the time and making a side absorption image, we can determine the position as a function of time. We can then fit the observed cloud in a way that is described in section 5.4.1. But the important parameter we are interested in is the centre of the cloud. We know that the distance of a fall in vacuum as a function of time is given by:

$$
\begin{equation*}
s(t)=s(0)+v_{0} t-g t^{2} / 2 \tag{5.4}
\end{equation*}
$$

where $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$ is the gravitational constant and $v_{0}$ a starting velocity that might be obtained by turning off the trap. If c is the distance between two pixels, then a falling distance $s$ is actually in the view of the camera that is tilted given by the following formula

$$
\begin{equation*}
s(p x)=\frac{\cos (\theta)\left(s(0)+v_{0} t-g t^{2} / 2\right)}{c} \tag{5.5}
\end{equation*}
$$

where $\theta$ is the angle by which the camera is tilted. This means we can write:

$$
\begin{equation*}
s(p x)=k\left(s(0)+v_{0} t+g t^{2} / 2\right) \tag{5.6}
\end{equation*}
$$

with $k=\frac{\cos (\theta)}{c}$. We can now make a least square fit for the starting velocity and $k$. This is done in Figure 5.5, we obtain $v_{0}=0.0483 \pm 0.0031 \mathrm{~m} / \mathrm{s}$ and $k=32896 \pm 608 \mathrm{px}^{-1}$ for a first and $v_{0}=0.03745 \pm 0.00082 \mathrm{~m} / \mathrm{s}$ and $k=30234 \pm 182 \mathrm{px}^{-1}$ for a second measurement. From $k$ we can use the fact that the imaging beam is tilted at an angle of $0.72 \pm 0.02$ radians to obtain that the distance per pixel is $23.053 \pm 0.734 \mu \mathrm{~m} / \mathrm{px}$.


Figure 5.5: Position of the condensate on the camera as a function of time.

### 5.3.2 Cross section

We now still need to know which absorption corresponds to which density. This is of course the Beer-Lambert law for transmission $T(x, z)$ :

$$
\begin{equation*}
T(x, z)=\exp \left(-\sigma \int n(x, y, z) d y\right) \tag{5.7}
\end{equation*}
$$

where $n(x, y, z)$ is the density. In this law the full cross section $\sigma$ is given by:

$$
\begin{equation*}
\sigma=\frac{3 \lambda^{2}}{2 \pi}[13, \text { eq } 3.12] \tag{5.8}
\end{equation*}
$$

At least this is the case if every transition is excited. In reality the absorption is less, because not every transition is excited. This means that the formula becomes:

$$
\begin{equation*}
\sigma=c \frac{3 \lambda^{2}}{2 \pi} \tag{5.9}
\end{equation*}
$$

with $c$ an unknown constant, that depends on the occupation of the spin states, frequency of the light and the magnetic fields. The magnetic fields however are set in a way which is suitable for the experiment, and it isn't known what the exact direction is when the magnetic trap is turned off. The way we solve this is by applying an external magnetic field to obtain the maximal
transition strength for this light and particles in a set spin direction. This turns out to be given by

$$
\begin{align*}
& \quad c_{\mathrm{id}}=\frac{10}{24}+j  \tag{5.10}\\
& \text { with } j \approx 0.005
\end{align*}
$$

So we end up with a factor of 0.4217 . We now use a relatively hot cloud in a harmonic trap, so the density profile is just a simple Gaussian. We then scan over the possible configurations of the magnetic field to find the maximal absorption. We can then integrate the Gaussian in one direction, which gives us a Gaussian too, to obtain the formula that should describe the cloud completely, as long as we know the trap frequencies. The only unknown parameter is the number of particles, but since we know the cross section in the optimal situation, it is given by formula (5.10). We then repeat the experiment, without the external fields optimized, but set in a way that is usually used in the experiment. This way we can again fit a Gaussian but this time the cross section is unknown. We can then use the number of particles that was given in the other experiment, so we can now calibrate the cross section. We repeat the experiment a few times to get an estimate of the uncertainty. We then find that $c$ in 5.9 is in our case $0.366 \pm 0.0083$ where it was calibrated on the 0.4217 of the optimal case.

### 5.3.3 Discussion

In the gravity experiment (section 5.3.1) we observe that the estimated variance is way bigger then one (2971), when we use the fit uncertainties as a reference. This suggests that the fluctuations from shot to shot are much bigger then the fluctuations that arise from the imaging itself. Further, in the determined pixel size we see that the partial uncertainty contribution of the angle is $0.43 \mu \mathrm{~m} / \mathrm{px}$ where the one coming from $k$ is $0.59 \mu \mathrm{~m} / \mathrm{px}$ meaning that although the relative uncertainty of the angle is significant, its partial uncertainty contribution is actually the smallest.

### 5.4 The actual experiment

Because of the high particle densities and the very small width of our dimple, measuring in situ would bring the dimple in the order of pixel size. This is why we first let the cloud expand. A description of the expansion is given in section 5.4.1 Next we do a scan over different starting temperatures and dimple depths to see if we can achieve a picture of the entire scope in section 5.4.2

### 5.4.1 Theoretical description of a time of flight experiment

When in absorption imaging a large portion of the light gets absorbed, the signal given by the light coming through can be lower than the noise of the camera. This occurs at high particle densities. Also, the size of our dimple is in the order of $20 \mu \mathrm{~m}$ in the radial, that is the same order of our imaging resolution, meaning we lack resolution. That is why we do our measurements in a time of flight experiment, in which we turn off both traps, and give our cloud time to expand. We will now try to describe this process accurately. We start off by looking at the expansion of the condensate in section 5.4.1.1 and then at the expansion of the thermal cloud in section 5.4.1.2 We need to know this behaviour so we can do a fit to our data. First we need a formula for the density. For simplicity we assume that in the static case the system can
be approximated by a Thomas Fermi distribution (see 2.3) for the condensate and a MaxwellBoltzmann distribution for the thermal cloud. Since we only measure our total particle density, and we know that in this experiment the condensate dominates in the centre of the trap and thermal cloud on its sides, and we know that at large distances from the centre thermal cloud approximates a Maxwell-Boltzmann distribution, we make the previous approximations. This gives us the following density distribution.

$$
\begin{equation*}
n(\rho, z)=\max \left(\frac{\mu}{U_{0}}-\frac{1}{U_{0}} V(\mathbf{r}), 0\right)+\frac{N_{\mathrm{ex}} e^{-\frac{\rho^{2}}{2 \sigma_{\rho}^{2}}-\frac{z^{2}}{2 \sigma_{z}^{2}}}}{2 \sqrt{2} \pi^{3 / 2} \sigma_{\rho}^{2} \sigma_{z}} \tag{5.11}
\end{equation*}
$$

where $N_{\mathrm{ex}}$ is the total number of excited particles, and $\sigma_{\rho}$ and $\sigma_{z}$ are the sizes of the thermal cloud. $V(\mathbf{r})$ in this case is given by (3.2).

### 5.4.1.1 Expansion of the condensate

To describe the expansion of the condensate we assume that the shape of the condensate does change, only the width (self-similar expansion). The method used in this section follows a similar way of thinking as the one used in [18], but uses different formulas, and has a different result because the potential is different. We introduce variable $R(t)$ and $W(t)$ as follows:

$$
\begin{align*}
& R(t)=\lambda_{\rho}(t) w_{0}  \tag{5.12}\\
& W(t)=\lambda_{z}(t) z_{R} \tag{5.13}
\end{align*}
$$

Here $R(t)$ and $W(t)$ will take over the role of the width of the focus and the Rayleigh length during expansion, because there is no longer a trap that has a Rayleigh length and a width of the focus. $\lambda_{\rho}(t)$ and $\lambda_{z}(t)$ are measures of how much the condensate has expanded compared to the beginning. Now the density profile for the condensate as a function of time becomes:

$$
\begin{equation*}
n_{0}(\rho, z, t)=\frac{1}{\lambda_{\rho}(t)^{2} \lambda_{z}(t)} n_{0}\left(\frac{\rho}{\lambda_{\rho}(t)}, \frac{z}{\lambda_{z}(t)}, 0\right) \tag{5.15}
\end{equation*}
$$

The potential as a function of time is given by:

$$
\begin{equation*}
V(\rho, z, t)=f(t) V(\rho, z, 0) \tag{5.16}
\end{equation*}
$$

with $f(t)$ being 1 for $t=0$ and 0 for $t>0$. Newton's second law then becomes:

$$
\begin{array}{r}
m \ddot{R}=m w_{0} \ddot{\lambda_{\rho}}=F_{\rho}\left[R=\lambda_{\rho} w_{0}\right]=-\partial_{\rho}\left(V+U_{0} n(t)\right) \\
m \ddot{W}=m z_{R} \ddot{\lambda_{z}}=F_{z}\left[W=\lambda_{z} z_{R}\right]=-\partial_{z}\left(V+U_{0} n(t)\right) . \tag{5.18}
\end{array}
$$

Here $F_{\rho}$ and $F_{z}$ represent the components of the effective force. We know from (5.15) and (5.11) that our density distribution is given by:

$$
\begin{equation*}
n_{0}(\rho, z, t)=\max \left(\frac{\mu-V\left(\frac{\rho}{\lambda_{\rho}(t)}, \frac{z}{\lambda_{z}(t)}, 0\right)}{\lambda_{\rho}(t)^{2} \lambda_{z}(t)}, 0\right) \tag{5.19}
\end{equation*}
$$

When $t>0$ then $V=0$, so we now solve the equations and obtain:

$$
\begin{gather*}
\frac{V_{0}+e m w_{0}^{2} \omega_{\rho}^{2}}{e m w_{0}^{2} \lambda_{z}(t) \lambda_{\rho}(t)^{3}}=\ddot{\lambda_{\rho}}  \tag{5.20}\\
\frac{\omega_{z}^{2}}{\lambda_{z}(t)^{2} \lambda_{\rho}(t)^{2}}=\ddot{\lambda_{z}} \tag{5.21}
\end{gather*}
$$

where $e$ is Euler's constant. These equations cannot be solved analytically, but if we neglect the harmonic potential, or if we study the case where the harmonic part of the potential is not present this simplifies to:

$$
\begin{gather*}
\lambda_{z}(t)=1  \tag{5.22}\\
\lambda_{\rho}(t)=\sqrt{1+\frac{t^{2} V_{0}}{e m w_{0}^{2}}} \tag{5.23}
\end{gather*}
$$

where we have set $\lambda_{\rho, z}(0)=1$ and $\dot{\lambda}_{\rho, z}(0)=0$. If the harmonic potential is present but the dimple is not, the system cannot be solved analytically either, but it can be approximated with an analytical solution quite accurately, as long as the trap frequency in the $z$ direction is small to the one in the $\rho$ direction:[18, eq $20 \& 21]$

$$
\begin{gather*}
\lambda_{z}(t)=\frac{\omega_{z}^{2} t}{\omega_{\rho}} \arctan \left(\omega_{\rho} t\right)-\ln \left(\sqrt{1+\omega_{\rho}^{2} t^{2}}\right)  \tag{5.24}\\
\lambda_{\rho}(t)=\sqrt{1+\omega_{\rho}^{2} t^{2}} \tag{5.25}
\end{gather*}
$$

where the approximation is made that $\omega_{z} \ll \omega_{\rho}$. In Figure 5.6 and 5.7 a graph is shown of the three different cases at 30 ms time of flight as a function of dimple depth. The red line depicts the case where there is only the harmonic, the blue one where there is only a dimple present ( 5.22 ) and $\sqrt{5.23}$ ) and the black line is a numerical solution to ( 5.20 ) and 5.21$)$. We see that in the $z$-direction there are no significant differences, and in the $\rho$-direction there is still quite a significant deviation from the case where we neglect the harmonic. This gives us reason to keep including the harmonic in the derivation on the next pages. Please note that the blue line in Figure 5.6 doesn't go to zero, it goes to one, as expected for no trap at all.

If we use 5.12 and 5.14 to eliminate $\lambda_{\rho, z}$ from 5.19 we can now see that we have the shape of our density distribution as a function of time. Because we do absorption imaging, we look at the column density. That means we have to integrate out one direction to find an appropriate fit function. And that means we have to find the edge of the condensate, since looking at (5.11) we realise that that this can give us a negative density if we do not account for that situation. So we solve (5.19) equal to 0 for $y$, and that gives two solutions, both a function of $x$ and $z$, and those two functions are our integration limits. This function is so complex that it is shown in appendix $D$. Doing the integrations gives us for the integrated density, which also can be found


Figure 5.6: The expansion parameter $\lambda_{\rho}$ for $t=30 \mathrm{~ms}$ as a function of the power of the dimple trap (proportional to the dimple depth). Three situations are shown: the dimple trap only (blue line), the harmonic trap only (red line) and the combined dimple and harmonic trap (black line). The parameters used are given by Appendix E. At zero power the situation with the dimple trap only gives an expansion parameter of one.


Figure 5.7: The expansion parameter $\lambda_{z}$ for $t=30 \mathrm{~ms}$ as a function of the power of the dimple trap (proportional to the dimple depth). Three situations are shown: the dimple trap only (blue line), the harmonic trap only (red line) and the combined dimple and harmonic trap (black line). The parameters used are given by Appendix E. The conclusion is that in the $z$ direction there is not much expansion.
in appendix D . Once we put this into the Beer-Lambert law we obtain our fit function. Right after the function a condition is given. There is a condition because also at some $x$ - and $z$-values the function can give a negative density, and we then need a zero density. However performing least squares fit yields large correlations between the different fit parameters. And that makes the result unreliable. To solve this, we realise that we already have determined a lot of the parameters in another way, see sections 5.1 , 5.2 and 5.3 . So we now make the choice to fix these parameters at set parameters during the fit. We do however realise that this makes propagating the uncertainties very hard. An error in one of the determined quantities in sections $5.1,5.2$ and 5.3 propagates as a systematic uncertainty. However this is a systematic uncertainty that is the same for every fit, so the error in trend is a lot smaller. ${ }^{1}$ We can however calculate some constants to make an estimate of the systematic uncertainty. We then see that the Rayleigh length has the largest partial contribution to the systematic uncertainty. ${ }^{2}$ This is of course an effect of the approximation of the potential as cylindrical, where section 5.1 showed it isn't, bringing an error in the Rayleigh length.

A last adaptation to the fit function is still made, to include a shift in the centre of the cloud, and a rotation in the camera image. Fixing the parameters we now have the following unknowns left: the rotation and $x$ and $z$ position, the chemical potential and $R$ and $W$. For the reference images at zero dimple depth the formulas are much simpler, and the fixing of parameters isn't

[^7]needed.

### 5.4.1.2 Expansion of the Thermal Cloud

As is discussed at the beginning of this section (5.4.1), we approximate the thermal cloud with a Gaussian. One must realise the dimple is very tight, and as was already shown by the simulations in section 3.1.1.3 the dimple has only a small influence on the thermal cloud. Further, the condensate is located in the centre, making the density very large in those regions, making the light completely absorbed there. Far away from the centre, a simple approximation for (2.20) in this potential shows that it goes to a Gaussian, and that is where we observe the thermal cloud.

The thermal cloud is subject to a normal ballistic expansion. This means it stays a Gaussian, but it gets wider. A simple derivation shows that the standard deviation of a Gaussian as a function of time is given by:

$$
\begin{equation*}
\sigma(t)=\sqrt{\frac{\mathrm{k} T}{m}} \sqrt{\frac{1}{\omega^{2}}+t^{2}} \tag{5.26}
\end{equation*}
$$

which can be rewritten to

$$
\begin{equation*}
T=\frac{m}{\mathrm{k}}\left(\frac{\sigma^{2}}{t^{2}+\frac{1}{\omega^{2}}}\right) \tag{5.27}
\end{equation*}
$$

yielding the temperature. The temperature can be measured in two directions, which are combined with a weighted average. One can now realise, looking at the speed of expansion of the condensate (see (5.23), that a very tight or deep dimple means a quick expansion of the condensate. And we know, our dimple is very tight compared to the harmonic trap. This means that the condensate will expand a lot faster then in the original harmonic trap, while with the thermal cloud there is no reason why there should be any difference in speed. We can also write down a $\lambda_{\mathrm{th}}$, comparable to $\lambda$ in 5.20 for the thermal cloud. If we now compare the two in Figure 5.6 , we see that for the deeper dimples the condensate has expanded two and a half times as much as the thermal cloud. In other words, the condensate has expanded 50 times its original size, whereas the thermal cloud has only expanded 18 times its original size. Now, in general the starting sizes of both clouds are not equal, but the size of the thermal cloud depends on temperature, meaning that at a lower temperature we have a smaller starting size. This means that at the lower temperatures there is a chance of the condensate overhauling the thermal cloud. In the outlook (section 6.3.1) this is discussed further. Because of the fast expansion of the condensate, there is only a small region at the side of the condensate where the thermal cloud is visible. This is because there may be no condensate in those regions, but the density still has to be high enough to get over the noise on the camera. Because of this we make the choice in section 5.4.2 to do a weighted average of all the temperature measurements at a certain starting temperature combined with dimple depth, to obtain a good estimate of the uncertainty in the temperature. In section 5.4.2. Figure 5.10 will illustrate this size difference.

### 5.4.2 Scanning over scope of dimple depths and starting temperatures

In this section we present the results of the experiment in which we varied the starting temperature and the dimple depth. In section 4.3 we discuss how we image the cloud, and in section 5.4 .1 we discuss how we fit the cloud. First, we obtain three images from every measurement. In one there is no cloud which is the reference for a second image with a cloud, and a third one where


Figure 5.8: The expansion factors after 30 ms TOF for both the thermal cloud and the condensate compared. We see that at a deeper dimple depth the condensate has multiplied its size way more than the thermal cloud.
there not even is light, to determine the offset of the camera. Second, the offset of the camera is subtracted from both images. This subtraction has a small risk of yielding negative values for when in the first two images the values were already very small. To avoid getting problems of dividing by negative numbers, and because of the fact that it is physically impossible to have less light than no light these few pixels are set to zero. Next we divide every pixel from the image with the cloud by the matching pixel from the image without the cloud, this way obtaining the fraction of the light that is absorbed. This should yield a range between zero and one. Values that are not a number, infinite or larger than two ${ }_{3}^{3}$ We now cut out the part of the image that does never capture light anyway. Next, we make an automatic estimate for the fit parameters such as the width of the cloud and the position. We then realise that the picture with the cloud might accidentally has captured more or less light than the one without. This results is a scaling factor (not an offset!) and we could of course use this as an extra fit parameter. We however think there is a more accurate way of determining it. We cut out a few times the width of the cloud and look at the pixels that are outside that window. Those pixels, should be one, and by making a complete average of the area we can very accurately determine the fraction of light between the two pictures. We can then multiply every picture with this fraction, removing the scaling factor as a fit parameter. An example of such a resulting image is shown in Figure 5.9. We then do our fit according to (D.3).

In section 5.4 .1 .2 it is discussed that we choose to average the temperature data from multiple

[^8]

Figure 5.9: An example of a processed image of a cloud of sodium atoms in a combined magnetic and dipole trap. The grey scale correspond to the amount of absorbed light, so the darker the more particles.
experiments to obtain an estimate for the uncertainty and because of the overhauling of the condensate. In Figure 5.10 the $R$ from section 5.4 .1 and five times the standard deviation in

Different temperatures at a dimple depth of 59.5 mW


Figure 5.10: The R of the condensate (blue points) and the standard deviation of the thermal cloud (red point) in a deeper dimple. The overhauling effect becomes clear.
the $\rho$-direction of the temperature is plotted at a chosen dimple depth. We see that indeed the thermal cloud is much smaller (in 30 ms time of flight expansion) than the condensate illustrating the overhauling effect.

First we determine our initial conditions. We cool the cloud, as explained in section 4.1 at first with an RF-knife. The RF-knife starts at a high value in MHz and has a final value in the range from 3.0 to 3.1 MHz . Varying the final value determines the initial temperature, the initial chemical potential and the initial number of particles. Fit results of measurements of the
initial conditions are shown in Figures 5.11 and 5.12 . We see that indeed starting at a lower


Figure 5.11: The number of thermal particles in red, the number of condensed particles in blue, and the total number of particles in green, as a function of the end value of the RF-knife. A lower ending value means a larger condensate fraction.
power means a lower starting temperature, and a larger number of condensed particles. Now we repeat the experiment, only this time we turn on the dimple adiabatically and image when it is at a certain power. By repeating the experiment many times for different powers at which we lower the dimple depth and different starting temperatures, we can make a scan over the entire scope of starting temperatures and dimple depths. The results of this are shown in Figure F. 1 in appendix F. A selection of those results is shown in Figures 5.13 and 5.14 . We see that we do not acquire a crossing between $\mu$ and $\mathrm{k} T$ at higher starting temperatures, although $\mu$ and $\mathrm{k} T$ do seem to move closer together as expected at higher temperatures. At lower temperatures we do see a crossing of $\mu$ and $\mathrm{k} T$, until we start at too low temperatures, where the chemical potential and the temperature were already almost equal at the beginning and here we seem no longer able to distinguish difference in behaviour between temperature and chemical potential. Actually the chemical potential doesn't even reach the value it did at the higher starting temperatures, while the temperature seems to go up more than in the previous experiments, suggesting a high correlation.

Now we also compare the data with the simulations in section 5.4.2. We do not have simulations available at exactly the same conditions, however in Figure 5.13 and 5.14 two simulations at similar circumstances are shown. The realisation what is going on becomes clear, the starting conditions are very similar, but in the simulation both the temperature and the chemical potential go up a lot faster than in the measurements. To make simulation and result still overlap a little


Figure 5.12: The temperature (in red) and the chemical potential in blue as a function of the ending value of the RF-knife. A lower final value means a lower temperature.


Figure 5.13: Comparison of simulation and experiment. The top axes and the line is the simulation, the bottom axes and the points represent the measurements. Red points represent the temperature $\mathrm{k} T$ and blue points the chemical potential $\mu$.


Figure 5.14: Comparison of simulation and experiment. The top axis and the line is the simulation, the bottom axes and the points represent the measurements.
bit, the power axis for the simulation data is scaled with a factor of three in both pictures.

### 5.4.3 Discussion and Conclusion

### 5.4.3.1 Discussion

Fixing some parameters at set values before the fit (discussed in section 5.4) brings a systematic uncertainty in the result. We expect this effect to be small. Since it is systematic, the qualitative results are not affected. The temperature has a large variation from shot to shot and is visible only because of a small signal to noise ratio on the side of the condensate where the condensate is absent and the thermal cloud gives the temperature, because of the overhauling effect illustrated in Figure 5.10. This makes the temperature data unreliable as soon as one comes to lower temperatures. A possible solution is discussed in the outlook (section 6.3.1). Because the overhauling happens during the time of flight, we could do two separate measurements, one where we image at the start (before expansion) and one where we image after the time of flight. In the image before expansion the condensate might not be properly imaged, but the thermal cloud has a width that can be imaged, meaning that we can determine the temperature from the first and the chemical potential from the second image. In Figure 5.15 the correlation between the width in the $\rho$ direction of the thermal cloud and the chemical potential, resulting from the fits, is shown for different measurements, as a function of dimple depth. We see that relatively high correlations do occur, implying that the value for the chemical potential and the temperature were not determined accurately in such measurements.

### 5.4.3.2 Conclusion

We did obtain a crossing between $\mu$ and $\mathrm{k} T$, meaning we obtained a $\chi$ higher than one, which was the goal of the experiment. We did not however obtain agreement with our simulation, only


Figure 5.15: Correlation between the chemical potential and the temperature for different measurements. We see that this correlation is still significant.
at the starting temperature. The effect our dimple seems a lot smaller than what one would expect from simulations.

## Chapter 6

## Discussion, Conclusion and Outlook

### 6.1 Discussion

In section 3.1.1.3, the accuracy of the simulations done could be improved by doing a more than one dimensional search, therefore reducing simulation time and improving accuracy if one simulates for the same time. This is especially the case in Figure 3.9, where we see the maximal temperature at which a crossing between $\mu$ and $\mathrm{k} T$ as a function of the dimple width. Also the simulations done in section 3.2.1.1, where we have tried to simulate continuously lowering the trap frequency of the harmonic trap, cannot be used, because it assumes thermal equilibrium in a regime where that is non-physical to assume. The results of section 3.2 .3 provide a meaningful insight on the effects of correlations between single-particle wave functions, but are again not useful experimentally. The approximation of the optical dipole potential as cylindrically symmetric is clearly bringing in a systematic uncertainty when looking at the (too high) estimated variance of doing that in section 5.1. Of course the observed non-Gaussian shape of the laser beam also brings in a systematic uncertainty. The effects of assuming the dimple in the centre of the harmonic potential, while of course this is subject to alignment of the laser, can actually be estimated. The depth of the dimple can be calculated. One can then look at which position the harmonic reaches this value. Actually at a depth of 30 mW this yields about $75 \mu \mathrm{~m}$, which is so large that we can exclude that as a significant contribution. The fixing of parameters at set values of course brings a systematic uncertainty in the result. The main challenge is the overhauling effect described in section 5.4. This makes the temperature data unreliable. A possible solution to this is discussed in the outlook.

### 6.2 Conclusion

The hypothesis we stated at the beginning was confirmed in a simulation of the experiment. A crossing between $\mu$ and $\mathrm{k} T$ was achieved in section 3.1.1.3 and the image of the thermal bath was confirmed. We learned in section 3.2.3.3 that neglecting the correlations between the single-particle wave function because of interactions has huge effects. Some important lessons about the properties of our experimental set-up were learned in sections 5.1 , 5.2 and 5.3 Finally we observed the crossings experimentally in section 5.4 although they seem to appear at higher dimple depths as in simulation. This means that in order to get an accurate estimate of the power
that is needed to use for obtaining a predefined $\chi=\mu / \mathrm{kT}$ one needs to rely on measurements of the $\mu$-T-power-RF-knife spectrum, and cannot rely on simulations. In the outlook this will discussed further.

### 6.3 Outlook

One of the goals of this experiment was to observe both so called first and second sound in the same measurement. In a cloud we actually have a two fluid system of super fluid (condensate) and thermal cloud (normal fluid). It turns out that around the point where $\mu$ equals kT first and second sound switch behaviour [10, Chapter 5], which is discussed in section 6.3.2. To be able to accurately set the system where $\mu$ equals $\mathrm{k} T$ one must know at which power the dimple must be set to achieve this point. The current measurements do not provide this accurately enough for the first and second sound measurements, this is discussed in section 6.3.1. Finally, since all equations simplify if we could use this experiment as an effective cooling step (this was already discussed in section 3.2 ) in the future this technique could might be done more effectively. This is discussed in section 6.3.3.

### 6.3.1 Improving temperature measurements

This thesis showed that even though there is agreement between simulation and experiment at zero dimple power, the crossing happens at three times as low powers in the simulation as in the experiment. This means that to get an accurate estimate of at what pre-set dimple power what $\chi$ is achieved we need a more accurate measurement of the points in figure F.1. Figure F. 1 shows that the chemical potential can be determined more accurately than the temperature. A solution to this might be to measure the temperature in a separate experiment before the time of flight, to avoid the so-called overhauling effect. The advantage to this is that top imaging can be used, which has a six times larger resolution but cannot be used in a time of flight experiment with a long time of flight like 30 ms , because the area imaged in top imaging is too small for that. An accurate measurement of the temperature and so the crossing between $\mu$ and $\mathrm{k} T$ is absolutely crucial before one can do the sound experiments, to set the chemical potential and the temperature at the right value.

### 6.3.2 First and Second sound

Now that we are able to reach the regime where $\chi$ becomes in the order of one, it becomes interesting to look at first and second sound. Theory predicts two sound modes with a different behaviour, one where the condensate and the thermal cloud move in phase, and one where they move out phase. [19] See for example chapter 5 in the Ph.D. thesis of Silvio Koller [10, Chapter 5] for an explanation. In Figures 6.1 and 6.2 which were taken from his thesis the difference in behaviour between first and second sound is shown. We see that where normal sound is always density driven, first and second sound can also be a temperature wave, or a mix of both. To generate the wave we make a density perturbation. To observe both sounds, it is crucial to get $\chi$ exactly right. That's why the experiments of the previous section must be done first before we can use this to measure first and second sound.

### 6.3.3 A second use of the RF-knife

As discussed in section 4.1, lowering the RF-frequency delivered by an antenna is used to cool the cloud. This is known as the RF-knife. The useful thing about the RF-knife is that it only


Figure 6.1: Image depicting the speed of sound for First (blue) and second sound (red). At $\mu=0$ the condensate vanishes, therefore the red line stops there. The avoided crossing is located where the chemical potential and the temperature get in the same order. Here the two sound modes switch behaviour. Modified version of image by former group member Silvio Koller.


Figure 6.2: In this is image it is shown how much a sound wave is a temperature wave, and how much a density wave. We see that at really low and at really high temperatures second sound (blue) is mostly a density wave, and in-between it is a mixture of a temperature and a density wave. First sound is mostly a density wave at higher temperatures and a density wave at lower temperatures. Modified version of image by former group member Silvio Koller.
acts on the magnetic part of the potential. This means that the dipole trap is unaffected by it. An issue with the RF-knife as used until now is that at one point one starts to cut into the condensate. But because the dimple would be unaffected by it, it would be an idea to first cool using the RF-knife, then adiabatically turn on the dimple, and finally re-use the RF-knife again, once the dimple is turned on. This way one can safely cut off more of the thermal cloud, without affecting the condensate in the dimple. This is basically the same concept discussed in section 3.2, but this might be a way to do it more efficiently.

## ACKNOWLEDGEMENTS

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research partner and fellow master student Martijn van 't Woud. Mostly because he was always prepared to listen to any problem that I had, and by letting me explain my problems to him also clarifying the situation to me and that way bringing the solution by just being there. He was able to keep my chaotic head on track, and was always there for support. We collaborated on the particle flux research, and the part about particle flux in this report was written in collaboration with him. The collaboration went further outside the group resulting in a train trip through the beautiful Belgian and German Ardennes, including a visit to the city of Liège. I would like to acknowledge P.hD. student Pieter Bons for the help when Alexander was absent, for the collaboration in the experiments to calibrate the cross section presented in section 5.3 .2 and for the supervision in the Particle Flux project. Of the other students in our group I would like to thank honours bachelor student Damaz de Jong in particular, for the productive discussions we had about both our projects, and the input we both gave to each others projects. Next, an atmosphere couldn't be provided without your fellow students. In my project the master students Christian te Riet (it is important keep thinking about other parts of physics than BoseEinstein condensates and you are just the person to do that with) and Peter Elroy (who is Toepen champion during lunch?) and the Bachelor students Jasper Smits (who secretly wants to live on Kanaleneiland) and Stefan Korenberg (one day it will turn out you haven't taken the time at home to study the rules of Tichu will prove useful, believe me) were crucial in this. Without a good technical support an experiment is impossible, so I would like to thank Frits Ditewig, Paul Jurrius, Cees de Kok and Dante Killian for that. I also would like to thank Dante Killian and Paul Jurrius for fixing my computer, which was a lot more difficult then originally thought. Finally I would like to thank my direct family: My mother Marina Gladines for being all right with it if she wasn't in the acknowledgements, my father for telling me that he should be in it and my big sisters Roos and Pieternel.

## Appendices

## Appendix A

## Theoretical description at zero temperature

In section 2.1 the effective interaction given by $U_{\text {eff }}\left(\mathbf{r}, \mathbf{r}^{\prime}\right)=U_{0} \delta\left(\mathbf{r}-\mathbf{r}^{\prime}\right)$, with $U_{0}$ given by [8, page 120 ]

$$
\begin{equation*}
U_{0}=\frac{4 \pi a \hbar^{2}}{m} \tag{A.1}
\end{equation*}
$$

is used. In the Hartree approximation (valid in most experiments, see [8] section 6.1]) one simply says

$$
\begin{gather*}
\psi(\mathbf{r})=\sqrt{N} \phi(\mathbf{r})  \tag{A.2}\\
n(\mathbf{r})=|\psi(\mathbf{r})|^{2} \tag{A.3}
\end{gather*}
$$

where $N$ is the number of particles and $n(\mathbf{r})$ is the density distribution.Now if one assumes this interaction, and does the Hartree approximation it can be derived that we get

$$
\begin{equation*}
E(\psi)=\int d \mathbf{r}\left(\frac{\hbar^{2}}{2 m}|\nabla \psi(\mathbf{r})|^{2}+V(\mathbf{r})|\psi(\mathbf{r})|^{2}+\frac{1}{2} U_{0}|\psi(\mathbf{r})|^{4}\right) \tag{A.4}
\end{equation*}
$$

for the total energy of a large number of particles. Here $V(\mathbf{r})$ is the potential. A derivation is shown in [8, section 6.1]. Using integration by parts for the first term we obtain:

$$
\begin{equation*}
\int_{V} d \mathbf{r}|\nabla \psi(\mathbf{r})|^{2}=\int_{V} d \mathbf{r} \nabla \psi(\mathbf{r}) \cdot \nabla \psi^{*}(\mathbf{r})=\int_{\partial V} d A \psi^{*}(\mathbf{r}) \nabla \psi(\mathbf{r})-\int_{V} d \mathbf{r} \nabla^{2} \psi(\mathbf{r}) \psi^{*}(\mathbf{r}) . \tag{A.5}
\end{equation*}
$$

If the volume $V$ becomes the entire space, the surface integral must vanish for physical reasons and we now obtain for the energy:

$$
\begin{equation*}
E(\psi)=\int d \mathbf{r}\left(-\frac{\hbar^{2}}{2 m} \nabla^{2} \psi(\mathbf{r}) \psi^{*}(\mathbf{r})+V(\mathbf{r})|\psi(\mathbf{r})|^{2}+\frac{1}{2} U_{0}|\psi(\mathbf{r})|^{4}\right) . \tag{A.6}
\end{equation*}
$$

To solve this we must minimize the energy, but the total number of particles must remain constant giving us a constraint, so we can use Lagrange-multipliers. We have the following constraint:

$$
\begin{equation*}
\int d \mathbf{r}|\psi(\mathbf{r})|^{2}-N=0 \tag{A.7}
\end{equation*}
$$

We now define the following function:

$$
\begin{equation*}
\Lambda=E+\lambda\left(\int d \mathbf{r}|\psi(\mathbf{r})|^{2}-N\right) . \tag{A.8}
\end{equation*}
$$

Minimizing $E$ with respect to the constraint is now the same as minimizing $\Lambda$. So minimizing $\Lambda$ with respect to the shape of $\psi$ gives us the solution of $\psi$. Once a suitable shape is found and we insert this shape back into A.8, we see that at the minimum $\Lambda$ obtains the same value as E. So if we would change the constraint, that is changing $N$, by dN , the resulting change in $\Lambda$ is $\frac{d \Lambda}{d N}=-\lambda$. The new $\Lambda$ will then again be equal to the E at this different optimum for this different constraint. This means that the energy will have changed because of the alteration of the constraint by $d E=d \Lambda=-\lambda d N$. We recognise this equation from known thermodynamics and conclude that $-\lambda=\mu$, the chemical potential. Now minimizing with respect to a small change $\delta \psi^{*}$ in $\psi^{*}$ we obtain:

$$
\begin{align*}
& \delta \Lambda=\Lambda\left(\psi^{*}(\mathbf{r})+\delta \psi^{*}(\mathbf{r})\right)-\Lambda\left(\psi^{*}(\mathbf{r})\right)= \\
& -\frac{\hbar^{2}}{2 m} \nabla^{2} \psi(\mathbf{r}) \delta \psi^{*}(\mathbf{r})+(V-\mu) \psi(\mathbf{r}) \delta \psi^{*}(\mathbf{r})+\frac{1}{2} U_{0}\left(2 \psi(\mathbf{r}) \psi^{*}(\mathbf{r}) \psi(\mathbf{r}) \delta \psi^{*}(\mathbf{r})+\psi(\mathbf{r})^{2}\left(\delta \psi^{*}(\mathbf{r})\right)^{2}\right) \tag{A.9}
\end{align*}
$$

We now neglect terms of $\left(\delta \psi^{*}\right)^{2}$ and divide by $\delta \psi^{*}$ and use that $\frac{\delta \Lambda}{\delta \psi^{*}}=0$ is the definition of the minimum to obtain:

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m} \nabla^{2} \psi(\mathbf{r})+V(\mathbf{r}) \psi(\mathbf{r})+U_{0}|\psi(\mathbf{r})|^{2} \psi(\mathbf{r})=\mu \psi(\mathbf{r}) \tag{A.10}
\end{equation*}
$$

the time independent Gross-Pitaevskii equation. For our purposes we now neglect the kinetic term (Thomas-Fermi approximation) ${ }^{2}$ and obtain:

$$
\begin{equation*}
V(\mathbf{r})+U_{0} n(\mathbf{r})=\mu, \text { or } n(\mathbf{r})=0 \tag{A.11}
\end{equation*}
$$

To remain consistent this means the energy becomes:

$$
\begin{equation*}
E(\psi)=\int d \mathbf{r}\left(V(\mathbf{r})|\psi(\mathbf{r})|^{2}+\frac{1}{2} U_{0}|\psi(\mathbf{r})|^{4}\right) \tag{A.12}
\end{equation*}
$$

So this give us the formula for our density distribution (2.3) and our total energy A.12). It becomes clear that when $V$ becomes too large compared to $\mu$, which it always does in a harmonic potential, no solution exists any more to the first part of A.11 and the second part is needed.

[^9]
## Appendix B

## Simulation methods

In Figure 3.1 equation (3.1) is plotted for three different values of $(\mu-V)$, basically $(\mu-V)$ moves the graph up and downward. We can see a clear regime where there is only one root, giving us the density directly. We also see a clear regime where there is no solution. No solution actually doesn't mean anything, only that there are no parameters to make the equation valid. Instead we will just assume that there is no condensate in that regime and use the equations describing the no-condensate situation discussed in section 2.2 Further there is an in-between regime, in which there is a second solution appearing. Again there is no good reason to abandon this second solution, but for continuity reasons, we will just assume the solution on the right as the valid one. Next the close observer sees that by defining $n_{0}$ as 0 outside the solution region that this results in a little jump in density. What is basically going on is that these problems occur in the regime where we are either really close to the side of the condensate, or really close to condensation temperature. Therefore the density of condensate $n_{0}$ is really low, and it is in this regions that the Hartree-Fock approach is actually no longer valid. It is an approximation, and the problems occur in its limits. Nevertheless the density is sufficiently low over there, having a low contribution to the rest meaning that the error is small. The reason the choices mentioned before are made is to keep our model complete. Because $n_{0}$ is already small in this region actually any choice would be valid. The method that is used for solving the root of (3.1) is an iterative one, meaning that we just make a good guess for $n_{0}$ (because we will scan over our potential, this is usually the last result for $n_{0}$, which is already a very good guess, keeping the simulation quick and efficient) and then enter it in the right-hand side of the equation, directly yielding a new value for $n_{0}$. This new value can then again be put into the right-hand side, once again yielding a new value for $n_{0}$. In most situations this converges to the solution as is illustrated in Figure B.1. By setting an accuracy, and comparing the old and the new $n_{0}$ for this accuracy we know when to end the iteration. It is crucial to realise that if slope however is too big it doesn't automatically converges to our choice of solution. That is why only the right-hand solution will be reported.

In section 3.1 we have a function in which we have the chemical potential as an input which gives the number of particles as an output, like it the other way around. This is solved using a binary search method. The binary search method is explained in the next section. We write a function in which we put our wanted number of particles, and the temperature, (and of course our potential shape). Next we make a guess for the chemical potential. This gives us a number of particles that is either too high or too low. Now if its too high we multiply our chemical potential by two. If its too low we divide by two. We keep on repeating this procedure until we


Figure B.1: Illustration of the convergence of the recursive approach. By starting at a value and repetitively inserting it back in again the formula converges to the solution, as long as the slope is not too large.
have both, a value for the chemical potential that gives a too high particle number $\left(\mu_{H}\right)$ and one that gives a too low one. $\left(\mu_{L}\right)$. If after a set number of iterations we still did not obtain a $\mu_{L}$ we take the negative of the lowest $\mu$ tried so far. We then start multiplying by two until we reach a $\mu_{L}$. Now that we have a $\mu_{H}$ and a $\mu_{L}$ we know that the $\mu$ we want is in between these two values. We take the average, and again check whether the average gives a particle number that is too low or too high. If it's too low, this $\mu$ becomes our new $\mu_{L}$, if it's too high this becomes our new $\mu_{H}$. We then repeat the procedure, so take again the average of $\mu_{L}$ and $\mu_{H}$. This way we get closer and closer to our wanted number of particles. Once a certain set accuracy is reached, so once $\mu_{L}$ and $\mu_{H}$ are close enough together, the function gives a $\mu$. This way we have successfully inverted our function, and we now have a function that yields $\mu$ as a function of temperature, trap shape, and particle number.

In chapter 3 the $\mathrm{Li}_{3 / 2}$ needs to be used. In this appendix we take a closer look on how we calculate this function. This so-called polylogarithm can be calculated by using the following formula:

$$
\begin{equation*}
\operatorname{Li}_{s}(z)=\sum_{k=1}^{\infty} \frac{z^{k}}{k^{s}} \tag{B.1}
\end{equation*}
$$

The Li-function needs to be evaluated many times in the simulation. Evaluating (B.1) that many times would be to time-intensive. So we need another method. First of all, we realise that in every instance this function occurs it is followed by the exponential inside. This is why we introduce a new function, $\mathrm{eLi}_{s}$ defined as follows:

$$
\begin{equation*}
\operatorname{eLi}_{s}(x)=\operatorname{Li}_{s}\left(e^{-x}\right) \tag{B.2}
\end{equation*}
$$

In Figure B. 2 this function is plotted. Now we realise that for large $x$ this function is very well approximated by $\exp (-x)$ (see Figure B.3). We start of by choosing an accuracy, in our case we now say that the result of our numerical function may only differ 1.006 from the actual value. We now see that $\exp (-x)$ and $\mathrm{eLi}_{3 / 2}(x)$ do not differ more than this value for all $x$ larger than 4. So this means we have a function in the square made up for $x$ between 0 and 4 and $\mathrm{eLi}_{3 / 2}(x)$ between 0 and $\operatorname{Zeta}(3 / 2)$. Now the point we will now make is that the accuracy of $x$ is not


Figure B.2: The $\mathrm{eLi}_{3 / 2}(x)$ function plotted. Please not that it has a finite value of Zeta(3/2) at zero.


Figure B.3: $\exp (-x)$ approximates the $\mathrm{eLi}_{3 / 2}(x)$ function very well at large values.
important, because $x$ is the input value, but it is the accuracy in the result that is important. What we now do is define a numerical list of coordinates which we can interpolate, and it is important to look at the accuracy in the result. We make this list of coordinates as follows. First we look at the value that the function has at $x=4$. Next we keep on multiplying this value by 1.006 until we reach $\operatorname{Zeta}(3 / 2) \approx 2.612$. Now we have our result coordinates. Now we calculate the inverse of the $\mathrm{eLi}_{3 / 2}$ function to find the matching $x$-coordinates, and this provides us with the $x$-range. This way accuracy in the result is guaranteed, which it wouldn't have been when we would just have chosen equidistant $x$-coordinates. The accuracy of this approach is shown in Figure B.4. A similar method is used for the $\mathrm{eLi}_{5 / 2}$-function.

Next is the case of the integral. Of course one could take equidistant points and simply multiply the function values with the distance between them. But it is important to realise that when a function goes to zero at the regions far from the centre, that the contribution to the integral is less, while in the centre it has a large contribution. This means that an error at the regions far from the centre has a small contribution to the accuracy of the integral calculation, but a fluctuation in the centre has a large contribution. To compensate for this we choose not to use an equidistant quadrature. Instead, we use the fact that we have knowledge of the shape and use a quadrature with more points in the centre. We do this using a coordinate substitution, in such a way, that if the function were a Gaussian the transformation would just be integrating the function 1 . We then obtain the following integral:

$$
\begin{equation*}
Q=\sqrt{2} \pi^{3 / 2} \sigma_{z} \sigma_{\rho}^{2} \int_{0}^{1} d y_{1} \int_{-1}^{1} d y_{2} q\left(\rho\left(y_{1}\right), z\left(y_{2}\right)\right) \exp \left(\frac{\rho\left(y_{1}\right)^{2}}{2 \sigma_{\rho}^{2}}+\frac{z\left(y_{2}\right)^{2}}{2 \sigma_{z}^{2}}\right) \tag{B.3}
\end{equation*}
$$

where $q$ is the function we want to integrate and we integrate equidistantly over $y_{1}$ and $y_{2}$ where we calculate the used values for $\rho$ and $z$ using:

$$
\begin{equation*}
\rho=\sigma_{\rho} \sqrt{2 \ln \left(\frac{1}{1-y_{1}}\right)} \text { and } z=\sqrt{2} \sigma_{z} \operatorname{Erf}^{-1}\left(y_{2}\right) \tag{B.4}
\end{equation*}
$$

The concept of this method was originally described by Peter van der Straten in [21] but the formulas here differ because the effect of a small contribution at small $\rho$ because of the extra $2 \pi \rho$


Figure B.4: The numerical accuracy of our approximation. Because at small distances a function is always very well approximated linearly, and we do an interpolation the accuracy is actually much better then the set accuracy. On the right we just see the pattern of Figure B. 3 appear again.
in a cylindrical integral is included. In the simulation we actually do not do this integral at small distances because we guess that the dimple might alter the shapes too much from a Gaussian, so we choose to do inside the dimple a high resolution equidistant integral instead. Outside the dimple however we do use the scheme described above.

## Appendix C

## Simulation Results

In this appendix a selection of simulation results that might be useful for other people are presented. First, in section C.1 the results of the simulation in section 3.1.1.3 are presented. Next in section C. 2 the energy levels in two directions of a dimple trap are presented.

## C. 1 Simulating at the crossing

In this section some extra results from section 3.1.1.3 are presented, with data at the crossing between the chemical potential $\mu$ and the temperature $\mathrm{k} T$. In Figure C. 1 this is done for $w_{0}=$ $10 \mu \mathrm{~m}$ and Figure C. 2 for $w_{0}=20 \mu \mathrm{~m}$ width of our dimple

## C. 2 Energy levels in two directions

In section 3.2.3.2 the energy levels for a trap shaped like 3.2 are presented, for a uncoupled system in two directions. Here the rest of these results are shown, in Figures C. 3 to C. 9


Figure C.1: Images showing a bunch of simulation results at the crossing between $\mu$ and $\mathrm{k} T$ all as a function of the temperature which we start with or $w_{0}=10 \mu \mathrm{~m}$.


Figure C.2: The same as Figure C. 1 but now for $w_{0}=20 \mu \mathrm{~m}$.


Figure C.3: Energies as a function of the $n^{t h}$ energy level for different dimple depths in a radial direction. The orange line represents the harmonic.


Figure C.4: Difference between energy levels as a function of the $n^{t h}$ energy level for different dimple depths in a radial direction. The orange line represents the harmonic.


Figure C.5: Difference between the values in Figure C.4 and the harmonic. For high energies the harmonic is reached again.


Figure C.6: Energy levels for the axial direction as a function of dimple depth.


Figure C.7: Energy levels for the axial direction as a function of dimple depth.


Figure C.8: Difference between energy levels as a function of the $n^{t h}$ energy level for different dimple depths in the axial direction. The orange line represents the harmonic.


Figure C.9: Difference between the values in Figure C.8 and the harmonic. For high energies the harmonic is reached again.

## Appendix D

## Missing formulas

In this section a few long formulas are presented that were missing in the text: First of all the integration limits of our condensate density function in section 5.4.1.1

$$
y_{\text {limit }}(x, z)= \pm \frac{\sqrt{\begin{array}{l}
m R^{2} w_{0}^{2} W^{2} \omega_{\rho}^{2} L(c)  \tag{D.1}\\
+m R^{2} w_{0}^{2} z^{2} \omega_{\rho}^{2} L(c) \\
-2 m R^{2} z^{2} z_{R}^{2} \omega_{z}^{2}-2 m w_{0}^{2} W^{2} x^{2} \omega_{\rho}^{2}+4 \mu R^{2} W^{2}
\end{array}}}{\sqrt{2} \sqrt{m} w_{0} W \omega_{\rho}}
$$

with

$$
\begin{equation*}
L(c)=L\left(\frac{4 V_{0} W^{4} \exp \left(\frac{2\left(m z^{2} z_{R}^{2} \omega_{z}^{2}-2 \mu W^{2}\right)}{m w_{0}^{2} \omega_{\rho}^{2}\left(W^{2}+z^{2}\right)}\right)}{m w_{0}^{2} \omega_{\rho}^{2}\left(W^{2}+z^{2}\right)^{2}}\right) \tag{D.2}
\end{equation*}
$$

where $L(x)$ is the Lambert W function, defined as the solution of $x=L(x) e^{L(x)}$. Doing the integrations gives us for the integrated condensate density:

$$
\left.\begin{array}{l}
\int_{-y_{\text {limit }}(x, z)}^{y_{\text {limit }}(x, z)} d y n_{0}(x, y, z)=-\frac{1}{6 \sqrt{2} R^{4} W^{4} \omega_{\rho} \sqrt{m\left(W^{2}+z^{2}\right)}} \\
\left(w _ { 0 } z _ { R } e ^ { - \frac { 2 W ^ { 2 } x ^ { 2 } } { R ^ { 2 } ( W ^ { 2 } + z ^ { 2 } ) } } \left(-6 \sqrt{\pi} \sqrt{m} R^{3} V_{0} w_{0} W^{4} \omega_{\rho} \operatorname{erf}\left(\frac{\sqrt{R^{2} w_{0}^{2} L(c) \omega_{\rho}^{2}-\frac{2\left(R^{2}\left(m z^{2} z_{R}^{2} \omega_{z}^{2}-2 \mu W^{2}\right)+m w_{0}^{2} W^{2} x^{2} \omega_{\rho}^{2}\right)}{m\left(W^{2}+z^{2}\right)}}}{R w_{0} \omega_{\rho}}\right.\right.\right.
\end{array}\right)
$$

under the condition:

$$
\begin{equation*}
2\left(R^{2}\left(m z^{2} z_{R}^{2} \omega_{z}^{2}-2 \mu W^{2}\right)+m w_{0}^{2} W^{2} x^{2} \omega_{\rho}^{2}\right)<m R^{2} w_{0}^{2} \omega_{\rho}^{2}\left(W^{2}+z^{2}\right) L(c) \tag{D.4}
\end{equation*}
$$

## Appendix E

## Used parameters

In section 5.4 a variety of parameters is used. Most are determined in the other sections of chapter 5. Here they have been summarized.

Magnetic trap frequencies:
$\omega_{z}=2 \pi \cdot 1.5 \mathrm{~Hz}$ and $\omega_{\rho}=2 \pi \cdot 196 \mathrm{~Hz}$
Potential depth scaling constant:
$\mathrm{vf}_{V_{0}}=4.55666 \mathrm{sm}^{2}$
Focus of the trap and Rayleigh length:
$18.74 \pm 0.36 \mu \mathrm{~m}$ and $z_{R}=1.012 \pm 0.057 \mathrm{~mm}$.
The power correction factor because of windows and mirrors is 0.85 .
The pixel size is $8 \mu \mathrm{~m} / \mathrm{px}$ but the effective distance corresponding to a pixel is $23.053 \pm 0.734$ $\mu \mathrm{m} / \mathrm{px}$.

The effective Clebsch-Gordan coefficient:
$c=0.366 \pm 0.0083$
Sodium mass:
$m=3.81754 \cdot 10^{-26} \mathrm{~kg}$
Euler's Constant:
$e \approx 2.71828$

## Appendix F

## Scan over the range of power depths and starting temperatures

In this appendix the results for the chemical potential and the temperature of a scan over the entire range of dimple depths and temperatures is shown in Figure F. 1


Figure F.1: The results for the temperature (red) and the chemical potential (blue) for different starting temperatures and dimple depth's. In some conditions indeed a crossing is observed.

## Appendix G

## Particle Flux

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In this appendix, some experiments involving the beam of particles that loads the Bose Einstein Condensate set-up are described. This small research project was done in collaboration with fellow master student Martijn van 't Woud, and was co-written with him. First, a fast flux experiment (in which the Zeeman slower is turned off) is performed (section G.3, in which the way the oven operates as a function of temperature, is investigated. The particle flux is determined by absorption measurements and these measurements are compared to calculations coming from the vapour pressure. We are able to show that scattering plays an important role in the way the oven operates. In slow flux experiments (section G.4) (in which the Zeeman slower is turned on), the power of the Zeeman laser is varied to find the behaviour of both flux and particle speed. Also, by varying the currents through the Zeeman slower coils, the behaviour of speed and flux of the sodium particles as a function of current is examined in section G.4.4 We show that varying the current through the coils yields a more reproducible result than varying the Zeeman laser power. Also, we provide a model to calculate the speed as a function of current.

## G. 1 General description

If a beam of particles (for example photons) is moving through a medium, one can wonder how much of that beam is scattered. If one assumes that every particle in the medium has a certain collisional cross section and that if one particle in the beam is scattered when it enters the area around the particle in the medium given by that cross section, one can set up a differential equation which, when it is solved, gives the transmission of particles. This results in the so-called Beer-Lambert law for the transmission:

$$
\begin{equation*}
T=\exp \left(-\sigma \int \rho(x) d x\right) \tag{G.1}
\end{equation*}
$$

where $T$ is the transmission, $\sigma$ is the cross-section, $\rho$ is the density of the medium and the integral runs over the path of the beam. In the case of a homogeneous density $\rho_{0}$ this becomes:

$$
\begin{equation*}
T=\exp \left(-\sigma \rho_{0} d\right) \tag{G.2}
\end{equation*}
$$

where $d$ is the distance traveled by the beam.

## G.1.1 Resonant light

In the case of a beam of resonant light this means: $I=I_{0} \exp \left(-\sigma_{r} \int \rho(x) d x\right)$, where $\sigma_{r}$ is the resonant absorption cross section. For two-level systems, this is given by $\frac{3 \lambda^{2}}{2 \pi}$ (see [13, Eq. 3.12]). However, since this is not the case, we multiply with the Glebsch-Gordan coefficient, which is different for various polarizations and transitions and dependent on the detuning of this transition. So we obtain:

$$
\begin{equation*}
\sigma=c(\text { transition; polarization; } \Delta f) \cdot \frac{3 \lambda^{2}}{2 \pi} \tag{G.3}
\end{equation*}
$$

In the case of sodium for $\pi$-polarized light wavelength 589.16 nm , resonant to the $3^{2} S_{1 / 2} \rightarrow 3^{2} P_{3 / 2}$ transition, $c$ is given by (see [13, Table 2.1 and Eq. 3.13] and [22, Appendix D]):

$$
\begin{align*}
& c\left(F_{g}=2 \rightarrow F=1,2,3 ; \pi ; \Delta f\right)= \\
& \frac{1}{30\left(\frac{16 \pi^{2} \Delta f^{2}}{\gamma^{2}}+1\right)}+\frac{1}{6\left(\frac{16 \pi^{2}\left(\Delta f-3.4339 \times 10^{7}\right)^{2}}{\gamma^{2}}+1\right)}+\frac{7}{15\left(\frac{16 \pi^{2}\left(\Delta f-9.2649 \times 10^{7}\right)^{2}}{\gamma^{2}}+1\right)} \tag{G.4}
\end{align*}
$$

for the transition $F_{g}=2 \rightarrow F=1,2,3$ used in Section G. 3 and $c$ is given by:

$$
\begin{align*}
& c\left(F_{g}=1 \rightarrow F=0,1,2 ; \pi ; \Delta f\right)= \\
& \frac{5}{18\left(\frac{16 \pi^{2} \Delta f^{2}}{9 \gamma^{2}}+1\right)}+\frac{5}{18\left(\frac{16 \pi^{2}\left(\Delta f+3.4339 \times 10^{7}\right)^{2}}{9 \gamma^{2}}+1\right)}+\frac{2}{9\left(\frac{16 \pi^{2}\left(\Delta f+5.0148 \times 10^{7}\right)^{2}}{9 \gamma^{2}}+1\right)} \tag{G.5}
\end{align*}
$$

for the $F_{g}=1 \rightarrow F=0,1,2$ used in Section G.4 This means that if one measures what percentage $A$ of light is absorbed, that one can determine the density of that medium by using:

$$
\begin{equation*}
1-A=\exp \left(-\sigma_{r} \rho d\right) \rightarrow \rho=\frac{\ln \left(\frac{1}{1-A}\right)}{\sigma_{r} d} \tag{G.6}
\end{equation*}
$$

If the medium is a moving beam of sodium particles with speed $v$ and diameter $d_{1}$ that flux will be given by:

$$
\begin{equation*}
F=\frac{\pi d_{1} v \ln \left(\frac{1}{1-A}\right)}{4 \sigma_{r}} \cdot \frac{\cos (\phi)}{n} \tag{G.7}
\end{equation*}
$$

where $n$ is the number of passes, and $\phi$ is the incident angle. In Section G. $3 \frac{\cos (\phi)}{n}$ is $\frac{1}{2}$ and in Section G. 4 it is $\frac{1}{2 \sqrt{2}}$.

## G.1.2 Beam of sodium particles

In the case of a beam of sodium particles with a flux (in particles per second) moving through a low density gas of sodium particles this means that the incoming flux $F_{0}$ is related to the outgoing flux $F$ by:

$$
\begin{equation*}
F=F_{0} \exp \left(-\sigma_{s} \rho_{0} d\right) \tag{G.8}
\end{equation*}
$$

where $\sigma_{s}$ is the sodium-sodium scattering cross section, given by $100 \cdot 10^{-18} \mathrm{~m}^{2}$. [23, Table 2.1]

## G. 2 Experimental setup

A schematic representation of the experimental setup is shown in Figure G. 1 The setup consists of a recirculating oven, a Zeeman slower and a vacuum chamber. The sodium particle beam emerges from the vapor pressure of a heated sodium source in the first chamber of the oven (see Section G.3.1) and then enters the second chamber through a diaphragm of 6 mm , and then the third chamber through a diaphragm of 10 mm . [13]

The measurements of the fast flux (see Section G.3) are performed in the vacuum chamber located between the BEC chamber and the Zeeman slower, whereas the measurements of the slow flux (see Section G.4) are performed in the BEC chamber. Only the sodium atoms which pass through the corresponding chamber (i.e. have the correct solid angle) can be used in the measurement for the flux. This angle is (in first order, larger orders can be neglected) determined by the relation $\theta=\arctan \left(\frac{d}{2 l}\right)$ where $d$ is the width of the limiting component and $l$ is the distance from the 6 mm diaphragm to that component. The component which yields the smallest angle determines the width of the beam. In Section G. 3 the width of the beam is determined by the width of the small pipe between the cubic chamber and the Zeeman slower (with diameter 3.5 cm ). In Section G. 4 it is determined by the 17 mm thick pipe between the vacuum chamber and the cubic chamber.

For small angles, the width of the beam can be calculated by a small angle approximation, so $d_{0} \approx L \theta$ where $L$ is the distance from the 6 mm diaphragm to the point where the measurement was performed. Inserting the appropriate values yields a beam width of 1.8 cm for the slow flux and 3.6 cm for the fast flux. The value of 3.6 cm is confirmed by a photograph of the window in the vacuum chamber, of which the diameter is known.

## G. 3 Fast flux varying the oven temperature

In the experiment described in this section, absorption measurements are performed while the temperature of the oven is varied. In that way, the operation of the oven can be understood thoroughly. Apart from the general setup described in Section G.2 in Section G.3.1 the setup relevant for this experiment will be described. In Section G.3.2 the Lorentzian formula will be introduced (Section G.3.2.1) and a calculation which converts absorption values into flux, including a prediction of the expected flux, will be described (see Section G.3.2.2). In Section G.3.3 the results will be presented and in Section G.3.4, they will be discussed and conclusions will be presented.

## G.3.1 Setup

During the measurements of the fast flux, the Zeeman slower is turned off. A fraction (see Section G.2) of the particles passes through the cubic vacuum chamber (labeled 'Fifth chamber' in Figure G.1), where the measurements take place. This is done using a probe laser beam (using the transition $F_{g}=2 \rightarrow F_{e}=1,2,3$ ), which is split into two separate light beams using a polarizing beam splitter cube (PBSC). The polarization of the light beams can be tuned using a waveplate. One of these light beams only passes through the chamber twice (without crossing the particle beam), before hitting a photo diode. The other light beam crosses the particle beam twice with an angle of 90 degrees before hitting another photo diode. For a schematic representation, see Figure G. 2 The two photo diodes are connected to a differential amplifier, which measures the difference between the two signals. This amplifier is connected to an oscilloscope, which is used


Figure G.1: Overview of the setup used. The effective beam used for the fast flux (Section G.3) is yellow, the effective beam used for the slow flux (Section G.4) is brown.


Figure G.2: The fast flux setup. The two beam trajectories are shown, together with the sodium atom beam. Image came from [24]
for obtaining the data for the experiment described in Section G.3.3. For this experiment, the particle flux as a function of oven temperature is measured. The oven consists of 3 chambers. In the first chamber (where the temperature is varied between 295 and $325^{\circ} \mathrm{C}$ ), a block of sodium is present. Through a 6 mm diaphragm, the atoms enter the second chamber, where also a recirculation pipe is present (this is also shown in Figure G.1). Only the atoms with the correct solid angle (see Section G.2 will then enter the third chamber, to which a diffusion pump is connected. The other atoms remain in the second chamber, will liquefy and go back to the first chamber via the small pipe connecting these chambers.


Figure G.3: Example of a fit to the absorption data of the fast flux. Based on Eq. G. 4 one would expect 3 peaks. One of these peaks is actually really small. The reason we see four peaks (which correspond to four Lorentzians), is because a Doppler shift can occur between the two passes.

## G.3.2 Theoretical description

## G.3.2.1 Lorentzians

In Figure G.3, an example of a fit to the absorption data of the fast flux is shown. In order to obtain this fit, a superposition of four Lorentzian line shapes is used, which obey the general formula:

$$
\begin{equation*}
L(\omega)=\frac{\left(\Gamma_{e f f} / 2\right)^{2}}{\left(\omega-\omega_{0}\right)^{2}+\left(\Gamma_{e f f} / 2\right)^{2}} \tag{G.9}
\end{equation*}
$$

where $\Gamma_{\text {eff }}$ is the effective linewidth, $\omega_{0}$ are the resonance frequencies of the transitions (for the fast flux $\left.F_{g}=1 \rightarrow F_{e}=0,1,2\right)$ and $\omega$ is the laser frequency. From this fit, the absorption can be found and compared with Eq. G.4 In turn, from the absorption the flux (particles per second) of sodium atoms can be calculated.

## G.3.2.2 Flux and vapor pressure

As discussed in Sections G.1.1 and G.1.2 one can calculate the sodium-flux in particles per second from the absorption. In Section G.3.3 the absorption and thus the flux is measured as a function of oven temperature. One can also make a prediction of this flux based on the vapor
pressure of the oven. This vapor pressure will determine the pressure in the first chamber. The vapor pressure of sodium is given by:

$$
\begin{equation*}
P=P_{0} \cdot 10^{A-\frac{B}{T}} \tag{G.10}
\end{equation*}
$$

with $A=4.51961, B=5202.12 \mathrm{~K}$, and $P_{0}=1 \mathrm{~atm}=101325 \mathrm{~Pa}$. Assuming the ideal gas law this gives the density as $\rho_{1}=\frac{P_{0}}{k T} \cdot 10^{A-\frac{B}{T}}$. In Section G. 2 it is mentioned that the pipe connecting the Zeeman slower and the measurement chamber determines the angle at which particles move to reach the measurement point. This angle can be converted into a solid angle, and since this angle is very small, the fraction $h$ of that to the total solid angle gives the amount of particles moving in the right direction. Further, we know that the particles leave the first chamber by a hole of diameter $d_{0}=6 \mathrm{~mm}$ [25]. The flux at the measurement point is now given by:

$$
\begin{equation*}
F=\frac{\pi d_{0}^{2}}{4} v \rho_{1} h \tag{G.11}
\end{equation*}
$$

The speed $v$ is the same as the speed used in Section G.1.1 and therefore, its value does not matter for comparison, but for the axes of the graphs we will just use the thermal velocity of $\sqrt{\frac{3 k T}{m}} \cdot 1$ The flux now becomes:

$$
\begin{equation*}
F=\frac{\pi d_{0}^{2}}{16} \sqrt{\frac{3 k T}{m}} \rho_{1} \arctan ^{2}\left(\frac{d}{2 l}\right) \tag{G.12}
\end{equation*}
$$

where $d=3.5 \mathrm{~cm}$ is the diameter of the pipe and $l=2.181 \mathrm{~m}$ is the distance of the pipe to the hole in the first chamber. This flux is a function of temperature in the first chamber, which is varied in the experiment.
The problem with this formula is that there is no account for the fact that the flux passes a second and a third chamber after leaving the first. There is no reason to assume that there is absolute vacuum in these chambers and that means that a part of the flux will be lost. This loss can be calculated using the Beer-Lambert formula from Section G. 1 If $F_{0}$ is the flux we expected at first, the flux at the measurement point will be given by:

$$
\begin{equation*}
F=F_{0} \cdot \exp \left(-\sigma_{s} \rho_{2} l_{2}\right) \cdot \exp \left(-\sigma_{s} \rho_{3} l_{3}\right) \cdot \exp \left(-\sigma_{s} \int \rho_{z}(x) d x\right) \tag{G.13}
\end{equation*}
$$

where $l_{x}$ is now the length of a chamber and the subscripts respectively refer to the $2^{\text {nd }}$ and $3^{\text {rd }}$ chamber and the Zeeman slower. The integral is still in the formula because there is a pressure difference between the right-hand and the left-hand side of the slower, but if we assume a linear gradient this will just result in the average of the two sides and then

$$
\begin{equation*}
F=F_{0} \cdot \exp \left(-\sigma_{s} \rho_{2} l_{2}\right) \cdot \exp \left(-\sigma_{s} \rho_{3} l_{3}\right) \cdot \exp \left(-\sigma_{s} \bar{\rho}_{z} l_{z}\right) \tag{G.14}
\end{equation*}
$$

Considering Figure G. 1 of the setup, the densities here originate probably mostly from the first chamber particles that did not have the angle needed to reach the measurement chamber, and taking into account that the third chamber has a diffusion pump to keep the density low, we can assume that the $\exp \left(-\sigma_{s} \rho_{2} l_{2}\right)$ term coming from the second chamber probably has the largest contribution, also because it is right next to the first chamber and the only way out for the

[^10]particles is to the third chamber and through the little recirculation pipe. Unfortunately we can only measure the pressure in the third chamber and the Zeeman slower directly, and not in the second chamber. In the third chamber we measure a temperature of 301 Kelvin, and a pressure of $1.45 \cdot 10^{-7} \mathrm{mBar}$, corresponding with a density of $3.49 \cdot 10^{15} \mathrm{~m}^{-3}$. Using that it has a length of 23.5 cm we see that the exponent gives 0.92 so about $8 \%$ of the particles passing through the third chamber gets lost due to scattering. In the Zeeman slower we measure a pressure of $1 \cdot 10^{-8}$ mBar on one side and $2 \cdot 10^{-9} \mathrm{mBar}$ on the other, yielding an average density of $1.48 \cdot 10^{14}$ $\mathrm{m}^{-3}$. Using the length of 1.88 m and assuming room temperature we see that about $3 \%$ of the particles going through it get scattered in the Zeeman slower. Both are not even close enough to account for the factor 20 . This strongly indicates that we should find out the density in the second chamber. Since it is in equilibrium via a 10 mm hole with the third chamber, we can calculate it from the third chamber. Let's take a closer look at the third chamber. There are 3 connections with the outside world. There is the pump, which has a pump capacity of $1200 \mathrm{l} / \mathrm{s}$, there is the hole to the second chamber and to the third chamber. It can be derived the time derivative of the density in the third chamber is given by:
\[

$$
\begin{equation*}
\partial_{t} \rho_{3}=\frac{C_{23} \cdot \Delta \rho_{23}}{V_{3}}-\frac{C_{p} \cdot \rho_{3}}{V_{3}}-\frac{C_{3 z} \cdot \Delta \rho_{3 z}}{V_{3}}, \tag{G.15}
\end{equation*}
$$

\]

where $C$ is the pump capacity through the holes in volume per time and $C_{p}$ the capacity of the diffusion pump. The subscripts indicate the second and third chamber and the Zeeman slower. We know $\Delta \rho 3 z \approx \rho_{3}$ and assuming equilibrium, this means that $\partial_{t} \rho_{3}=0$ so since $\Delta \rho_{23}=\rho_{2}-\rho_{3}$ we obtain:

$$
\begin{equation*}
\rho_{2}=\frac{\left(C_{p}+C_{3 z}\right) \cdot \rho_{3}}{C_{23}}+\rho_{3} . \tag{G.16}
\end{equation*}
$$

Using a working formula for the $C$ of a circular opening, that is $C=0.113 \cdot R^{2} \sqrt{\frac{T}{M}} \cdot \mathrm{l} / \mathrm{sec}$ and that of a short tube, $C=0.31 \cdot \frac{R^{3}}{2.7 R+L} \sqrt{\frac{T}{M}} \cdot \mathrm{l} / \mathrm{sec}$ where $R$ is the radius, $L$ is the length of the short tube, all in mm and $M$ is the molar mass, and realizing $C_{23}$ is a series resistance of a hole of 10 mm and a short tube of diameter 34 mm and length 60 mm , we can now calculate the expected density in the second chamber. [26] This turns out to be $4.6 \cdot 10^{17} \mathrm{~m}^{-3}$. Realizing the length of the second chamber is 7.6 cm , we now see that fraction of particles that get through the second chamber $\exp \left(-\sigma_{s} \rho_{2} l_{2}\right)$ is 0.35 , indicating that we expect about $97 \%$ (!) of particles to get lost in the second chamber.

## G.3.3 Measurements \& results

In the experiment we measure the absorption of the light. As said before in Section G.3.2.2 we can calculate the flux from that. By varying the oven temperature we can obtain the flux of particles as a function of oven temperature. Next we can compare this with the expected flux of particles that leave the first chamber and have the correct angle to reach the measurement point based on the vapor pressure. Measurements were done in a temperature range of $290-330^{\circ} \mathrm{C}$. When looking at our data, we get about 55 times as less particles as expected, corresponding to absorption percentages of depending on temperature $2-5 \%$ where one would expect $55-95 \%$ absorption (a percentage to our knowledge never measured before in this setup).


Figure G.4: The particle flux versus the temperature of the bottom of the oven. The values in the frames indicate the fraction of saturation on which is measured. The solid line is the prediction, the dashed line is the fit. It can be seen that the values of the flux are highly sensitive to the density in the second chamber, as the difference in density is only $10 \%$. The difference in the graph is a factor 1.4 , because we are now in the highly nonlinear part of the exponent.

But this is of course without the scattering losses discussed before. Considering the numbers discussed in the previous section the expected flux should be lowered by a factor of 38 . This 'expected' ${ }^{2}$ line is plotted as the solid line in Figure G. 4 There is a large difference between the data and the expected line. But what we should take into account is that the solid line is for a huge part determined by the density in the second chamber, which was estimated in the previous section as $4.6 \cdot 10^{17} \mathrm{~m}^{-3}$ because this is calculated from the values in the second chamber. Its temperature was measured quite roughly, and its pressure was measured against a background pressure which can be nonzero (making the density in the third chamber higher, and so the density in the second chamber), so the value of $4.6 \cdot 10^{17} \mathrm{~m}^{-3}$ has an unknown uncertainty. Taking our densities in the third chamber and the Zeeman slower as correct for their part in the scattering processes (realizing that their influences where small anyway), we can make a fit to the data with the density in the second chamber as a parameter. This results in a value of $(3.8061 \pm 0.048) \cdot 10^{17} \mathrm{~m}^{-3}$. This fitted line is shown as a dashed line in Figure G.4 One thing to consider is if this second chamber density does not contribute to the flux, because it gives again a pressure itself. Calculating this shows that it does not significantly contribute, not surprising realizing that the densities in the first chamber depend on temperature in the order of $10^{20}-10^{21}$ $\mathrm{m}^{-3}$.

## G.3.4 Discussion

The expected value of $4.6 \cdot 10^{17} \mathrm{~m}^{-3}$ is not accurate because there are a certain number of values coming from the third chamber which are used but are not that accurately known, while we do not have a value to estimate an uncertainty in this.
The fit that was made originally had an estimated variance of 7954 , way larger than the expected 1. The uncertainty in the data points for the absorption however comes itself from a fit of four Lorentzian functions (see SectionG.3.2.1), and we see that the uncertainties coming from that in no way correspond to the fluctuations in the measurement. For example, for the temperature of 295 degrees Celsius where a lot of measurements were done we see a spread of $1.54 \cdot 10^{12}$ around an average of $9.52 \cdot 10^{12}$ particles per second, orders of magnitude larger than the uncertainties that come from the Lorentzian fit (which are in the order of $10^{10}$ ). To correct for these way underestimated uncertainties we multiply them with the square-root of the estimated variance, to get a corrected uncertainty for the final fit. Better would be to do more measurements at a certain temperature to get the spread in absorption from there.

## G.3.5 Conclusion

We can conclude that our model seems to correspond with the data in order. The order of the density that comes out for the second chamber is calculated in two distinct ways and seems to match within $30 \%$. We cannot tell how significant this difference is, because the uncertainty on the value of $4.6 \cdot 10^{17}$ particles per second is unknown. Due to conservation of mass, it is impossible to measure a higher particle flux than the solid line in Figure G. 4 if the data it is based on, is exact. This suggests that the uncertainty in the value $4.6 \cdot 10^{17}$ is quite large. But the most important conclusion to draw from these measurements is that we have to deal with enormous scattering losses. We lose $3 \%$ of our particles do to scattering in the Zeeman slower, $8 \%$ in the third chamber, and a much larger number of $\mathbf{9 7 \%}$ in the second chamber, proven by the densities determined in two separate ways in the second chamber.
We can conclude that if there is a way to lower the density in the second chamber, that it must

[^11]be possible to increase the flux with an order of magnitude. For example if the density in the second chamber is reduced by a factor of 10 , the flux increases with a factor of more than 20 . Also an increase in the density in the Second Chamber corresponds to a steep decrease in the flux, increasing this density with a factor of 10 brings the flux to about 1 particle every 10 seconds, so virtually none.

## G. 4 Slow flux

In the slow flux experiments, Doppler shift starts to play a role (see Section G.4.1). The setup components which are specific for these experiments are described in Section G.4.2 In Section G.4.3 an experiment describing the absorption as a function of Zeeman power is described. In Section G.4.4 the speed is being tweaked by varying the values of the compensation coils of the Zeeman slower.

## G.4.1 Theoretical description

Because the probe beam passes the atom beam with an angle of 45 degrees (see Section G.4.2), this induces a Doppler shift. The first time the probe beam crosses the atoms, the component parallel to the atoms is in the same direction as the velocity of the atoms (moving away from the source) so the observed frequency is smaller than the non-Doppler shifted frequency. When the probe beam comes back, the component parallel to the atom beam is in the opposite direction of the velocity of the atoms, so the observed frequency is larger than the non-Doppler shifted frequency. Together, this leads to a double Doppler shift which can be expressed as $\Delta f=$ $\frac{2 v \sin (\theta)}{c} f_{0}$ with $\Delta f=f_{2}-f_{1}$. An example of a fit (with Doppler shift) can be seen in Figure G. 5 Again, four Lorentzians were used for this fit, but two of them are just the other two Doppler shifted. So, we can only distinguish two Lorentzians in the characteristic of this transition. As discussed in Sections G.1.1 and G.1.2 one can calculate the sodium-flux in particles per second from the absorption. In this case the $\frac{\cos (\phi)}{n}$ is $\frac{1}{2 \sqrt{2}}$. The difference with the fast flux measurements is that we now have a lower speed. This speed is now no longer taken as the thermal velocity, but instead determined from the Doppler shift as mentioned above.

## G.4.2 Setup

During the experiments described in this section, the Zeeman slower is turned on. The Zeeman slower consists of a laser beam which slows the atoms, and of magnetic coils. Together, these give a Zeeman shift to the atoms which are on resonance, which causes them to slow. Now, the measurements are performed in the vacuum chamber (labeled 'BEC chamber' in Figure G.1), where (as can be seen in the picture in Figure G.6) the probe beam (now with the transition $\left.F_{g}=1 \rightarrow F_{e}=0,1,2\right)$ is again split into two beams using a PBSC. One beam is immediately directed into a photo diode. The other beam first passes the PBSC and then passes the atom beam with an angle of 45 degrees (two times, back and forth) before it is directed into another photo diode. These photo diodes are connected to a differential amplifier which also measures the difference between the two signals. These three signal outputs are connected to two oscilloscopes to provide data for the experiments described in Sections G.4.3 and G.4.4

File: ALL0029 $\mathrm{V1}_{\text {maxn }}=1.24 \pm$ sdmeanvoltage1[1] $\mathrm{V}_{\operatorname{man}}=1.23 \pm$ sdmeanvoltage2[1]


Figure G.5: Example of a fit to slow flux data with Doppler shift and an offset function, which has to be subtracted from the curve in order to get correct absorption values.


Figure G.6: The slow flux setup. 24]

## G.4.3 Experiment 1: Varying the Zeeman power

## G.4.3.1 Measurements \& results

In this experiment the power of the Zeeman laser is varied to measure the effects. Measurements were done in a range from about 40 to 120 mW . The effect of the laser power on the speed seemed to be marginal, though a small insignificant slope was observed, making the speed about $4 \%$ smaller at 120 mW than at 40 mW (see Figure G.7). As will be described in Section G.4.4 the speed can be varied by tuning the current in the coils in the second part of the Zeeman laser. Varying the Zeeman power does seem to have an effect on the absorption percentage. As said before, from these absorption percentages we can directly calculate the flux. Because of the fact that at these low absorption percentages $\ln \left(\frac{1}{1-A}\right) \approx A$ in very good approximation, and because of the fact that the speed is nearly constant, the flux versus Zeeman power will have a similar shape as the absorption percentage versus Zeeman power. That said, only the flux versus Zeeman power is shown (see Figure G.8). A raise in flux when increasing the Zeeman power is shown. But we also see a strange variation in the flux. This variation was also observed in a different experiment (done before the absorption measurement) where the intensity of the laser was varied. There, the variation in the maximal absorption was about $0.2 \%$ around an average of $0.6 \%$, about the same as we see here at the higher Zeeman values.


Figure G.7: A graph of the speed versus the Zeeman power. The speed seems to be nearly constant with a slight decrease and fluctuations that are larger than expected based on the uncertainty of the data.


Figure G.8: Graph showing the flux as a function of the Zeeman power. The data is noisy. We can conclude however that the Zeeman power influences the flux. The red uncertainties are estimated, the black in the Zeeman power are based on before and after measurements. The uncertainties in the fluxes are calculated from the Lorentzian fits taking into account the covariances.

## G.4.3.2 Discussion

As mentioned in Section G.3.4 the uncertainty in the data points for the absorption comes from a fit of two Lorentzian functions (or actually four, but these are Doppler shifted so two are relevant, see Section G.4.1, and we see that the uncertainties coming from that in no way correspond to the fluctuations in the measurement. In this measurement it is even more extreme than before with fluctuations of about 0.2 percent around an average of 0.6 when varying the power of the measurement laser. The same kind of fluctuations are observed when varying the Zeeman laser power. These variations don't seem to be that extreme in the experiment in Section G.4.3.18o there seems to be going something wrong when touching the lasers.

## G.4.3.3 Conclusion

We can conclude that we observe an increase in flux when increasing the Zeeman power. Large fluctuations make the result not significant. A lot more measurements are needed if one would want to eliminate them.

## G.4.4 Experiment 2: Speed tuning

## G.4.4.1 Setup

In this experiment, the effect on the Doppler shift (and velocity of the atoms) due to the settings of 2 different coils on the Zeeman slower is investigated. These coils include the 'Master' and 'Compensation' coils. In the experiment, only the current of both coils was varied (the voltage was kept constant). Both of these coils are located at the second part of the Zeeman slower. The Master coil induces a magnetic field at the second part of the Zeeman slower (next to the MOT-chamber, which is labelled BEC-chamber in Figure G.1), but the magnetic field lines are also present at the exact spot of the MOT (magneto optical trap), which is unfortunate because in this trap, the field should be zero. To achieve this, the compensation coil is present.


Figure G.9: Doppler shift as a function of the master at 7.0 A of the compensation coil. On the right-hand side the speed is shown. A clear negative slope is seen with a much larger impact than obtained when varying the compensation coils. A higher master means a lower speed. The fitted line is linear and the grey area is two times the uncertainty of where the line should be.

This component consists of one coil, which produces a second magnetic field which cancels the magnetic field induced by the Master coil at the spot where the MOT is. In this way, the second part of the Zeeman slower can be used for further slowing and at the same time, there is no magnetic field in the MOT. In this section, the focus will be on the effect of the Master coils. The effects of the compensation coils will also be described.

## G.4.4.2 Measurements \& results

As described in Section G.4.4.1 the speed of the particles can be tuned by varying the current in the coils in the second part of the Zeeman slower. It turns out that when varying the current in the Master, a huge effect on the speed is observed. The higher the current, the lower the
speed. Varying the current in the compensation coil has a smaller effect on the speed, going the opposite way, a higher current there means a higher speed.

Speed versus current In the measurements we varied two things, the current in the Master coil and the current in the compensation coil. The effect on the speed is measured. In Figure G.9, the speed as a function of the value of the current in the master is shown (at the right-hand axis, the left-hand axis shows the Doppler shift), at a constant amperage of 7A in the compensation coil. It seems to go almost linear. In general, the compensation coil is there to compensate the Master and cannot be chosen free in value as in this experiment. For that, it is interesting to see the whole plane of varying both the Master coil and the compensation coil. In Figure G.10 the Master is kept constant at 7A, and the compensation coil is varied. Again, the behavior seems linear. Next the Master is set constant at 10A and the compensation coil is varied. But now a problem appears. As described in Section G.4.1 the speed is determined by looking at the shift of the transition.


Figure G.10: Doppler shift when varying the compensation coil, at a constant Master of 7.0 A . The question marks correspond to measurements where the values were not fitted to Lorentzians, but directly read from the oscilloscope. The line seems linear again, but small compared to Figure G. 9

But now, this shift becomes so small that the two start to overlap. It is now no longer possible to distinguish more than two Lorentzians. The question now is if the distance between these two comes from the internal characteristics of the transition or from the Doppler shift. To determine this we look at the other measurements where we do have four Lorentzians and realize that the distance between the two internal peaks must be constant. We now calculate from all other measurements this short peak distance. It gives a weighted average of 38.52 MHz with an Standard Deviation On Mean of 0.62 MHz and a weighted standard deviation of 2.25 MHz . The
value for the internal peak distance could quite well differ two SDOM for a single measurement, but two times the Standard Deviation itself is quit improbable. That means that if a value for the distance is larger than 43 MHz we can quite surely say that it can't be an internal peak distance, so it must be a Doppler shift.


Figure G.11: Peak distances when varying the compensation coil, at a constant Master of 10 A . If a point is outside the light-gray area, we are sure it is a Doppler shift and it corresponds to a speed. If it is in there, it can be a characteristic of the peak.

In Figure G. 11 it becomes clear that 4 of the 6 points are surely Doppler shifts. The other two can well be internal peak distances so they will not further be included. Now one observes linear lines in Figure G.10 in Figure G.11 and Figure G.9. We now generalize this linear behavior to the idea that all can be approximated linear in the $[6 \mathrm{~A}, 10 \mathrm{~A}] \mathrm{x}[6 \mathrm{~A}, 10 \mathrm{~A}]$ plane. If every line in one variable must be linear while keeping the other constant it can be proven that there is only one function to fit these conditions. That function is: $v=C_{1} I_{M} I_{S}+C_{2} I_{M}+C_{3} I_{S}+C_{4}$. Fitting this to our data turns out to work quit well. The resulting function is:

$$
\begin{align*}
& v=\left(-0.266 I_{C} I_{M}+3.635 I_{C}-14.862 I_{M}+160.99\right) \\
& \pm \sqrt{\begin{array}{c}
0.0170252 I_{C}^{2} I_{M}^{2}-0.318035 I_{C}^{2} I_{M}+1.51059 I_{C}^{2}-0.258096 I_{C} I_{M}^{2} \\
+4.82355 I_{C} I_{M}-22.915 I_{C}+0.998313 I_{M}^{2}-18.6634 I_{M}+88.6668
\end{array}} \mathrm{~m} / \mathrm{s} \tag{G.17}
\end{align*}
$$

where $I$ must be inserted in amperes. Here, $I_{M}$ is the current going through the master in ampere, and $I_{C}$ the current through the compensation coil in ampere.


Figure G.12: Flux versus speed. The line seems quite linear but is not when looking closer at it. The linear component comes from the fact that the flux scales with the speed. The nonlinear component is shown in Figure G. 13 The uncertainties are calculated using the Lorentzians with appropriate covariances.

Absorption versus speed Also one could be interested what the effect of this changing speed has on the flux. This is shown in Figure G.12. It shows the flux versus the speed. It seems mostly linear, which is not that surprising since the flux scales with the speed in the original formula as explained in Section G.3.2.2 But looking closer at the linear line is only visual, as it would imply that the absorption is constant, which it is certainly not, as can be seen in Figure G.13. This figure shows the maximal percentage of absorption versus the speed. Since $\ln \left(\frac{1}{1-A}\right) \approx A$ at these absorption percentages this scales almost directly with the flux divided by the speed, giving the number of particles per length unit, this is given at the right-hand axes. The behavior seems to give a lower number of particles at a lower speed, making the effect of lower flux at lower speed even more extreme. The behavior turns out to be described quite well on this range by the following working formula:

$$
\begin{align*}
& A_{\max }=0.167 \ln \left(\frac{v-17 \mathrm{~m} / \mathrm{s}}{0.011 \mathrm{~m} / \mathrm{s}}\right) \\
& \pm \frac{\sqrt{\begin{array}{r}
0.0273127 v^{2}+0.000441967 v^{2} \ln ^{2}(92.81 v-1578.52)-0.00689588 v^{2} \ln (92.81 v-1578.52) \\
-0.960201 v-0.015034 v \ln ^{2}(92.81 v-1578.52)+0.12785 \ln ^{2}(92.81 v-1578.52) \\
+0.238384 v \ln (92.81 v-1578.52)-2.05965 \ln (92.81 v-1578.52)+8.44071
\end{array}}}{v-17.0081} \% \tag{G.18}
\end{align*}
$$

but there seems to be no physical reason for this.


Figure G.13: Absorption versus speed. Since at these absorption percentages $\ln \left(\frac{1}{1-A}\right) \approx A$ this scales with the flux/v = particles per length unit too. This is shown on the right-hand axis. Uncertainties in the data come from the Lorentzian fits. The working function is also shown, the gray area is the two times uncertainty in where the line should be.

## G.4.4.3 Discussion

Although the behavior of the speed versus the current seems fairly linear, fitting a linear line in Figure G. 9 corresponds with an estimated variance of 80 . This may indicated that the uncertainties in speed were underestimated, which seems reasonable, since they only come from the original fit of Lorentzians, while accounting for the correct covariances, and do not include any other variations. Still it makes the claim that it is linear only valid to a certain approximation, making that we can only trust the model in a certain range of master currents and of compensation coil currents. It does however seem valid (to a certain approximation) to use the model in the entire plane between 6 and 10 A for both the master and the compensation. The model for the absorption is a true working formula and only valid in the measured range. Further one must realize that we have to do with working formulas and not with theoretical models in this section.

## G.4.4.4 Conclusion

We can conclude that we obtained a model of getting the speed of the particles from the values of the currents. Further we see that lowering the speed costs flux, not only because the flux scales with the speed, but also because the number of particles per meter drops. The working function of this seems to suggest a negative flux for speeds lower than $17 \mathrm{~m} / \mathrm{s}$, something that is completely non-physical, so it would be interesting to see what happens at lower speeds. Unfortunately, as explained in 'Speed versus current' it is impossible to measure speeds (and distinguish them from the characteristics of the transition) below $18 \mathrm{~m} / \mathrm{s}$.

## G. 5 Conclusion

The most important conclusions from the measurement of the fast flux (with varying oven temperature) is that the loss in the Second Chamber of the oven is by far the most determining factor in terms of particle loss. If the density in the Second Chamber is lowered, then the particle flux from the oven can become much larger. Another way to increase the flux is to increase the Zeeman power (this follows from the slow flux measurements). Also, as illustrated in Figure G.12, because the flux scales with the speed, lowering the speed also lowers the flux, since it can be seen in Figure G. 13 that lowering the speed does not increase the density.

## Bibliography

[1] Oliver Penrose and Lars Onsager. Bose-einstein condensation and liquid helium. Phys. Rev., 104:576-584, Nov 1956.
[2] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell. Observation of bose-einstein condensation in a dilute atomic vapor. Science, 269(5221):198-201, 1995.
[3] K. B. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle. Bose-einstein condensation in a gas of sodium atoms. Phys. Rev. Lett., 75:3969-3973, Nov 1995.
[4] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet. Evidence of bose-einstein condensation in an atomic gas with attractive interactions. Phys. Rev. Lett., 75:1687-1690, Aug 1995.
[5] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet. Evidence of bose-einstein condensation in an atomic gas with attractive interactions [phys. rev. lett. 75, 1687 (1995)]. Phys. Rev. Lett., 79:1170-1170, Aug 1997.
[6] Wolfgang Ketterle. Nobel lecture: When atoms behave as waves: Bose-einstein condensation and the atom laser. Rev. Mod. Phys., 74:1131-1151, Nov 2002.
[7] E. A. Cornell and C. E. Wieman. Nobel lecture: Bose-einstein condensation in a dilute gas, the first 70 years and some recent experiments. Rev. Mod. Phys., 74:875-893, Aug 2002.
[8] C. Pethick and H. Smith. Bose-Einstein Condensation in Dilute Gases (Second edition of 2008). Cambridge University Press, 2002.
[9] D.J. Griffiths. Introduction to quantum mechanics. Pearson Prentice Hall, 2005.
[10] S.B. Koller. Experiments on hydrodynamic transport in ultra-cold bose gasses. PhD thesis, Universiteit Utrecht, 2012.
[11] J. Stenger, D.M. Stamper-Kurn, M.R. Andrews, A.P. Chikkatur, S. Inouye, H.-J. Miesner, and W. Ketterle. Optically confined bose-einstein condensates. Journal of Low Temperature Physics, 113(3-4):167-188, 1998.
[12] B.R. Johnson. New numerical methods applied to solving the one dimensional eigenvalue problem. Journal of Chemical Physics, 67(486), 1977.
[13] R. Meppelink. Hydrodynamic excitations in a Bose-Einstein condensate. PhD thesis, Universiteit Utrecht, 2009.
[14] S. Hooker and C. Webb. Laser Physics. Oxford Master Series in Condensed Matter Physics Series. OUP Oxford, 2010.
[15] Rudolf Grimm, Matthias Weidemeller, and Yurii B. Ovchinnikov. Optical dipole traps for neutral atoms. 1999. cite arxiv:physics/9902072 Comment: Review Article, to appear in Advances in Atomic, Molecular and Optical Physics; REVTeX, 39 pages, 21 figures, 2 tables.
[16] CVI Melles Griot. Gaussian beam optics, 2009.
[17] Martijn van 't Woud BSc. Spin drag below the bose condensation temperature. Master's thesis, Universiteit Utrecht, 2013.
[18] Y. Castin and R. Dum. Bose-einstein condensates in time dependent traps. Phys. Rev. Lett., 77:5315-5319, Dec 1996.
[19] Laszlo Tisza. The theory of liquid helium. Phys. Rev., 72:838-854, Nov 1947.
[20] David Foster. Partial derivatives treating a complex number and its complex conjugate as independent variables: Why the trick works. David Foster's Various Notes on Math and Physics, Oct 2012.
[21] Peter van der Straten. Trap averaging. May 2011.
[22] H.J. Metcalf and P. Van Der Straten. Laser Cooling and Trapping. Graduate Texts in Contemporary Physics. Springer-Verlag GmbH, 1999.
[23] Wilbert Lucas Kruithof. Laser trapping of sodium isotopes for a high-precision $\beta$-decay experiment. PhD thesis, Rijksuniversiteit Groningen, 2012.
[24] Luuk Holt MSc. Master thesis. Master's thesis, Universiteit Utrecht, 2012.
[25] K.M.R. van der Stam. Superradiant scattering of laser light from a Bose-Einstein condensate. PhD thesis, Universiteit Utrecht, 2006.
[26] Dr P.K. Rol. Inleiding tot de vacuümtechniek.


[^0]:    ${ }^{1}$ Box shaped, harmonic, Gaussian as long as it is a trap
    ${ }^{2}$ Not exactly zero, but right above it, because particles in higher states also have interaction energy and contribute to the positive term in the chemical potential that way, but this effect is small.

[^1]:    ${ }^{1}$ It is actually $2 n-n_{\mathbf{p}}$

[^2]:    ${ }^{2}$ In a trap that does not confine for all energies, but up to a maximum, as the trap in section 3.2 one would be required to evaluate this up to the maximum energy allowed in the trap

[^3]:    ${ }^{1}$ It can be proven that any reversible adiabatic process is an isentropic process. By executing the process slowly we obtain this reversibility.

[^4]:    ${ }^{2}$ Completely filled is used a lot in the following sections, the expression is associated with potentials that do not go to infinity. In other words there is a maximum energy at which particles are still trapped. A completely filled trap is a trap that is filled up to this energy

[^5]:    ${ }^{1}$ scheduled for release in October 2013
    ${ }^{2}$ see for example exercise 3.1 in 14

[^6]:    ${ }^{3}$ For a perfect Gaussian beam the Rayleigh Length is given by $f r a c \pi w_{0}^{2} \lambda$, but we don't not assume that in this report, since we do not necessarily have a perfect Gaussian beam

[^7]:    ${ }^{1}$ It is of course possible to propagate these uncertainties. One would have to redo the fit over and over again, with the uncertainties of the fixed parameters added and subtracted, and every combination over and over again. However since fitting with the condition in D. turns out to be quite time-intensive, and the number of combinations with all the added and subtracted parameters is large, we do not do this. We expect the resulting systematic error to be small.
    ${ }^{2}$ To be clear we are talking to the contribution to the uncertainty in the density, so this won't help with the uncertainty in the chemical potential

[^8]:    ${ }^{3}$ Two to allow fluctuations around the one, but still get out non-physically large numbers

[^9]:    ${ }^{1}$ Here we treat $\psi$ and $\psi^{*}$ as if they are independent from each other. It can be proven that this yields the correct result 20].
    ${ }^{2}$ When the number of atoms is large and the interaction is repulsive, the kinetic term becomes small compared with other energies. The neglecting of this term is the Thomas-Fermi approximation.

[^10]:    ${ }^{1}$ Note that this velocity occurs on both equations and its value is of no importance. Further, $\sqrt{\frac{8 k_{B} T}{m \pi}} \approx$ $\sqrt{\frac{3 k_{B} T}{m}}$.

[^11]:    ${ }^{2}$ Of course this expected flux includes measurements of the pressure and temperature at different places in the setup.

