Dynamic pricing at Theater Bellevue

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Contents

1 | Introduction

Ik hoef alleen maar even zo te doen
Subsidie-stop
En de hele cultuur
Houdt in een keer op

't Is jammer

— Annie M.G. Schmidt
Ik hoef alleen maar even zo te doen
From the musical *En nu naar bed*

1.1 Background and motivation

The city council of Amsterdam is going to decrease their expenses on the arts [6]. There is only money left for four metropolitan theatres (theatres in the centre of Amsterdam) of which one is a debating centre [9]. This means that five theatres have to close and that the remaining theatres have to economise. The theatres that have to compete with each other are De Kleine Komedie, Jeugdtheater de Krakeling, MC Theater, Theater Bellevue, Theater de Engelenbak, the Tropentheater, De Balie, Felix Meritis and Pakhuis de Zwijger. Furthermore it will get more difficult to receive subsidies from private funds since due to other municipal and national cost reductions more theatres and theatre companies are fishing in the same waters.

With the increase of the ageing population there are more people with spare time and at the same moment their disposal income and their purchasing power decreases [20]. On the one hand we have a new source of money since the group is growing, but on the other hand they have a smaller income and will not spend much money on theatre. There is a slight decrease in purchasing power for other parts of the population although there is a small increase in the average income. These developments and the decrease in occupation degrees encourage people in the arts sector to find new ways to attract new public and generate more revenue.

1.2 Theater Bellevue

Bellevue was founded in 1840. The building hosted the Gentleman's society Concordia and the Ladies society Bellevue. In later years political meetings, boxing matches, concerts and performances all took place in Bellevue. The building was changed into the largest hall complex of Amsterdam in 1938 and in the subsequent years Bellevue became the most popular hall complex of Amsterdam.
After being transformed into television studios and being transformed in a theatre, Theater Bellevue started cooperating with the Nieuwe De La Mar in 1987. This cooperation holded until the Nieuwe De La Mar closed in 2006 and since then Theater Bellevue has been an independent theatre.

Theater Bellevue is a studio theatre, which means that the distance between stage and public is very small. These kind of theatres are often smaller than other theatres like municipal theatres. Theater

Bellevue consists of three halls: the Grote Zaal, Klein Bellevue and the Paloni Zaal.
The Grote Zaal can host approximately 250 people. Many different genres like drama, dance, musical theatre, mime, puppet theatre and youth theatre are shown in the Grote Zaal. This hall is used for performances of both known theatre companies and young talent. Seats are allocated to the public at the moment tickets are bought, but there are no ranks. So people get a row and seat number when buying tickets, and all seats are priced the same since the seats are ascending and the view on the stage good no matter where someone is seated.
Klein Bellevue is a smaller hall which can host approximately 100 people and most performances in Klein Bellevue are cabaret performances. Klein Bellevue is a link for starting comedians between the community theatres and the larger municipal theatres, but it is also a place where established comedians come to try new material. The seats in Klein Bellevue are not allocated in contrast with the Grote Zaal, so the public will not get seat and row numbers, but as soon as the doors open any seat can be taken. People arriving earlier have an advantage over people arriving last-minute.
The Paloni Zaal is used to show the Bellevue Lunch performances. There performances are produced by Theater Bellevue and sometimes co-produced with other producers. The writers and actors are both young talent and established actors and well known writers. The seat plan of the Paloni Zaal can be changed for each and every production and the maximal capacity differs between 50 till 70 people.

Theater Bellevue should cut costs in the next years, like many other theatres in The Netherlands. The Amsterdamse Kunstraad advised the city council to still grant Theater Bellevue a subsidy, but the subsidy is cut with 24% with respect to the previous year. Theater Bellevue will remain to exist as a metropolian theatre together with Jeugdtheater de Krakeling, De Kleine Komedie and debating centre De Balie. The advice of the Amsterdamse Kunstraad is adopted by the city council in November 2012 and Theater Bellevue has to deal with a much smaller subsidy starting from January 2013.
To face this, cost reductions are made and Theater Bellevue is looking for other ways to yield more revenue. Dynamic pricing and revenue management could be promising means to deal with a smaller subsidy.

1.3 Dynamic pricing and revenue management

Dynamic pricing is a part of revenue management. Revenue management deals with the decisions a salesman has to make before he can actually sell his products. When should he sell his product and for what price? He wants to sell his product at the most favourable moment, but what will the future do? The price should be such that the revenue is as high as possible, but customers should not be chased away by the price. Revenue Management is about selling the right product to the right customer for the right price on the right time.
Dynamic pricing is an element of revenue management which deals mainly with the price: what is the price of the product for different periods for different customers [21].

Revenue management in itself is an old idea, but it is a quite new branch of science and technology and it developed in the 70s of the previous century in the airline industry. The Airline Deregulation Act of 1978 was the start for Revenue Management. This act caused a loss of control of the prices for the U.S. Civil Aviation Board (CAB). Airlines could now choose their own prices instead of standard prices of the CAB. Low-cost airlines were set up and flying became quite cheap. One of these low-cost airlines was PeopleExpress. They sold tickets for prices that were 50 to 70% lower than the prices of established airlines. The established airlines lost many of their customers to the low-cost airlines. A new strategy was needed for the established airlines to survive. Robert Crandall, vice president of marketing of American Airline recognised that his airline had already budget seats for a very low price, because the air planes regular flew with a low occupation degree. The seats that were not occupied in the first place could be sold for a low price. These tickets had some restrictions. The tickets should be bought 30 days before departure and there were only a limited number of tickets available. This strategy worked

as it should, but American Airlines had problems with implementing the strategy. They realised that not all flights are the same and that the number of available budget tickets differs per flight.
American Airline launched the DINAMO system in 1985. This system made it possible to offer tickets for prices that could match the ticket prices of PeopleExpress or that were even lower. The introduction of DINAMO was a great success for American Airline. Their income and profit increased considerably, while PeopleExpress went bankrupt in 1986.
Revenue management has been applied often since then. Not exclusively in the airline industry, but almost every branch uses revenue management. The hotel branch, car rental, casinos, cargo transport, ticket sales and many other branches use revenue management [5].

Research in revenue management continues. Important research fields are forecasting, overbooking research and pricing [18].

- Forecasting is a quite important part of revenue management in the airline industries, since it is of direct influence on the booking limits and therefore on the expected profits. As a part of forecasting research is done in demand distributions, arrival processes and unconstraining. Especially unconstraining is an important field within forecasting, since all historical data is censored due to booking limits and capacitiy limits. Unconstrained data shows the real demand and can be used for forecasting.
- Overbooking is one of the largest problems in the airline industry. The early research done was mostly about controlling the probability of denied boarding, while nowadays it is mostly about determining a booking limit that maximises the expected revenue.
- Most of the articles on pricing are about pricing at industry level, instead of at revenue management level, but both type of article are helpfull. In the airline industries it is now common to include pricing in their revenue management strategies and not approach it as a separate process.

According to Cross [5], revenue management can be applied to almost any business

- if the business has products that have a perceived value to their customers,
- if there is a limited supply of these products,
- if there is a limited demand,
- and if there are alternatives.

Performances at Theater Bellevue have a fixed capacity so there is a limited supply of tickets, not everyone wants to visit the performances and if a performance is sold out there are other performances to visit at Theater Bellevue or other theatres. Furthermore performances have a certain value for each customer.
According to Andersen [1] revenue management can be applied to any business if there is:

- *"Perishable inventory and/or seasonal demand, so that the timing of a sale is important;*
- *High fixed or sunk costs, and relatively low marginal costs of selling an additional unit;*
- *Fixed capacity, either overall or in the short term;*
- *Advance purchase (or, at least, reservation) of products or services;"*

There are similarities between both sets of requirements. Theater Bellevue also satisfies the requirements from Andersen and theoretically revenue management and dynamic pricing can be applied to Theater Bellevue.

A revenue management system includes in general the following steps [21]:

1. Data collection
2. Estimation and forecasting
3. Optimisation
4. Control

When implementing revenue management one cycles through these steps and revenue management is a continuous process.

1.4 Dynamic pricing in the arts sector

Dynamic pricing is gaining interest in the arts sector. It has already been applied to many organisations in the United States, like the Chicago Symphony Orchestra[17], the Center Theatre Group and the L.A. Opera[15] both in Los Angeles. It has been applied in London at dancing centre The Place as well and also in The Netherlands it is gaining interest. The Nederlands Philharmonisch Orkest are researching the possibilities of implementing dynamic pricing and revenue management [3], Albert Verlinde uses dynamic pricing for the musical Shrek [4], Orkater and the Stadsschouwburg Amsterdam have already experimented with pricing [22] and Cees Langeveld and the Bureau Promotie Podiumkunsten have been writing a book about dynamic pricing [12], which is published in January, 2013 [11].

The Place

The most interesting example for Theater Bellevue is the centre for contemporary dance The Place, which is situated in the centre of London. There are 300 seats and there are approximately 150 performances a year. In October 2001 the centre reopened after a renovation. Before the renovation there were two types of tickets, namely the full price and a concession. These prices varied for every performance and they depended on the experience of the performer. The more established the performer, the higher the price. In general the prices varied between £8 and £12. Research told them that some people would have paid more for the tickets, while others would have bought tickets if the prices were lower.
The Place is a studio theatre so it does not really matter where you are seated, the view on the stage is always good and the visitors can decide for themselves if they want to be closest to the action, sweat and saliva on the front row or more to the back for the overview. The tickets are unreserved tickets and this should be preserved. After some budget airline flights director John Ashford realised that dynamic pricing could be used to solve the pricing problem. That dynamic pricing could solve other problems like the notorious late-booking audience was a fortuitous coincidence. Approximately 50% of the tickets was sold on the day of the performance [24].
Now there are five main pricing categories with prices varying between £5 and £15. Every price has it own advantages and disadvantages. The cheapest ticket should be bought at least a week in advance and cannot be returned or changed while the most expensive tickets could be bought last-minute and could be returned or changed and children could go for half the price. If a performance is ready for sale then there are at least 20 tickets available for each price. The remaining tickets can be allocated between the prices as wanted.
The strategy was very effective for The Place. The ticket yield increased by 14%, the ticket sales increased by 28% and the net income by 44%. The customer rating improved as well, now being 85% compared to 75% earlier.

Center Theatre Group

The Center Theatre Group in Los Angeles is a non-profit organisation that produces and develops performances in three theatres: the Mark Taper Forum, the Kirk Douglas Theatre and the Ahmanson Theatre.

After a pricing study with The Pricing Institute in 2010, the Mark Taper Forum changed from a two-price house into a three-price house and the house was divided into 28 zones within these three pricing categories. Each zone within a categorie was assigned a set of rules stating if seats could be discounted or used for complimentary seats. For seats with the highest occupation degree at the prime locations consessions were not possible and they could not be comped. The price of the zones could be changed for every performance such that the revenue could be maximised and the best seats were only available to those who became a donor as well.
By re-scaling and re-zoning the seats and by requiring a donation the ticket price increased on average by 9%, the subscription renewal rate became 83% and the subscription revenue increased by 28% which is 2.5 million dollar.

1.5 Research questions and plan of approach

In this Master's thesis I will investigate the possibility to implement dynamic pricing at Theater Bellevue. The main questions will be:

- Is it possible to enlarge the revenue of Theater Bellevue by implementing dynamic pricing?
- And if it is possible, in which way?

The following subquestions will help to answer the main question:

- How is the current pricing situation at Theater Bellevue organised?
- What information can we extract from the historical data?
- Is it possible to estimate the real demand? And if so, how?
- Is it possible to find an optimal pricing strategy? And if so, how?

The plan of approach to answer these questions is as follows:

1. Investigate the current pricing situation by reading earlier research reports and interviewing staff.
2. Perform data analysis on the historical data and use this data to estimate the demand, by unconstraining the data
3. Construct a model that is based on the estimation of the demand.
4. Validate the model.
5. Give recommendations based on the model validation and the data analysis.

2 | Present situation at Theater Bellevue

2.1 Present pricing policy

At the moment it is possible to buy tickets in different ways. Tickets can be bought on-line and these tickets have to be paid directly. The tickets can be sent to the buyer's home address, can be picked up at the ticket booth of the theatre or can be printed as an e-ticket. If tickets are bought by making a phone call, the tickets are not bought yet and only a reservation is made. The tickets have to paid at the ticket booth within 30 days. The reservation is cancelled if tickets are not paid within this term and the seat can be booked again. If a performance is sold out it is possible to apply for the waiting list. If reservations are not picked up, the tickets are sold to people on this waiting list. It is also possible to buy a ticket directly at the ticket booth. Right now it is more expensive to buy tickets on-line since an extra administration fee is required which is not the case when paying at the ticket booth.

It is also possible to buy the so called 'uitgaansabonnementen' at the Amsterdam Uitburo (AUB). These season tickets can contain all kind of performances from all affiliated theatres and can only be bought during pre-sale. The AUB makes the reservation and collects the payment and transfers the clients to the relevant theatres. Theater Bellevue will finish the process and makes sure that the tickets are sent to the clients. These tickets have their own ticket type label in the database.

Owners of a CJP card, culture card, XXXS card or people older than 65 can get a discount on the ticket price. The discount differs for each hall, so for performances in the Grote Zaal these group gets a discount of € 2.50, for performances in Klein Bellevue they will get a discount of € 1.50, and for performaces in the Paloni Zaal it is a € 2.– discount.
Students can get an extra discount of € 3.50 if they watch a performance on Thursday evenings and students whose studies are related to theatre can visit any performance for only € 10.–
If a performance is not sold out, there may be tickets for sale for this performance at the Last Minute Ticketshop of the AUB at the Leidseplein. These tickets are sold for half price. So if someone would have a discount for some reason, he would pay half of the discount price.
There are at most 20 tickets for performances in the Grote Zaal for sale and at most 10 tickets for performances in Klein Bellevue.
It is possible for employees to see a performance for free. Employees can bring one guest for free as well. Critics can see and review a performance for free as well, but companions have to pay the discount price.

Theater Bellevue has in comparison to many other theatres and other cultural organisations no other structural last minute campaigns except for the Last Minute Ticketshop. It occasionally happens that a specific group is approached with a discount campaign but this is a small group and always incidental.

The prices of the tickets are based on the price of equivalent performances, historical prices and the prices of the same performance in other theatres. The ticket prices vary between €10.– for the cheapest performance and approximately €21.– for the more expensive performances. The price of the tickets has not really increased over the years and it is the philosophy of Theater Bellevue that tickets should stay affordable.

2.2 Available data

The information of the customers and the performances is collected in different datasets.
There are three types of datasets available for the analysis of the historical data:

1. a dataset that shows for every performance the data per ticket that is sold.
2. a dataset that contains the data for every customer per performance.
3. a dataset that contains the occupation degrees

All three datasets are available for the last five theatre seasons (2007-2008, 2008-2009, 2009-2010, 2010-2011, 2011-2012). There are many similarities between the first two datasets, but the information is ordered in a different way.

2.2.1 Data ordered per performance per ticket

The datasets that show for every performance the data per ticket sold contain the following relevant information:

- **Customer number**
  Every customer that buys a ticket for the first time, is assigned a customer number. This number is used every time a ticket is bought and remains the same for every season.

- **Customer type**
  The customer type describes if the tickets are bought by a company, individual or someone else. The customer types that appear the most are: *anders (else), bedrijf (company), Collega (colleague), collega-bedrijf (colleage-company), diversen (miscellaneous), groep (group), impresariaat (impresario), Individu (individual), Medewerker (employee), onderwijs (education), pers-bedrijf (press company), Pers-persoon (press individual)* and *Theater collega (theatre colleague)*.
  The customer types are also shown with an abbreviation. The abbreviation of the customer type is the first letter of the customer type, so the most appearing abbreviations are: *a, b, C, c, d, g, i, I, M, o, p, P* en *T*.

- **Name and address**
  The database contains the name, address, postal code, city, country and telephone number of the customer.

- **Seasons**
  A theatre season starts in August or in the beginning of September and ends in the middle of July. The season is shown by a three or four digit code: 708, 809, 910, 1011 or 1112.

- **Performance code**
  A unique performance code is given to each performance every season. This code consists of six digits, where the first digit shows the hall of in which the performance takes place (two for performances in the Grote Zaal, three for performances in Klein Bellevue and four for the performances in the Paloni Zaal). If Theater Bellevue sells tickets for third parties (for theatre alliances or festivals), then the first digit will show other numbers. The following four digits show the date of the performance (mmdd) and the last digit is in general a zero, unless there are more than two performances in the same hall at the same day and in that case the last digit will become a serial number. If a performance is cancelled and a new performance is scheduled this new performance will get a serial number as well as the code of the cancelled performance will stay occupied.
  The performance code contains no information about the theatre seasons, so the codes are reused

every season. There are five separate databases, one for every season and a unique combination can made by combining the season and the performance code.

- **Date, starting time and day of the performance**
  The date and time of the performance. The day of the performance is added to the dataset. Performance are scheduled almost every day in the Grote Zaal and in Klein Bellevue, while lunch performances in the Paloni Zaal are usually not scheduled on Mondays and Saturdays.

- **Price**
  The price that is paid for a ticket.

- **Ticket type**
  Most tickets have a *Normaal (normal)* ticket type. Not every ticket type is used each season. The most common ticket types are: *50% korting, Actie250, Actie300, Actie350, Actie400, Actipact1, Actipact2, AUB abonnement kinderen, AUB abonnementen, AUB ticketshop, AUB ticketshop actie 1, AUB ticketshop kinderen, AUB ticketshop reductie, Calandlyceum, CJP, CJP/aktieprijs, CKV bon, Diverse reducties, Genodigden, Holland Festivalactie, Internet reductie, Internet volle prijs, Internet Actie 1, Internet Actie 2, Interred+oppas, Intervol+oppas, Kinderen, KK500, KK550, Lastminute ticketshop, Normaal, Pakket 1, Pakket 2, Pakket 3, Pakket 4, Schr.res.kortingspassen, Schriftelijke.res.normaal, Senioren, Stadspas, Stadspas kinderen, Stapelaktie 1, Stapelaktie 2, STDP Aktie, Theaterbon5, Theaterbon 7,50, Theaterbon 12,50, Theaterbon 25, Theaterschool, Volwassenen, Webkind* and *XXXS-pas*.

- **Hall**
  The hall of the performance is showed as well by name as by abbreviation. The abbreviation are not always the same. The Grote Zaal can be abbreviated by *BVGN, BELG* and *2RAN*, Klein Bellevue can be abbreviated by *3BVK* and the Paloni Zaal can be abbreviated by *4BVP, BVP_* and *BELP*. If Theater Bellevue sells tickets for alliances or festivals other abbreviations are used for those performances.

- **Row and seat number**
  Seats in the Grote Zaal are allocated so for these performances row and seat numbers are shown.

2.2.2 Data ordered per customer per performance

The datasets that contain the data that is ordered per customer per performance contain the following relevant information:

- **Customer number**

- **Name and address**

- **Performance code**

- **Date and starting time of a performance**

- **Number of tickets**
  A customer can buy more than one ticket for a performance. The total number of tickets that are bought are shown.

- **Total ticket costs**
  If more than one ticket is sold the ticket prices are added and make up the total ticket costs.

- **Transaction date**
  The transaction date is the date on which the tickets are paid. If a reservation is made, then the tickets should be paid within 30 days. The transaction date is not the reservation date and this can give a wrong premise, because it is not clear when the reservation was made.

- **Number of days before the performance**
  The number of days before the performance is the number of days between the transaction date and the performance date and is calculated by subtracting the transaction date from the performance date.

- **Number of weeks before the performance**
  The number of weeks before the performance is calculated by dividing the number of days before the performance by 7 and rounded off downwards.

- **Ticket channel**
  A ticket can be bought either on the internet or at the ticket booth. The most common ticket channels are: *kassa klant p1, VIA DE KASSA, internet, mailing list p1* en *pre pass p1*.

2.2.3 Occupation degrees

There is a seperate database for every season that contains the following information:

- **Information about the performance**
  The database contains the performance code and beside it also the title of the performance or the troupe that is playing the performance. It contains information about the date and starting time as well.

- **Capacity**
  The capacity is in general the maximum possible capacity of a hall.

- **Blockade**
  A blockade is put in the system if there is no zeroth row available, if sight lines should be taken into account, if a première is scheduled and ticket should be reserved for the press. The blockade can decrease if a zeroth row is needed or if the seats for the performer are not used.

- **Occupied tickets**
  The number of occupied tickets is equal to the number of sold tickets plus the number of reserved tickets. This database is used daily during the sales period of a performance to see if extra publicity is needed.

- **Sold tickets**
  The number of tickets that have been paid.

- **Reserved tickets**
  The number of reserved tickets that have been booked but not paid yet. If a reserved ticket is paid it will move from ‘Reserved tickets’ to ‘Sold tickets’.

- **Waiting list**
  If a performance is sold out, it is possible to be put on a waiting list. If a reserved ticket is not picked up or paid on time it is sold to the people on the waiting list.

- **Free tickets**
  The number of free tickets that are given away for a performance. These tickets can be given to employees of Theater Bellevue or family and friends of the performer. Reviewers can get free tickets as well.

- **Available tickets**
  The number of available tickets is given by substracting the number of occupied tickets and the blockade from the capacity.

- **Rate**
  The percentage of occupied seats is given by dividing the number of occupied seats by the capacity minus the blockade. The percentage gives in general a better insight in the occupation rates than the number of occupied seats, but it can also distort the figures.
  A première is in general only visited by guests like family, friends and press. So the occupation rate will be 100%, but almost no seats have sold. So although it may look like a sold out performance, but it is not.

- **Sales**
  The amount of money yielded per performance.

- **Genre**
  This column is only added in the databases of the seasons 2009-2010, 2010-2011 and 2011-2012. It was not possible to determine the genres for the performances in the other seasons.

3 | Data analysis

Investigations about Theater Bellevue's audience are made every two or three years. Early investigations told us who is visiting Theater Bellevue and what means of communication they use [2] and about the visit frequencies, the overlap between the halls and the moment of acquisition [14]. It is known that over 80% of the audience lives in or near Amsterdam, approximately 35% of the tickets is bought in the last week before the performance and that approximately 80% of the audience watches a performance only once a year.
The focus of the data analysis will be on the number of tickets sold instead of on the number of customers. The analysis will be split into three parts, because each hall has its own capacity and its own audience.

3.1 Box plot

A box plot is a graph that shows how the data is distributed and can be used to compare several data samples when their distribution functions are unknown.
A box plot contains five important values: the mininum, the first quartile, the median, the third quartile and the maximum of the sample.
The median is the number that is in the middle if all data in the sample is ordered from small to large and if there is an odd number of data. If there is an even number of data the median will be given by the average of the two central data. The median splits the data into two parts; 50% of the data is smaller than the median and 50% is larger. The first quartile is the median of the lower dataset and the third quartile is the median of the higher dataset.
We will show the average of the sample with a dot.

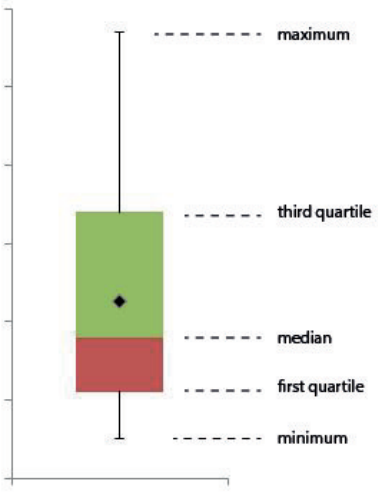


Figure 3.1: An example of a box plot

3.2 Seasons

First we will have a look at the ticket sales per theatre season to get an idea about the influence of the seasons on the ticket sales. Figure 3.2 shows that there is a decline in the ticket sales over the last theatre seasons, but there are different patterns for each hall. The number of tickets sold for performances in the Grote Zaal has strongly decreased (17%) between season 2007-2008 and season 2008-2009 and there is also a sharp decline (10%) in the ticket sales between season 2008-2009 and season 2009-2010, while the sales remained approximately constant in the last three seasons.
Season 2009-2010 has been an exceptional good season for Klein Bellevue. The ticket sales increased by 23% in comparison with the previous season, while they decreased in the seasons afterwards to the level of the season 2007-2008.
The ticket sales in the Paloni Zaal is quite constant except for the seasons 2007-2008 and 2009-2010 in which the ticket sales were somewhat lower than in the other seasons. The total number of tickets sold

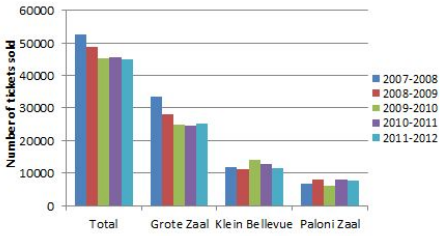


Figure 3.2: The number of tickets sold per hall

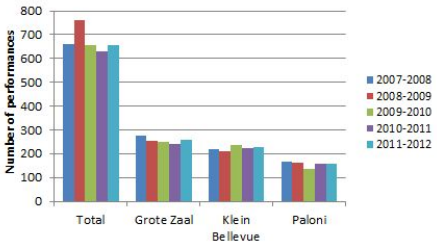


Figure 3.3: The number of performances per hall

is of course related to the number of performances. Figure 3.3 shows that the number of performances per hall is more or less constant. The decline in the ticket sales has not been caused by scheduling less performances. In season 2008-2009 more performances have been scheduled than in season 2008-2009, but less tickets have been sold.

Pre-sale

The pre-sale for the new theatre season starts approximately three months before the start of the season. It starts in general in May or in the beginning of June. Figure 3.4 confirms that the audience of Theater

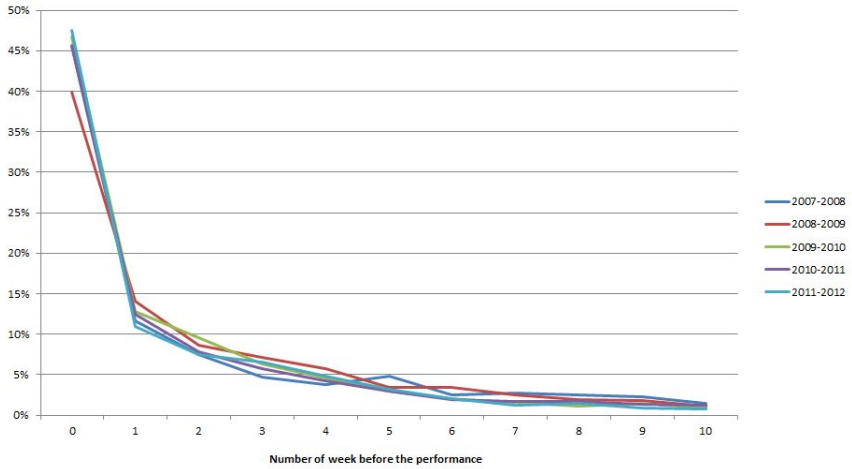


Figure 3.4: Percentage of the number of tickets sold before the performance.

Bellevue is a real last minute audience. It shows that almost 50% of the tickets is sold in the last week before the performance. The percentage of tickets sold in the last week is except for season 2008-2009 higher than 45%. Season 2008-2009 is in this respect the best season, since there were less last minute tickets sold.

If we have a look at the average week in which the tickets were sold we see that season 2011-2012 was the best year. The average week in which tickets were sold in season 2007-2008 was 3.94, in season 2008-2009 it was in week 3.98, in season 2009-2010 in week 3.68, in season 2010-2011 in week 4.23 and in season 2011-2012 the average week number had increased to 4.76. So although there is still a high last minute sale, more tickets are bought earlier. This can be caused by the scheduling of popular performances or well-known actors.

The problem with this pre-sale curves is that it is not approximating the real pre-sale curve. Tickets can be reserved by calling the ticket booth and are registered as 'reserved' and have to be paid within a month. When the tickets are paid, which can be on the day of the performance, they are registered as

bought on that day and the information about the reservation is deleted from the system. So the high number of tickets sold in the last week, can also be caused by the paying of reserved tickets and not only by buying last minute tickets.

Sales channels

It is possible to buy tickets on-line or at the ticket booth. Figure 3.5 show the ticket sales per sales channel. We see that the on-line ticket sales has more than doubled between season 2007-2008 and season 2011-2012. In season 2007-2008 12.76% of the tickets were sold on-line, while in season 2011-2012 34.98% of the tickets were sold on-line. When looking at the percentages, we see that the on-line ticket sales has increased by almost 300%.

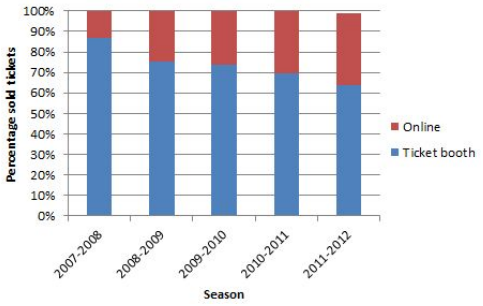


Figure 3.5: The percentage of tickets sold split per sale channel

Pre-sale per sales channel

If we split the pre-sale data per channel, we see that the average sales week increases for on-line ticket sales and that the average sales week decreases for ticket sales at the ticket booth.
Tickets that are bought on-line are bought earlier every season. The average sales week was 0.6 in season 2007-2008 and it has been almost quadrupled in season 2011-2012 to week 2.35. The average sales week for tickets bought at the ticket booth has decreased. The average sales week was 3.33 in season 2007-2008 and it has decreased to week 2.40 in season 2011-2012.
This can be explained by the increase of the on-line ticket sales and therefore there are less people that buy their tickets at the ticket booth and the data of ticket sales at the ticket booth is more and strongly influenced by the paying of reservations.

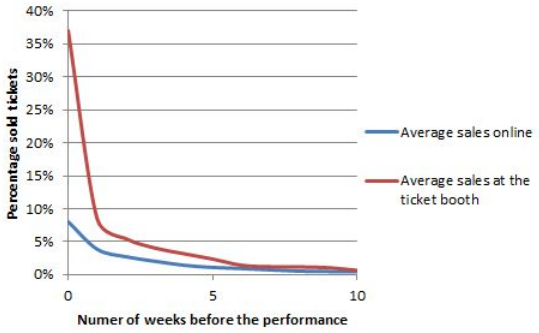


Figure 3.6: Percentages of the average number of tickets sold before a performance, split per sales channel.

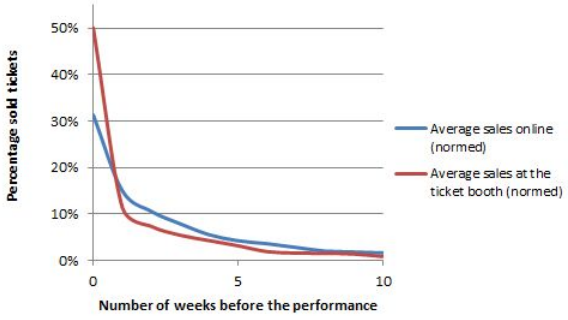


Figure 3.7: Normed percentages of the average number of tickets sold before a performance, split per sales channel.

If we have a look at the percentages of the average number of sold ticket that are split per channel (Figure 3.6), we see that it is difficult to compare the curves because of the differences in quantities. We norm both curves to make a better comparison possible. Figure 3.7 shows the normed curves. Now it is clear that 30% of the tickets bought online are sold in the last week against 50% of the tickets bought at the ticket booth. There are also differences in the steepness of both curves. The curve of the on-line sales is much less steep than the curve of the ticket booth sales. This figure confirms the data of the average sales week of both sales channels.

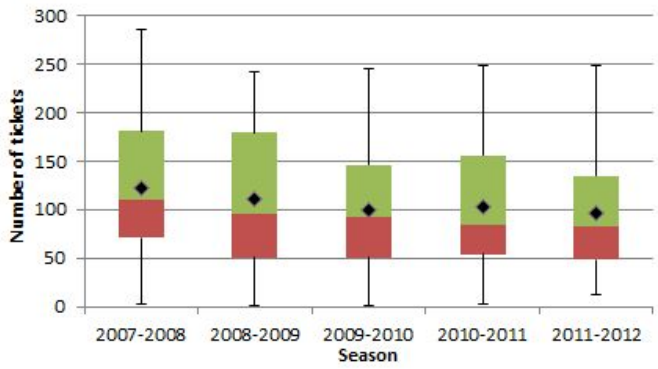
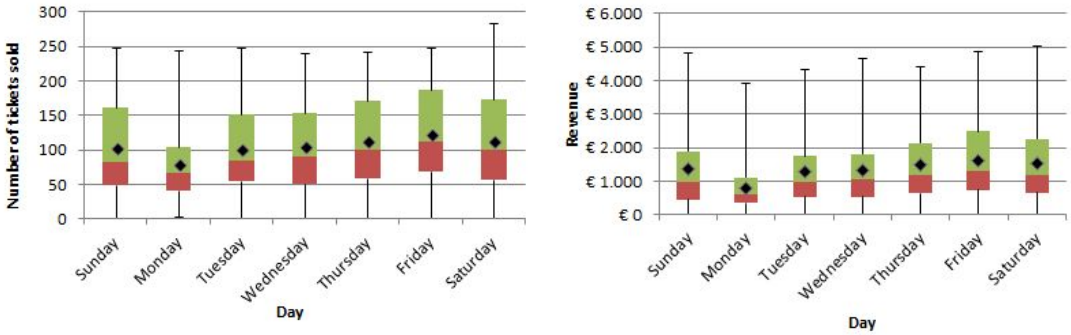


Figure 3.8: Distribution of the number of tickets sold per performance per season in the Grote Zaal

3.3 Grote Zaal

We will split the sales per hall, in order to get a better insight in the ticket sales. Figure 3.2 and Figure 3.3 show the total ticket sales and the total number of performances in the Grote Zaal. However, they do not show information about the number of tickets sold per performance. Figure 3.8 shows the distribution of the total number of tickets sold per performance per season. It tells us that there have been one or more performances in season 2007-2008 that had a higher capacity than normal. Furthermore we notice that, except for season 2007-2008, more than 50% of the performances sell less than 100 tickets per performance. The average number of tickets sold per performance is still approximately 100 tickets, which can be caused by the sold out performances which sell a lot more tickets.

Days of the week



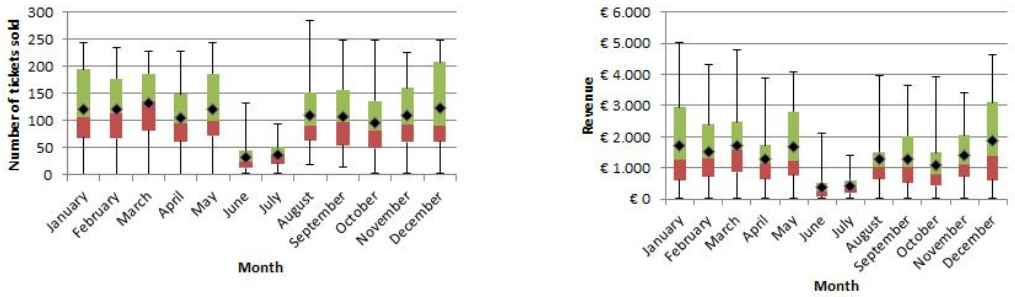
(a) (b)

Figure 3.9: Distribution of the number of tickets sold per performance (3.9a) and the revenue per performance (3.9b) per day of the week in the Grote Zaal over all seasons.

Figure 3.8 shows only the distribution of the number of tickets per season. It can occur that some days are more popular to visit theatre, because customers have more leisure time in the weekends to visit theatres.
Figure 3.9 shows that performances that are scheduled on Monday are less visited than performances on other days. 75% of the performances on Mondays sell less than 105 tickets per performance, while 50% of the performances on Friday sell more than 112 tickets per performance and 25% of the performances sell even more than 186 ticket per performance.
The distribution of the revenue shows the same pattern and the same differences between the different days. 75% of the performances on Monday yield less than € 1,100.–, while 50% of the performances on Friday yield more than € 1,300.–.
When we look at the distribution of the ticket prices in Figure 3.14 we notice that every day almost

75% of the tickets are sold for € 16.– or more, but on Monday 75% of the tickets are sold for less than € 15.–.

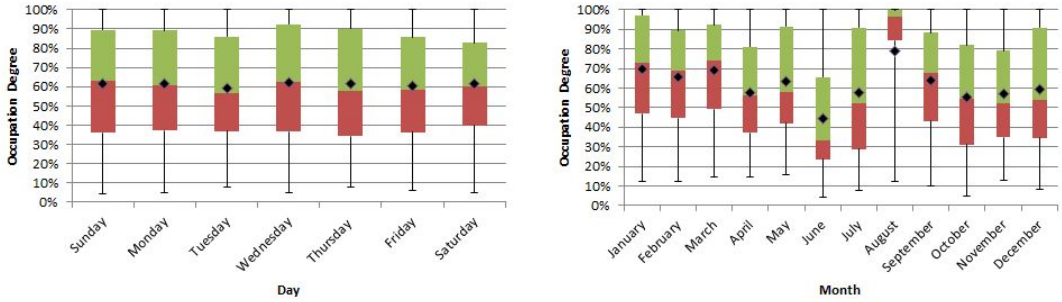
Months of the year



(a) (b)

Figure 3.10: Distribution of the number of tickets sold per performances (3.10a) and the revenue of the performance (3.10b) per months in the Grote Zaal over all seasons.

When the sun starts shining and the temperature increases, people flock to terraces and out of theatres. The warmer months can be hard, especially for theatres as Theater Bellevue, since they depend heavily on a last minute audience. The presence of the sun in spring and summer indeed influences the ticket sales as you can see in Figure 3.10.
Just before the summer starts in June performances sell less tickets. 75% of the performances sell less than 45 tickets per performance in June and 75% of the performances in July sell less than 51 tickets per performance. Less performances are scheduled July, but this is not the cause for June since the number of performances in December is less than the number of performances in June (see Figure 3.17) and December performs better. The winter months - January, February and March- perform good. 50% of the performances sell more than 100 tickets per performance in this period and 25% of the performances sell between 186 and 228 tickets per performance. Not all performances in this period sell out, but the performances have on average a higher occupation degree than the performances in October. The month December is a good month as well, but there are more performances that sell less than 100 tickets per performance than in January. However, the top performances have higher occupation degrees: 25% of the performances sell between 208 and 248 tickets per performance.
If we have a look at the revenue we see that 50% of the peformances in December, January, February and March yield more than € 1,350.– per performance. January and December perform the best, since 25% of the performances yield even more than € 3,000.–. The months that sell less tickets, yield less too. The performance with the highest revenue in July still yields less than slightly less than 50% of the performances in December.
Something else that is quite remarkable is that there have been performances in August which have sold a lot of tickets, but the maximal revenue collected is not exceptionally high. The maximal revenue in August is lower than the maximal revenue in March, while the maximal number of tickets sold in March is lower than in August. We have to guess the reasons, but it can be caused by the price of the tickets which could have been lower in August than in March.
The distribution of the ticket prices in Figure 3.15 shows that the prices in June have the largest dispersion. There are a couple of outliers, but 50% of the tickets has cost less than € 15.50 and 25% even less than € 10.–. The prices in September are not really high as well and 50% of the tickets are sold for less than € 16.–, but 25% of the prices is less than € 14.40. There are less tickets sold for a low price in September than in June. Apart from these two months there are only minor differences and the average price is approximateley € 15.–.



(a) (b)

Figure 3.11: Distribution of the occupation degree per performance per day of the week (3.11a) and per month of the year (3.11b) in the Grote Zaal over all seasons.

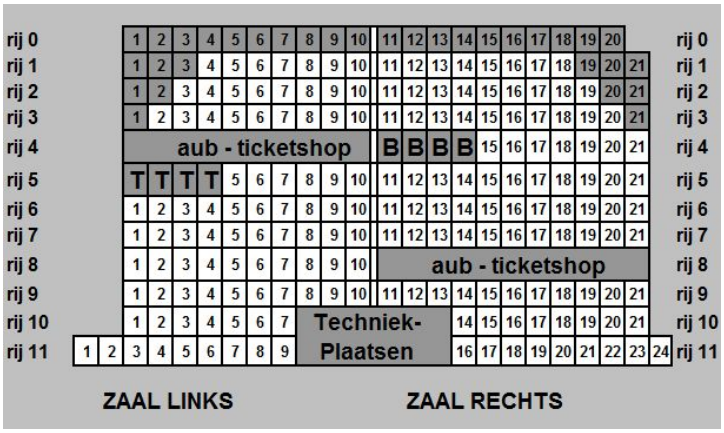


Figure 3.12: The seat plan of the Grote Zaal with the possible blockades. The seats with a T are the seats for the Theatre and the seats with a B are the seats for the actors.

Occupation degrees

Figure 3.11a shows the distribution of the occupation degrees and we see that, inter alia, August was one of the best months. The occupation degree of 75% of the performances was higher than 84%. Further more we see that June is again not performing well. 75% of the performances have an occupation degree that is lower than 65%. We also notice that the distribution of the occupation degrees per day of the week have much more similarities than the occupation degrees per month of the year. Figure 3.9a showed us that there were not many tickets sold on Monday, but the performances on Monday are not performing worse than any other performances according to Figure 3.11a. The performances on Monday will probably have a low capacity.

It is interesting to split the occupation degrees per day and per month, but it is also interesting to look at the occupation degrees of the seats in the hall. Splitting the hall into (more) ranks can be an option to increase the revenue.

It is only possible since August 2012 to pick your own seats, if you are booking your tickets on-line. This was not possible before this season and seats were allocated according to the best seat available principle. Furthermore the system was ordered to centre the audience as much as possible.

Beside this, seats were blocked before tickets could be bought for the Last minute Ticketshop at the Leidseplein (these seats could also be sold for the normal price if needed), for the actors, the employees, technicians and the so-called capacity seats. These capacity seats are only sold when all other seats are sold and form the zeroth row. Figure 3.12 shows the seat plan of the Grote Zaal and the seats that get usually blocked.

If we look at the occupation degrees of the seats in Figure 3.13 we see the same pattern as in Figure 3.12. The seats on the zeroth row have a clearly lower occupation degree than for example the middle of row 2.

Something striking is the descending of the occupation degree from the middle. The seats in the middle have the highest occupation degree, which can be caused by the fact that these seats have the best view on the stage, but also because these seats were the first seats to be allocated by the system.

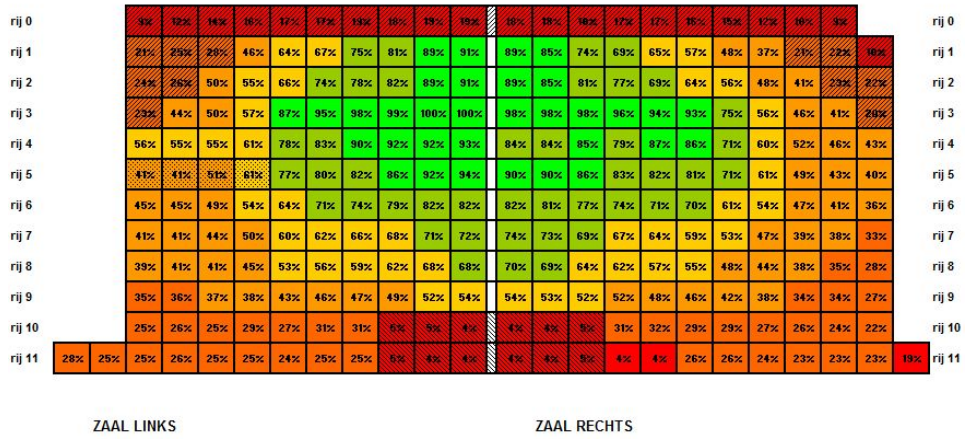


Figure 3.13: The seat plan of the Grote Zaal that contains the occupation degrees of the last five seasons. Red seats have a low occupation degree, yellow seats have an average occupation degree and green seats have a high occupation degree.

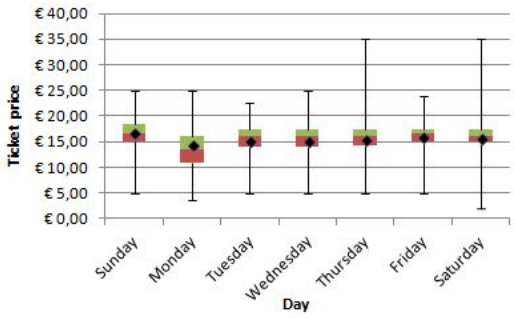


Figure 3.14: Distribution of the ticket price per day in the Grote Zaal over all seasons.

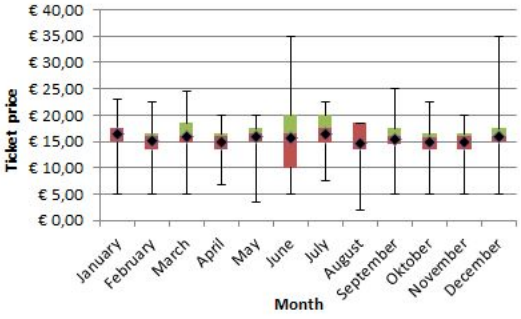


Figure 3.15: Distribution of the ticket price per month in the Grote Zaal over all seasons.

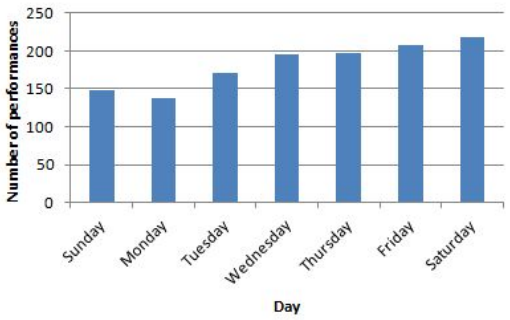


Figure 3.16: The number of performances per day in the Grote Zaal over all seasons.

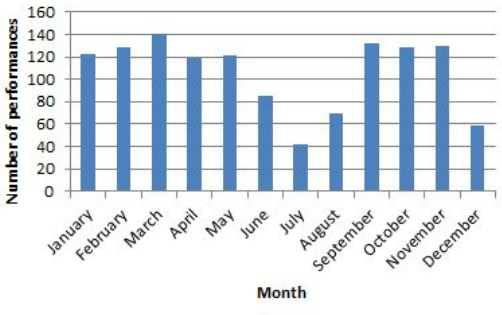


Figure 3.17: The number of performances per month in the Grote Zaal over all seasons.

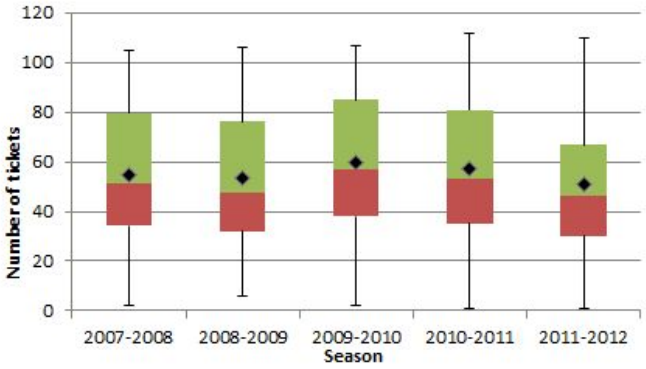
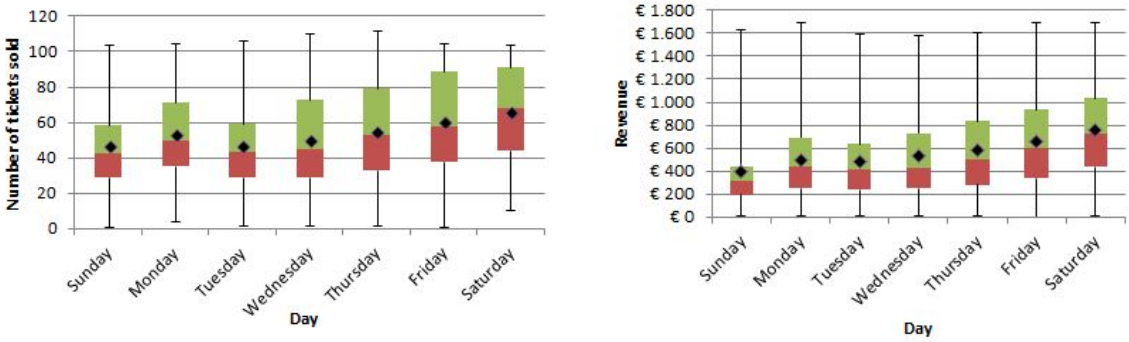


Figure 3.18: Distribution of the number of tickets sold per performance per season in Klein Bellevue.

3.4 Klein Bellevue

When we have a look at the distribution of the ticket sales for Klein Bellevue in Figure 3.18, we see that season 2009-2010 was the best season of the last five years. 50% of the performances in this season sold more than 57 tickets per performance and 25% of the performances sold between 85 and 107 tickets per performance.
The performances in season 2010-2011 perform not as well as the performances in season 2009-2010 but they peform only slightly less. The other seasons peform clearly less; the data in season 2011-2012 is for a large part less than the data in 2009-2010 and the average number of tickets sold has decreased too. 75% of the performances in 2011-2012 has almost sold less tickets performance than the average number of tickets sold per performance in 2009-2010.

Days of the week



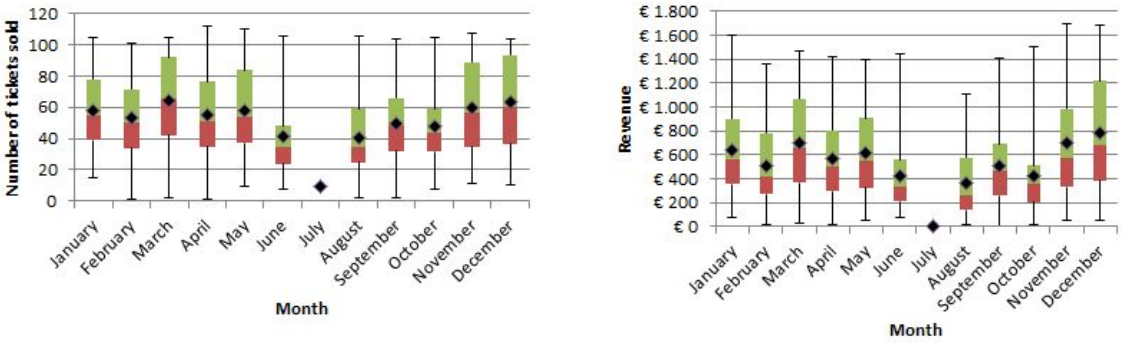
(a) (b)

Figure 3.19: Distribution of the number of tickets sold per performance (3.19a) and the revenue per performance (3.19b) per day of the week in Klein Bellevue over all seasons.

Figure 3.19 shows the distribution of the number of tickets sold and the distribution of the revenue per performance per day over the last five years. We see that Saturday is a pretty good day. 50% of the performances on Saturday sell more than 68 ticket per performance and 25% of the performances sell between 91 and 104 tickets per performance.
When we look at the distribution of the performances on Sunday we see that the whole weekend is not as popular as in the Grote Zaal. 75% of the performances on Saturday sell more tickets than 50% of the performances on Sunday. There have been less performances scheduled on Sunday in the last five years (see Figure 3.24), so this can cause a wrong premise.
If we take a look at the distribution of the revenue on Saturday and Sunday we see the same pattern again. 75% of the performances on Saturday has a higher revenue than 75% of the performances on

Sunday. When we compared the distribution of number of the tickets sold, it was 75% against 50%. This can be caused by a lower price of the tickets on Sunday. Furthermore we see that everyday has one or more outliers of a maximal revenue of € 1,600.–.
When we compared the days of the Grote Zaal we found out that Monday was not a good day. It seems that Monday is quite a good day for Klein Bellevue. The distribution of the revenue and the number of tickets sold is not as good as on Saturday, but it is not as bad as on Sunday. This can be caused by the choice of the performances on Monday. Performances of *BIES* and *Het Nieuwe Lied* are monthly scheduled on Mondays in Klein Bellevue and these performances have partly their own audience. Thursday, Friday and Saturday are for both halls and for the number of tickets as for the revenue good days.
Figure 3.22 shows the distribution of the ticket prices per day and we see that 75% of the performances on Monday, Friday and Saturday are sold for than € 16.–, while 75% of the performances on Sunday are sold for less than € 16.–. Furthermore we see that the average price of a ticket for a performance on Sunday is lower than on the other days.

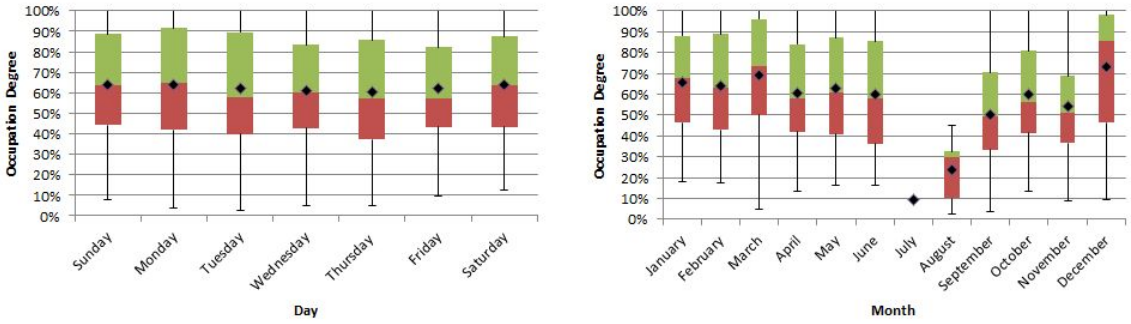
Months of the year



(a) (b)

Figure 3.20: Distribution of the number of tickets sold per performance (3.20a) and the revenue per performance (3.20b) per months in Klein Bellevue over all seasons.

When we look at the total number of performances per month over the last five years in Figure 3.20, we see that there has been only one performance in July in the last five years and we will not take this month in consideration.
Figure 3.20 shows the distribution of the number of tickets sold and the revenue per month and it shows that March is one of the best months. 50% of the performances in March sell more than 66 tickets per performance and 25% of the performances sell between 92 and 105 tickets per performance. Almost 75% of the performances in March sell more tickets than almost 75% of the performances in June.
In general we see that peformances in the Summer and Autumn sell less tickets than performances in Winter and Spring. 50% of the performances in December sell more tickets than 75% of the performances in June, August and October.
Although the differences between the months are smaller, we see the same results when we look at the distribution of the revenue. March is still a good month, but the differences compared to June are smaller. The revenue in January and February is not as good as could be expected when we looked at the distribution of the number of tickets sold.
More than 75% of the performances in August and October yield less than 50% of the performances in December. 50% of the performances in August yield even less than € 256.–. Not many performances have been scheduled in August, but the number of performances is approximately equal to the number of performances in December, and this is quite a good month.
If we compare the results of the Grote Zaal with the results of Klein Bellevue wee see that December is a good month for both halls. This can be caused by the scheduling of popular performances and



(a) (b)

Figure 3.21: Distribution of the occupation degrees per performance per day of the week (3.21a) and per month of the year (3.21b) in Klein Bellevue over all seasons.

well-known actors and comedians in the Christmas period.
Figure 3.23 shows the distribution of the ticket prices per months. We see that performances in August cost indeed less than performances in the other months and this can explain the low revenue. Furthermore, we see that the ticket prices remain more or less constan except for December. The average ticket price in December is higher than in the other months and 75% of the ticket prices in December is higher than in the rest of the year.

Occupation degrees

We see that there are only minor differences between the distributions of the occupation degrees per day (Figure 3.21a), while there are many differences between the distributions of the number of tickets sold per day of the week (Figure 3.19a). So although it seemed that Sunday was a bad day, it is a better day than expected.
If we look at the distribution of the occupation degrees per month (Figure 3.21b) and compare this with the distribution of the number of tickets sold per performance (Figure 3.20a), we see that there are many differences between the two graphs. August did not seem a worse month when we looked and compared the number of tickets that were sold in that month, but if we look at the occupation degrees we see that 75% of the performances has an occupation degree that is lower than 32% and the performance with the highest occupation degree in August still has a lower occupation degree than at least 50% of the performances in any other month.

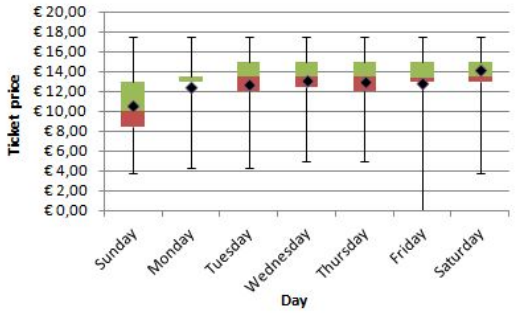


Figure 3.22: Distribution of the ticket price in Klein Bellevue per day over all seasons.

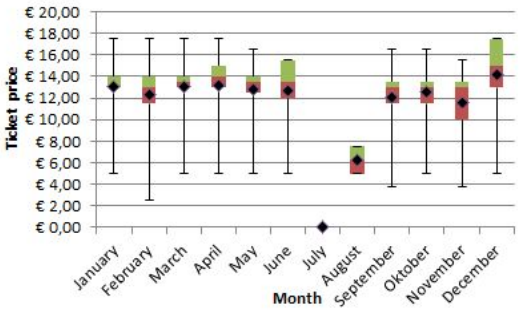


Figure 3.23: Distribution of the ticket price in Klein Bellevue per month over all seasons.

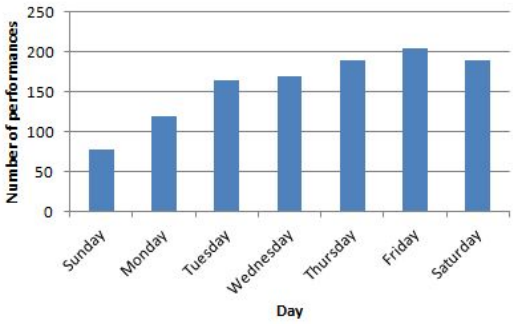


Figure 3.24: Number of performances per day in Klein Bellevue over all seasons.

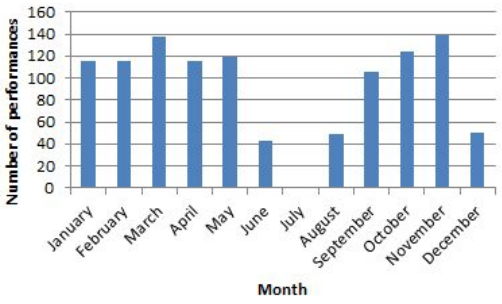


Figure 3.25: Number of performances per month in Klein Bellevue over all seasons.

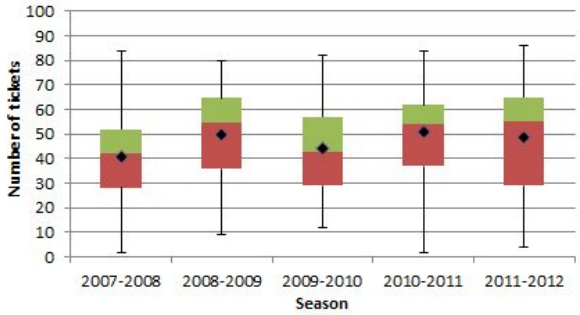
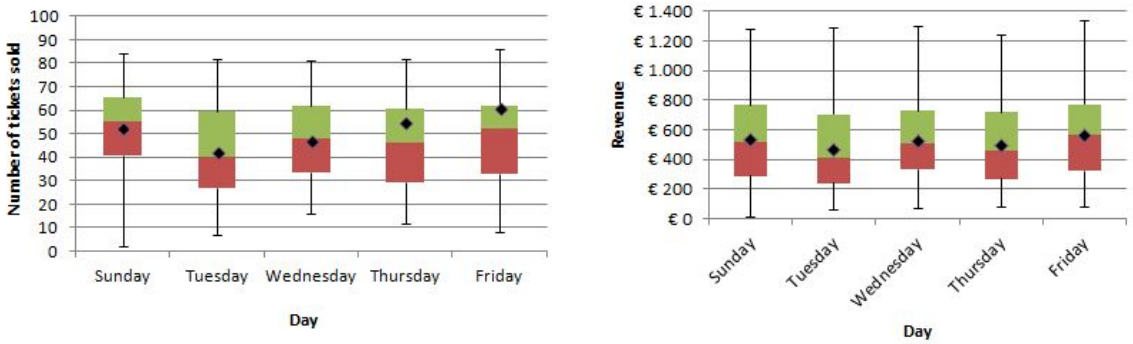


Figure 3.26: Distribution of the number of tickets sold per performance per season in the Paloni Zaal.

3.5 Paloni Zaal

Figure 3.26 shows that seasons 2008-2009, 2010-2011 and 2011-2012 have been good seasons for the Paloni Zaal. 50% of the performances in these seasons sold more than 48 tickets per performance and 25% even more than 62 tickets per performance.
The season 2007-2008 performed not as good. 75% of the performances sold less than 52 tickets per performance, while in the three good season 50% of the performances sold more than this amount of tickets.
This is not all since the maximal capacity in the Paloni Zaal can change per production. This could have been the case in season 2007-2008. We will see the occupation degrees later on and then we can make a better comparison between the seasons.

Days of the week

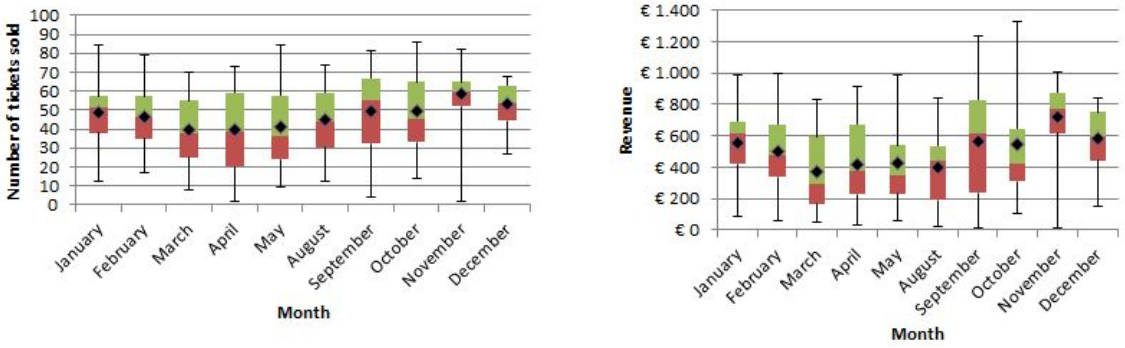


(a) (b)

Figure 3.27: Distribution of the number of tickets sold per performance (3.27a) and the revenue per performance (3.27b) per day of the week in the Paloni Zaal over all seasons.

The lunch performances in the Paloni Zaal are in general only scheduled on Sunday, Tuesday, Wednesday, Thursday and Friday. It can however occur that performances are scheduled on Monday or Saturday (see Figure 3.32), but this does not happen often and we will not analyse the performances on these days. Figure 3.27a shows the distribution of the number of tickets and the revenue and we see that there are more similarites between the days than by the distributions of the other halls. Tuesday performs not as good as for example Sunday and Friday. 75% of the performances on Sunday sell more than 41 tickets per performance, while on 50% of the performances on Tuesday sell less than 40 tickets per performance. The average number of tickets is also lower on Tuesday in comparison to the other days.
If we look at the distribution of the revenue there is even more overlap between the box plots than between the box plots of the number of tickets. Performances on Tuesday have the lowest average revenue and 50% of the performances yield between € 56.– and € 408.– while 50% of the performances on Friday yield betwee € 78.– and € 570.–. The differences are still much smaller than for example in the Grote Zaal. So it does not really matter on which day the lunch performances are scheduled.
If we look at the distribution of the ticket prices in Figure 3.30 we see that there are again a lot of similarities. 50% of the prices is between € 12.– and € 14.– and the only minor differences between the days are outliers.

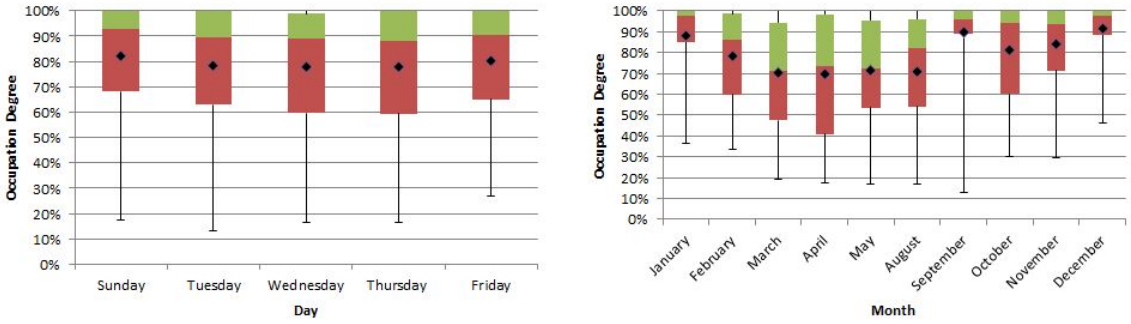
Months of the year



(a) (b)

Figure 3.28: Distribution of the number of tickets sold per performance (3.28a) and the revenue per performance (3.28b) per months in the Paloni Zaal over all seasons.

Lunch performances are not scheduled in July and August (see Figure 3.33) and we will not show these months in the other graphs. Figure 3.28 shows the distribution of the number of tickets per performance and the revenue per month. The month November is quite a good month. There are some outliers towards the minimum, but 75% of the performances in November sell more than 52 tickets per performance. The month March performs less. 75% of the performances sell less than 55 tickets per performance and the maximum is also lower than in many other months. It is possible that the lunch performances that were scheduled in March had a lower capacity than performances in other months, but the number of tickets sold in December is lower than the number of tickets sold in March, but the distribution is much more compact (the worst selling performance in December still sells better than 25% of the performances in March).
The distribution of the revenue shows that the month November is a pretty good month. The maximal revenue in November is not the highest revenue, and although there are a couple of lower outliers, 75% of the performances yield a revenue between € 617.– and € 1,006.–. The month september performs good as well. 50% of the performances yield a revenue between € 614.– and €1,237.–. The month March performance better than the expectations raised by the number of tickets that were sold in this month. 50% of the performances yield less than € 296.–, but there are high outliers. 25% of the performances yield between € 596.– and € 834.–.
The distribution of the ticket prices (Figure 3.31) shows that 50% of the ticket prices are priced between € 12.– and € 14.–. In some months, even 75% of the ticket prices are priced between € 12.– and € 14.–.



(a) (b)

Figure 3.29: Distribution of the occupation degrees per performance per day of the week (3.29a) and per month of the year (3.29b) in the Paloni Zaal over all seasons.

Occupation degrees

If we have a look at the distribution of the occupation degrees per day of the performances in the Paloni Zaal (Figure 3.29a) we see that the occupation degrees are high and that there are almost no differences between the days, just as in Figure 3.27a.

Figure 3.29b shows the distribution of the occupation degrees per month and it shows different patterns than the distribution of the number of tickets sold per month (Figure 3.28a). While November sells a lot of tickets it does not have the highest occupation degrees. January, September en December are really good months. 75% of the performances have an occupation degree that is higher than 87%.

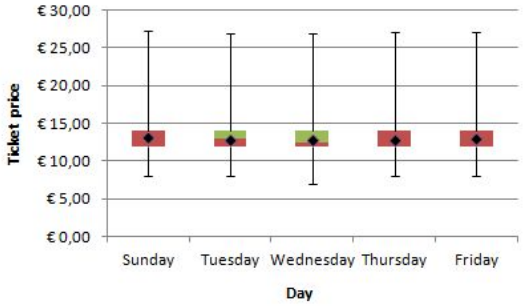


Figure 3.30: Distribution of the ticket price in the Paloni Zaal per day over all seasons.

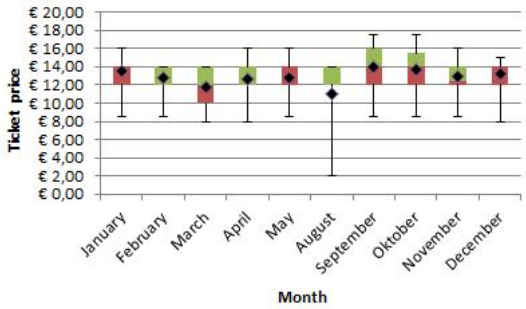


Figure 3.31: Distribution of the ticket price in the Paloni Zaal per month over all seasons.

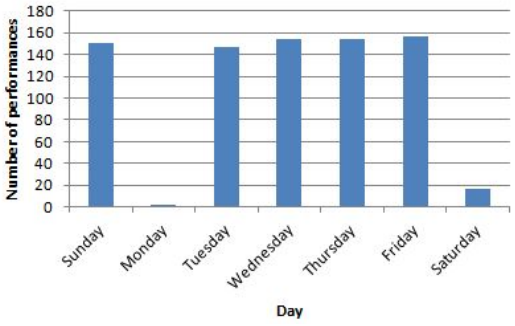


Figure 3.32: Number of performances per day in the Paloni Zaal over all seasons.

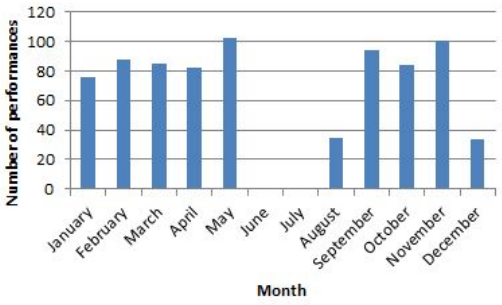


Figure 3.33: Number of performances per month in the Paloni Zaal over all seasons.

3.6 Observations

We can conclude this chapter by making the following observations:

- Every year less tickets have been sold by Theater Bellevue. Still, there are many diferences between the halls. Klein Bellevue sells more tickets in season 2009-2010 compared with the previous seasons, while the other halls sell less tickets in this season compared to the previous season.
- Almost 50% of the tickets are bought in the last week, but this can be a wrong premise as the reservation date is overwritten by the date of payment once a ticket is paid. Many tickets that are reserved within a month before the performance are collected at the date of the performance, so although the visit was planned earlier this is not shown by the data.
- The online ticket sales has increased over the last five years. It has increased by more than 50%. Furthermore we noticed that people that book online book their tickets earlier.
- For performances in the Grote Zaal we see that:
  - except for season 2007-2008, 50% of the performances in the other seasons sell less than 100 tickets per performance.
  - performances on Monday do not perform well. Almost 75% of the performances sell less than 100 tickets per performance.
  - performances in winter (January, February, March) perform well, since 50% of the performances sell more than 100 tickets per performance and the top 25% have high sales numbers too.
  - the occupation degrees do not differ much per day, but they do differ a lot per month. Furthermore we see that the seats in the center of the Grote Zaal have the highest occupation degrees.
- For performances in Klein Bellevue we notice the following:
  - The distribution of the number of tickets shows the same pattern as the total number of tickets.
  - The performances on Sunday and Tuesday do not perform well, while the performances on Saturday do. 50% of the performances on Saturday sell more than 75% of the performances on Sunday and Tuesday.
  - Performances in March and December perform well, 50% of the performances in this month sell more tickets than 75% of the performances in October.
  - The distribution of the occupation degrees of the performances per day and per month show different patterns than the distribution of the number of tickets sold and there are almost no differences per day.

  For performances in the Paloni Zaal we see that:
  - performances in seasons 2007-2008 and 2009-2010 performed a bit less than performances in the other seasons.
  - performances on Tuesday perform less than performances on the other days. 50% of the performances on Tuesday sell less than 75% of the performances on Sunday.
  - November, December and January are good months for Theater Bellevue, since they have high averages and at least 50% of the performances sell more than 50 tickets per performance.
  - the occupation degrees are in general very high.

4 | Estimation of the demand

It is important to know the real demand of a performance when revenue management or dynamic pricing is implemented. At this moment the demand is censored and it is not known how many people wanted to buy a ticket for a performance that is sold out or the price people wanted to pay for a performance. The first question is mainly important in revenue management while the second question is of importance in dynamic pricing. It does not happen that often that performances are sold out in Theater Bellevue so the demand for seats for a performance is often equal to the actual amount of tickets sold. The second question is far more interesting.
If a customer is prepared to pay € 20.– for a ticket and a ticket is sold for € 15.– then this customer will buy the ticket, but if the ticket is sold for € 25.– then he will not buy it.
In this chapter we will try to estimate the underlying price that a customer is willing to pay by calculating the probability that a customer buys a ticket for a certain price.

4.1 Kaplan-Meier estimators and unconstraining

Edward L. Kaplan and Paul Meier published an article in 1958 [7] in which they described a way to estimate the survival probabilities based on life-time data even if some data is censored. The Kaplan-Meier estimator is often used in medical studies to estimate the survival probabilities of different treatments and to compare the different treatments. The data of these experiments is often censored since patients skip follow up appointments and disappear from the database or the experiment is stopped after a couple of years.

The Kaplan-Meier estimator is calculated as follows:
Let $P(t)$ be the probability that the survival time is longer than t. If $0 \leq t \leq \infty$ then $1 \geq P(t) \geq 0$ and theoretically, $P(t)$ is a right-continuous function since there can be an infinite amount of steps in time, the experiment can run forever and there are no people lost. Let the observed life time for a sample size N be ordered as follows:

$$t_1 \leq t_2 \leq \cdots \leq t_N.$$

$n_i(t_i)$ is the number of people or objects that are at risk of dying just before time t_i, let $d_i(t_i)$ be the number of people or objects that die during t_i and let T be the random variable for a person's survival time. If people or objects for some reason disappear from the research data, then let $l_i(t_i)$ be the lost people at time t_i. If people or objects disappear we speak of censored data.

In the case of censored data the number of people at risk of dying is given by the number of survivors minus the lost people: $n_i(t_i) - l_i(t_i)$. If the data is not censored, the number of people at risk is given by the number of survivors. The Kaplan-Meier estimator $\hat{P}(t) = \mathbb{P}(T > t)$ for $0 \leq t \leq \infty$ is given by:

$$\hat{P}(t) = \prod_{t_i \leq t} \frac{n_i'(t_i) - d_i(t_i)}{n_i'(t_i)}, \qquad \text{with } n_i'(t_i) = \begin{cases} n_i(t_i) & \text{if the data is not censored} \\ n_i(t_i) - l_i(t_i) & \text{if the data is censored} \end{cases}$$

where $\frac{n_i'(t_i) - d_i(t_i)}{n_i'(t_i)}$ is the probability of surviving at time t_i, so the probability that someone is alive at time t is calculated by multiplying all the probabilities that this person has survived the previous years or

months.
The Kaplan-Meier estimator can also be written as:

$$\begin{aligned}\hat{P}(t) \approx \mathbb{P}(T > t) &= \mathbb{P}(T > t \text{ and } T > t-1) \\ &= \mathbb{P}(T > t | T > t-1) \cdot \mathbb{P}(T > t-1). \\ &\approx \mathbb{P}(T > t | T > t-1) \cdot \hat{P}(t-1).\end{aligned}$$

When we rewrite the Kaplan-Meier estimator we use the conditional probability of given that someone has survived till time $t-1$, he will survive time t as well. By using this method we have created a recursive formula for the Kaplan-Meier estimator, since the estimator is also based on the previous estimator. The recursive formula is easier to use as it does not need as much computations as the original formula.

The Kaplan-Meier curve (from now on abbreviated by KM curve) $\hat{P}(t)$ is a step function so it is not smooth. Furthermore t is between 0 and $\tilde{t} < \infty$, where $\tilde{t}$ is the end of the running time of the experiment. $\hat{P}(t)$ is now between 1 and $\hat{P}(\tilde{t}) \geq 0$. If $\tilde{t} \longrightarrow \infty$ then $\hat{P}(\tilde{t}) \longrightarrow 0$, but in general this will not happen since for example experiments stop because a lack of time and therefore we have $\hat{P}(\tilde{t}) > 0$. If there is enough data and there is no censored data the KM curve $\hat{P}(t)$ will approximate the survival probability $P(t)$.

Switching to prices

To use the Kaplan-Meier estimator for estimating the probability that someone is willing to pay a ticket we need to transform the current set-up.
Suppose that the observed life-time t_i corresponds with the price p_i that is paid for a ticket. Let n_i (the number of people at risk of dying) be the number of people that have not bought a ticket for price p_i yet and let the number of dying people d_i at time t_i correspond with the number of people that buy a ticket for price p_i.
The Kaplan-Meier estimator then becomes $\hat{P}(p_i)$ and gives the probability that someone will buy a ticket for price p_i or higher. We assume a finite population and if a performance is not sold out we assume that the tickets that are not sold are sold for € 0.–.

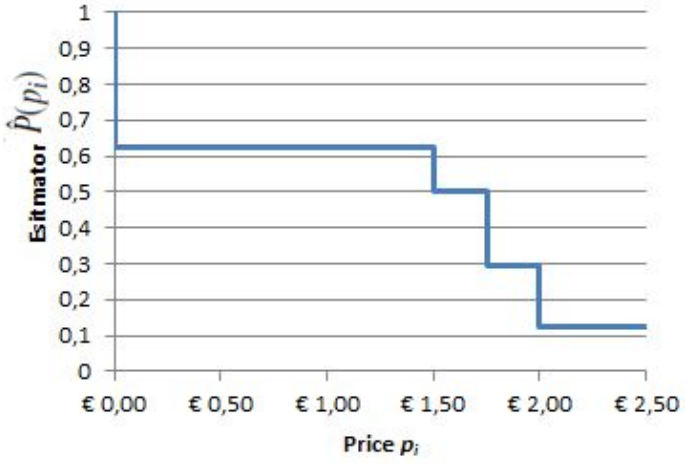


Figure 4.1: The KM curve for $i = 0, 1, 2, 3, 4$.

Example
Suppose that the following number of tickets are sold for performance 1:

1. 20 tickets for € 2.– and
2. 15 tickets for € 1.50.

There were still tickets available for performance 1 and in total there were 60 seats to be sold, so 25 tickets have not been sold. We suppose that these tickets were sold for € 0,–.
We cannot estimate the probability that someone buys a ticket for € 2.50 or more using this data, since there is no data avalaible, so we will need more data.

The following number of tickets are sold for performance 2:

1. 15 tickets for € 2.50,
2. 25 tickets for € 1.75 and
3. 20 tickets for € 0.– (the tickets that were not sold).

The data of both performances can be summarised in the following table:

p_i	n_i	d_i	$\hat{P}(p_i)$
€ 0.-	120	45	1
€ 1.50	75	15	0.625
€ 1.75	60	25	0.5
€ 2.-	35	20	0.292
€ 2.50	15	15	0.125

If we now plot p_i against $\hat{P}(p_i)$ we will obtain a typical KM curve. Figure 4.1 shows the graph of $\hat{P}(p_i)$.

Statistical equivalence

A log rank test is used to show that two (or more) KM curves are statistically equivalent [8]. Statistically equivalent means that by using a test statistic that compares the curves we do not have evidence that the curves are different. So we are comparing the curves to get an idea if for example different sales type have comparable curves.

A log rank test uses a statistic that gives an overall comparison of the KM curves and it is a large-sample Chi-square test. This test was developed in the 1970s by R. Mantel and D. Cox but was named the log rank test by Richard and Julian Peto [16] because the rank scores are made up by the logarithm from the survival estimator. Other statistical tests for survival analysis are often based on the log rank test but for example weights can be added. Since the 'survival analysis' of the ticket prices is relatively easy I will focus on the log rank test.

The null hypothesis and alternative hypothesis are as follows:

$$\begin{aligned} H_0 &: \textit{Both KM curves are statistically equivalent.} && (4.1)\\ &: \hat{P}_1 = \hat{P}_2 \\ H_1 &: \textit{Both KM curves are statistically inequivalent.} && (4.2)\\ &: \hat{P}_1 \neq \hat{P}_2 \end{aligned}$$

To test whether the null hypothesis should be accepted or rejected the log rank test statistic is used. To calculate this test statistic the expected number of dying people (so the expected number of tickets bought) is needed.

The expected number of deaths in group 1 (e_{1j}) and group 2 (e_{2j}) is defined as:

$$\begin{aligned} e_{1j} &= \frac{n_{1j}}{n_{1j}+n_{2j}} \cdot (d_{1j}+d_{2j}) \\ e_{2j} &= \frac{n_{2j}}{n_{1j}+n_{2j}} \cdot (d_{1j}+d_{2j}), \end{aligned}$$

where d_{ij} is equal to the number of deaths in group i in period j (with $1 \leq j \leq N$) and n_{ij} the total number of susceptible people in group i in period j. So the expected number of deaths in a group at time j is calculated by the proportion of the number of people in this group compared to the total number of people at time j, multiplied by the total number of deaths at time j.

Furthermore we need the sum of the observed deaths minus the expected deaths to calculate the log rank test statistic:

$$O_i - \mathbb{E}(O_i) = O_i - E_i = \sum_{j=1}^{N} (d_{ij} - e_{ij}), \qquad i = 1, 2.$$

The log rank test statistic is determined for a group by squaring this sum and dividing it by its variance:

$$\text{log rank test statistic} = \frac{(O_i - E_i)^2}{Var(d_i)}$$

and the sample variance is given as follows:

$$Var(O_i) = \sum_{j=1}^{N} \frac{n_{1j} n_{2j} (d_{1j}+d_{2j})(n_{1j}+n_{2j}-d_{1j}-d_{2j})}{(n_{1j}+n_{2j})^2 (n_{1j}+n_{2j}-1)} \qquad i = 1, 2.$$

Since given that the null hypothesis holds d_{ij} has the hypergeometric distribution with parameters $N = n_{1j} + n_{n2j}$, $n = n_{ij}$ and $K = d_{1j} + d_{2j}$ so we obtain $Var(d_{ij}) = n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}$ and if we use our parameters we will find the given variance.

The log rank test statistic will achieve a value x. Furthermore it is known that the log rank test statistic will approach a χ^2 distribution with one degree of freedom assuming that the null hypothesis is true [8]. Now suppose that we have a test statistic Y, that is given by:

$$Y = \frac{O_i - E_i}{\sqrt{Var(O_i)}}.$$

Then under the null hypothesis and by the central limit theorem this will approach a standard normal distribution and by squaring Y we obtain the log rank test statistic and by definition this will be a χ^2 distribution. The p value is the probability of, assuming the null hypothesis is true, observing a result at least as extreme as the value of the log rank test statistic. The p value can be calculated as follows:

$$p = \mathbb{P}(\chi^2 \geq x).$$

Using distribution tables or χ^2 calculators on the internet one can calculate the p value. The null hypothesis is rejected if the p value is smaller than the assumed level of significance α and the alternative hypothesis will be accepted and the two KM curves are not statistically equivalent.

The log rank test statistic can also be approximated by a Chi-squared test statistic, since the log rank test is a type of a Chi-squared test. Instead of the log rank statistic one can use

$$X^2 = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}.$$

This approximation is easier to use when one wants to compare two or more different KM curves and no software is available. The reason is that the log rank test statistic will become a matrix formula when dealing with three or more curves , since the variance will turn into a covariance matrix.

Example
Suppose there are three different sales type (CJP, Senioren and Stadspas) with the corresponding KM curves and we want to test whether we can merge them into one curve.
The three different KM curves are plotted in figure 4.2. If one has a good look, one will see that there is a larger correspondence between the curves of the Senioren and Stadspas sales, than between the curves of the Senioren and CJP sales. We shall prove this now.

First we shall demonstrate that the KM curves of the CJP and Senioren sales are statistically non-equivalent. The corresponding null hypothesis is

$$\begin{aligned} H_0 \quad &: \quad \text{the KM curves of the CJP and Senioren sales are equal.} \\ &: \quad \hat{P}_{CJP} = \hat{P}_{Senioren} \end{aligned}$$

and in the end we will reject it.

Figure 4.2: The three different KM curves that correspond to the sales types.

Example (continued)
We need the expected number of deaths of both sales types to calculate the Chi-squared test statistic and we will obtain a table that will look like this:

j	p_j	d_{1j}	d_{2j}	n_{1j}	n_{2j}	e_{1j}	e_{2j}	$d_{1j}-e_{1j}$	$d_{2j}-e_{2j}$
1	€ 0.00	400	1251	725	2270	400	1251	0	0
2	€ 7.70	6	1	325	1019	1.70	5.31	4.31	-4.31
3	€ 10.27	1	0	319	1018	0.24	0.76	0.76	-0.76
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
26	€ 20.00	1	34	1	34	1	34	0	0
Totals:						639.00	2355.57	85.14	-85.14

The log rank test statistic and the Chi-squared test statistic can be calculated using the totals:

$$\begin{aligned} \text{log rank test statistic} &= \frac{(O_2-E_2)^2}{Var(O_2-E_2)} = \frac{(-85.14)^2}{277.83} = 26.09 \\ X^2 &= \sum_{i=1}^{N}\frac{(O_i-E_i)^2}{E_i} = \frac{85.14^2}{639.00}+\frac{(-85.14)^2}{2355.57} = 14.42 \end{aligned}$$

The corresponding p values are

$$\begin{aligned} p_{lr} &= \mathbb{P}(\chi^2 \geq 26.09) = 0 \\ p_{X^2} &= \mathbb{P}(\chi^2 \geq 14.42) = 0.0001. \end{aligned}$$

For any of the most frequently picked significance levels ($\alpha = 0.10; 0.05; 0.01; 0.005$ and 0.001) it holds that $p \leq \alpha$.
We conclude that the KM curves of the CJP and Senioren sales types are significantly different.

Example (continued)
The Senioren and Stadspas sales types can be compared in the same way. We will obtain the following test statistics:

$$\begin{aligned} \text{log rank test statistic} &= \frac{(O_2 - E_2)^2}{Var(O_2 - E_2)} = \frac{(40.87^2}{485.65} = 3.439 \\ X^2 &= \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i} = \frac{(-40.87)^2}{2311.30} + \frac{40.87^2}{1436.36} = 1.885, \end{aligned}$$

and the corresponding p value are:

$$\begin{aligned} p_{lr} &= \mathbb{P}(\chi^2 \geq 3.439) = 0.0636 \\ p_{X^2} &= \mathbb{P}(\chi^2 \geq 1.885) = 0.1697. \end{aligned}$$

It applies for any likely significance level α that $p_{X^2} \geq \alpha$, so the null hypothesis is accepted, but we have that $p_{lr} \leq 0.10$, so the null hypothesis is not accepted for all signifance levels, but if we take $\alpha \leq 0.05$ then we can conclude that the KM curves of the Senioren and Stadpas sales types are significantly equivalent.

4.2 Kaplan-Meier curves per genre

It does not often occur often that a performance is completely sold out in Theater Bellevue. From now on, we will assume that a performance is sold out if more than 90% of the tickets are sold. It was possible to find the genres for the performances in the seasons 2009-2010, 2010-2011 and 2011-2012 was het mogelijk om de genres terug te halen. The following genres were assigned during these three seasons:

T	Drama (Toneel)
C	Cabaret (Cabaret/Kleinkunst/Stand-up Comedy)
P	Muppet Theatre (Poppentheater)
MT	Musical Theatre (Muziektheater)
M	Music (Muziek)
J	Youth Theatre (Jeugdtheater)
D	Dance (Dans)
DIV	Miscellaneous (Diversen)

Multiple genres can be assigned to one performance. All performances during the last three seasons are sorted per genre and performances with more than one genre are sorted into the genre which is more characteristic than the other genres. Performances that have been assigned for example the Muppet Theatre genre have often been assigned the Drama or Youth Theatre genre as well and will be sorted into the Muppet Theatre genre since this is the most characteristic genre of the two.

To compare the prices of the different performances in the different seasons we take into account the inflation. The inflation in September 2011 was 2.7% and in September 2010 1.6%. The prices of the performances in 2009-2010 are increased by 1.6% and 2.7% and the prices of the performances in 2010-2011 are increased by 2.7%.

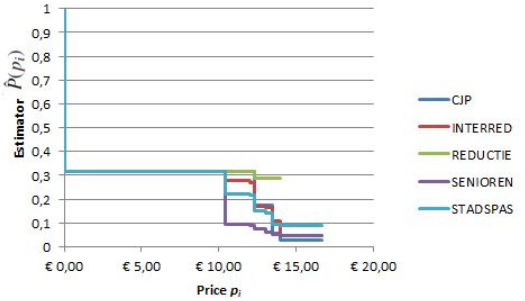
(a) Full Price Drama in the Grote Zaal (not sold out)

(b) Discount Price Drama in the Grote Zaal (not sold out)

(c) Full Price Drama in the Grote Zaal (sold out)

(d) Discount Price Drama in the Grote Zaal (sold out)

(e) Full Price Drama in Klein Bellevue (not sold out)



(f) Discount Price Drama in Klein Bellevue (not sold out)

(g) Full Price Drama in Klein Bellevue (sold out)

(h) Discount Price Drama in Klein Bellevue (sold out)

Figure 4.3: The KM curves per genre and hall (1)

We will make different KM curves for the full prices and the discount prices of a performance and we will split these prices even further, since Theater Bellevue assigns different ticket types which tells us where tickets are bought and by whom and we can discover differences in buyer behaviour. We will use the following ticket types since they are used every season and give us the most information:

1. **CJP**
   The *CJP* ticket type can only be bought by owners of a CJP membership card. A CJP card can only be bought by people younger than 30 years, and owners can buy tickets for a discount price in Theater Bellevue. These tickets cannot be bought on the internet and have to be bought at the ticket booth or by making a reservation.

2. **Interred**
   This ticket type is used for all discount price tickets bought on the internet.

3. **Intervol**
   The *intervol* ticket type is used for all full price tickets that are bought online.

4. **Normaal**
   The full price tickets bought at the ticket booth are given the label *normaal*.

5. **Reductie**
   This label is assigned to customers if they buy there tickets at the ticket booth and for some reason they have the right to buy a ticket for the discount price, but they do not get one of the other labels.

6. **Senioren**
   Elderly people can buy tickets for the discount price and this label will be assigned when they buy their tickets at the ticket booth.

7. **Stadspas**
   The Stadspas is a card for people and families in Amsterdam if they have an income that is equal or less than the minimal income. Owners of a Stadspas can buy tickets for the discount price.

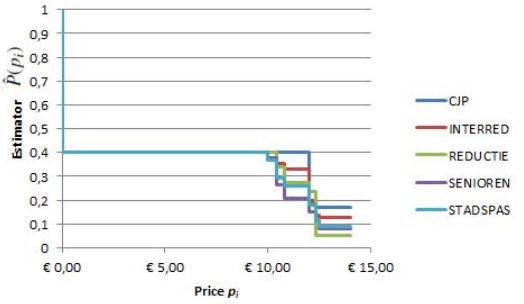
Furthermore we will compare the curves that have the most similarities by sight (so we will not compare the curves that come from different price categories, for example Normaal and CJP) and show these result in tables that are published in Appendix A.

Drama

Most performances in Theater Bellevue are Drama performances. The Drama performances are scheduled in all three halls and the data is split for each hall. Therefore we obtain six seperate databases for the Drama performances.

The KM curves for the Drama performances in the Grote Zaal are shown in Figures 4.3a and 4.3b (not sold out performances) and Figures 4.3c and 4.3d (sold out performances). We see that there are similarities between the *normaal* and *intervol* curves in Figure 4.3a and between the *CJP*, *reductie* and *interred* curves in Figure 4.3b. These observations are confirmed by the results in Table A.1a for a confidence level $\alpha = 0.1$. We see that the *CJP* curve is statistically equivalent with the *interred* and *reductie* curves, that the *interred* curve is statistically equivalent with the *reductie* and *stadspas* curves, that the *intervol* curve is statistically equivalent with the *normaal* curve, that the *reductie* curve is statistically equivalent with the *stadspas* curve and furthermore that the *senioren* curve is statistically equivalent with the *stadspas* curve.
When we look at the sold out Drama performances in the Grote Zaal we see that looking at Figures 4.3c and 4.3d and using Table A.1b that almost all discount KM curves are statiscally equivalent except for the *interred* and *reductie* curve and that the full price KM curves *normaal* and *intervol* are statistically inequivalent as well.

(a) Full Price Drama in the Paloni Zaal (not sold out)



(b) Discount Price Drama in the Paloni Zaal (not sold out)

(c) Full Price Drama in the Paloni Zaal (sold out)

(d) Discount Price Drama in the Paloni Zaal (sold out)

(e) Full Price Cabaret in Klein Bellevue (not sold out)

(f) Discount Price Cabaret in Klein Bellevue (not sold out)

(g) Full Price Cabaret in Klein Bellevue (sold out)

(h) Discount Price Cabaret in Klein Bellevue (sold out)

Figure 4.4: The KM curves per genre and hall (2)

The KM curves for the Drama performances in Klein Bellevue are shown in Figures 4.3e, 4.3f, 4.3g and 4.3h. We notice that there are great differences between the KM curves in the discount price figures (Figures 4.3f and 4.3h). There is not enough data available to make good estimations of the probabilities (in the case of the sold out Drama performances in Klein Bellevue, there are only 18 CJP tickets sold out of 2292 tickets in total). However we will still compare the curves. For the not sold out out performances we find that all discount prices KM curves are statistically equivalent with each other and that the full price KM curves are also statistically equivalent with each other.
For the sold out performances we find that the *normaal* and *intervol* curves are statistically equivalent and that the discount price curves are statiscally equivalent except for the *reductie* curve and *senioren* curve and the *interred* curve and *reductie* curve.

The KM curves for the Drama performances in the Paloni Zaal are plot in Figures 4.4a, 4.4b, 4.4c and 4.4d. We see that that there are much more differences between the curves, than for example for the Drama performances in the Grote Zaal. Still, when we look at the p-values of the KM curves of the not sold performances in Table A.3a we see that a lot of KM curves are -perhaps quite unexpected- statiscally equivalent. The only exception are the *intervol* curve and the *normaal* curve, the *senioren* curve and the *CJP* and *interred* curves.
The results of the equivalence tests of the sold out performances in Table A.3b correspond more to our expectations. The only statistically equivalent KM curves are the *CJP* curve and the *senioren* and *stadspas* curves and the *senioren* curve and *stadspas* curve.

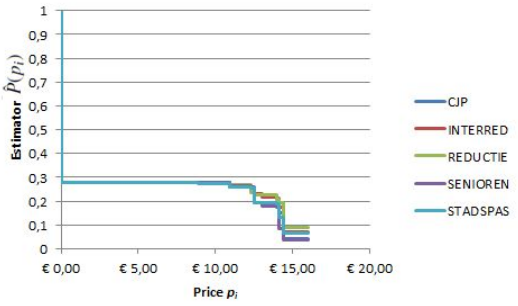
Cabaret

Cabaret is in general scheduled in Klein Bellevue. It can happen that a cabaret performance is scheduled in the Grote Zaal, but this is incedently. There are too less cabaret performances in the Grote Zaal to obtain enough data, so the cabaret KM-curves are only made for Klein Bellevue.
The KM curves for the Cabaret performances are shown in Figures 4.4e, 4.4f, 4.4g and 4.4h. We see that there are differences between the curves especially for the discount price curves. When we compare the curves of the not sold out Cabaret performances we find that the *normaal* curve and the *intervol* curve are statistically inequivalent, while the discount price curves are all -except for the *CJP* curve and *reductie* curve- statistically equivalent (Table A.4a).
If we compare the curves of the sold out Cabaret performances we see that all discount price curves are statistically equivalent and again we find that the *intervol* curve and the *normaal* curve are statistically inequivalent.

Muppet Theatre

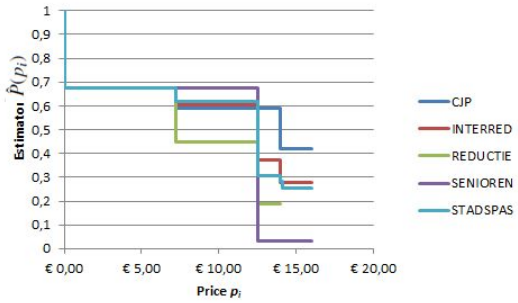
There are not enough Muppet Theatre performances in each hall to split the data per hall. So we will calculate, plot and compare all Muppet Theatre performances at once. Since Muppet Theatre does not sell out often, there is not enough data to give a good estimation of the probability curves and the probability curves do not give an approximation that is good enough. Still we will show the curves, but we will not use them afterwards.
The KM curves of the Muppet Theatre performances are shown in Figures 4.5a, 4.5b, 4.5c and 4.5d. Table A.5a shows the results of the comparison of the KM curves of the not sold out Muppet Theatre performances. We find that the full price curves are statiscally equivalent as are almost all discount price curves except for the *reductie* and *senioren* curves. When we compare the curves of the sold out performances (Table A.5b) we see that the difference between the *reductie* curve and the *CJP*,*interred* and *stadspas* is too large, so these curves are statistically inequivalent, while the other discount price curves are statiscally equivalent. The full price curves are statistically equivalent too.

(a) Full Price Muppet Theatre (not sold out)



(b) Discount Price Muppet Theatre (not sold out)

(c) Full Price Muppet Theatre (sold out)



(d) Discount Price Muppet Theatre (sold out)

(e) Full Price Musical Theatre (not sold out)

(f) Discount Price Musical Theatre (not sold out)

(g) Full Price Musical Theatre (sold out)

(h) Discount Price Musical Theatre (sold out)

Figure 4.5: The KM curves per genre and hall (3)

Musical Theatre

Musical Theatre performances are scheduled in all three halls, but most performances are scheduled in the Grote Zaal. It differs from year to year if performances get scheduled in the Paloni Zaal and how much performances are scheduled in Klein Bellevue. So in this case we will use all data to get an idea about the probability curves, but we need to keep in mind that it is mostly about the performances in the Grote Zaal.
The KM curves of the Musical Theatre peformances are shown in Figures 4.5e, 4.5f, 4.5g and 4.5h. We see that the curves coincide more than for example the curves in Figure 4.5d. Table A.6a confirms our assumptions for the not sold out Musical Theatre performances. All discount price curves are statiscally equivalent and so are the full price curves.
The same applies for the sold out Musical Theatre performances.

Music

All Music performances of the last three years are scheduled in Klein Bellevue. So these curves and the estimations of the probabilities can only be applied to performances in Klein Bellevue.
The KM curves are shown in Figures 4.6a, 4.6b, 4.6c and 4.6d. Figures 4.6c and 4.6d show and especially Figure 4.6d that there is too less data to obtain good estimations of the probabilities for sold out performances. So we will compare the curves for now, but we will not use the sold out curves anymore.
Table A.7a shows us for the not sold out Music performances that all discount price curves are statistically equivalent and the full price curves too. For the sold out performances we find that the discount price curves and the full price curves are statistically equivalent too (Table A.7b).

Youth Theatre

There are too few Youth Theatre performances each year to obtain enough data to get an estimation. Furthermore there are apart from the premières no sold out performances and the genre Youth Theatre is in general applied as an extra genre to performances. For this reason I have decided that I will not do a Kaplan Meier analysis for this genre.

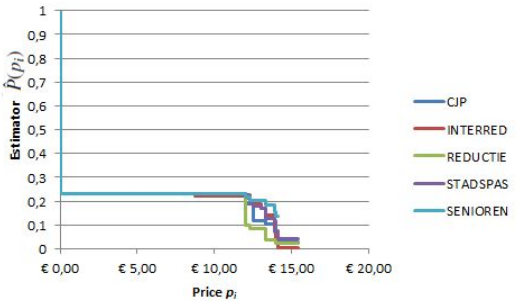
Dance

All Dance performances are scheduled in the Grote Zaal. The corresponding KM curves are shown in Figures 4.6e, 4.6f, 4.6g and 4.6h. We see that there are similarities between the curves and this is confirm by Table A.8a and Table A.8b. We find that the discount price KM curves for both the not sold out and sold out Dance performances are all statiscally equivalent and that the full price KM curves are statistically inequivalent.

Miscellaneous

There are not many Miscellaneous performances per year, but it is enough to get an estimation for the not sold out performances, while there are too less sold performances to get a good estimation. This can be seen in Figures 4.7a, 4.7b, 4.7c and 4.7d. Especially for the sold out performances it is clear that there is too less data available as the curves do not approximate a survival function.
However we see that for the not sold out performances that the curves lie close to each other and this is confirmed by the results of the Chi-squared test in Table A.9a. All discount price KM curves are statistically equivalent and the full price curves are statistically equivalent too.
For the sold out performances we find that the full price KM curves are not statistically equivalent, but that the discount prices are statiscally equivalent (Table A.9b).

(a) Full Price Music (not sold out)



(b) Discount Price Music (not sold out)

(c) Full Price Music (sold out)

(d) Discount Price Music (sold out)

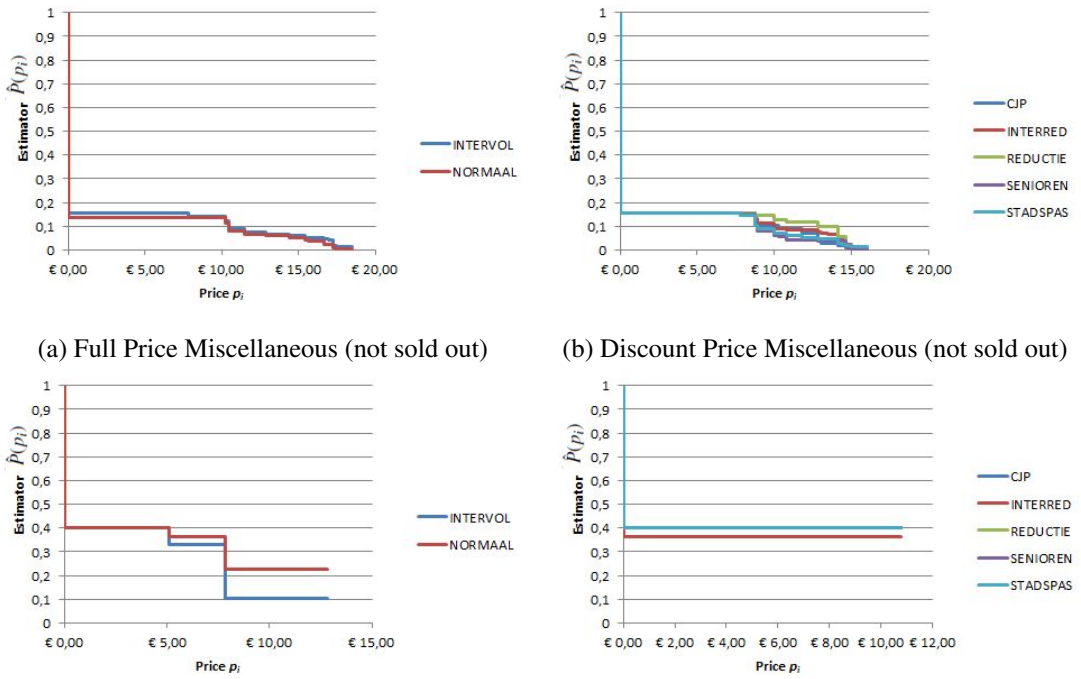
(e) Full Price Dance (not sold out)

(f) Discount Price Dance (not sold out)

(g) Full Price Dance (sold out)

(h) Discount Price Dance (sold out)

Figure 4.6: The KM curves per genre and hall (4)



(a) Full Price Miscellaneous (not sold out)

(b) Discount Price Miscellaneous (not sold out)

(c) Full Price Miscellaneous (sold out)

(d) Discount Price Miscellaneous (sold out)

Figure 4.7: The KM curves per genre and hall (5)

4.3 Observations

In the present situation there are two price categories: the full price and a discount price. Tickets can be bought on the internet and at the ticket booth and discount tickets can be bought by many different people.

The analysis of the Kaplan-Meier curves shows us that although now there are only two categories it may be wise to introduce more pricing categories. The tickets with the *normaal* and *intervol* labels are sold for the full price, but for Cabaret (both sold out and not sold out) performances, sold out Drama performances in the Grote Zaal, Drama performances in the Paloni Zaal (both sold out and not sold out) and for Dance performances the curves belonging to these labels are statistically inequivalent and something similar holds for the discount price tickets.

For example, for sold out Drama performances in the Grote Zaal we find that the *CJP* and *senioren* curves are statistically inequivalent, so it may be wise to introduce a price for elderly people which is a bit lower than the full price, but higher than the *CJP* price.

And the KM curves for sold out Dance performances show us that customers who buy their full price tickets online are willing to pay more than customers who buy their tickets at the ticket booth.

5 | Modelling and optimisation

We now know the pre-sale curve from the data analysis and the Kaplan-Meier curves and by combining the curves we can make a model that gives the optimal price for a certain time and the number of seats that are still available that maximises the revenue. As we have seen in the previous chapter, we have many different KM curves and we will have as many price strategies, since we will have matrices for the full prices and for discount prices. To develop a model we will first approximate all the curves so that we can use them in the model and then develop and evaluate the model.

5.1 Approximations of the curves

To use the pre-sale and the Kaplan-Meier curves we need to make approximations of all the curves.

Pre-sale curve

To approximate the pre-sale curves of the seasons we will first calculate the average pre-sale curve of all the pre-sale curves as shown in Figure 3.4. This average pre-sale curve has the shape of the following function:

$$f(t) = \frac{a}{b+t}, \qquad (t \text{ in weeks}).$$

The variables a and b are now calculated by using least squares and the *Solver* function of *Microsoft Excel*. For the pre-sale curve as obtained in Chapter 3 we get the following results:

$$a = 0.169676, \qquad b = 0.376362.$$

We will use the values for a and b right now, but it would be better to update them if the reservation data are known and the pre-sale curve gives a better approximation of the time at which tickets are bought.

Kaplan-Meier curves

We have Kaplan-Meier curves for many genres and for different halls. They differ substantially that we can not use one approximation but should use different approximations. There are however similarities in the KM curves. All KM curves are of the form

$$P(p) = \begin{cases} 1 & p = 0 \\ c & \text{for } 0 < p < p' \text{ where } p' \text{ is the breakpoint} \\ g(p) & \text{for } p' < p < \tilde{p} \end{cases}$$

with some constant c, which satisfies $0 < c < 1$, and a function $g(p)$, which satisfies $g(p') = c$ and $g(\tilde{p}) \geq 0$. So the approximation of the KM curves is mainly about approximating the function $g(p)$. The function $g(p)$ is approximated by using *trend lines* in *Microsoft Excel* or if the approximation of the trend line is worthless, by guessing the type of the function and approximating its constant by using least squares and the *Solver* function of *Microsof Excel*. The approximations of the KM curves can be found in Table B.1 and Table B.2

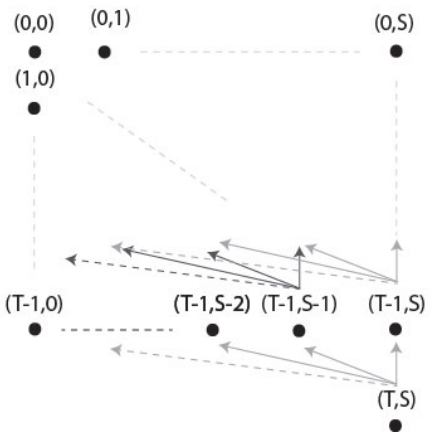


Figure 5.1: An image of a discrete time Markov Decision Process, where each point is a state and each arrow is an action with a certain probability.

5.2 The model

To implement dynamic pricing in Theater Bellevue we need to know the optimal price at each moment for the number of seats that are still available. The model is based on Markov Decision Processes (MDP's). A Markov Decision Process consists of a quadruple that contains:

- a set of states $\mathbb{S}$,
- a set of actions $\mathbb{A}$,
- a reward function $R(s, a)$ for state s and action a,
- and a set of descriptions L.

The set of descriptions L is not really needed, so the triple $(S, A, R(s, a))$ is sufficient.

In our case we have:

- a statespace $\mathbb{S} = \{(t, s) : T \geq t \geq 0,\ S \geq s \geq 0\}$, where t is the remaining time in weeks till a performance and $T \leq 52$, and s is the number of seats available with $s \leq S$, where S is the maximum number of seats available at the start of the sales period.
- a set of actions $\mathbb{A} = \{\text{buying of } a \text{ tickets} : 0 \leq a \leq S, \text{ with a certain probability of buying } a \text{ tickets: } \mathbb{P}(a)\}$,
- and a reward function $R((t, s), a) = p(t, s) \cdot \min(a, s)$, where $p(t, s)$ is the price asked for state (t, s).

So if we start in state (T, S) at the start of the sale and when no seats have been sold, we move backward in time and we can move to state

$$\begin{cases} (T-1, S) & \text{with probability } \mathbb{P}(0) \\ (T-1, S-1) & \text{with probability } \mathbb{P}(1) \\ \vdots & \vdots \\ (T-1, 0) & \text{with probability } \mathbb{P}(S), \end{cases}$$

which can be seen in figure 5.1. We now want to find the pricing strategy $p(t, s)$ that maximises $\sum_a R((t, s), a)$.

So we can formulate the problem as follows:

$$\begin{array}{ll} \max & \sum_a R((t,s),a) \\ \text{with} & t \text{ the remaining time till a performance} \\ & s \text{ the number of seats still available} \\ & p(t,s) \geq 0 \\ & p(t,s) \geq p(t+1,s) \\ & p(t,s) \in \mathbb{N} \cup 0.5\mathbb{N} \end{array}$$

The fifth line in the problem is given by Theater Bellevue. They do not want to encourage the customers to wait before buying their tickets and see if it gets cheaper. The public of Theater Bellevue is a quite last-minute public (see Chapter 3) and this condition makes sure that buying you tickets early in the season guarantees the cheapest tickets.
If we want to program this using dynamic programming and we know the probabilities of which actions happen and we know the reward function we can use the following formulation for $p(t,s)$, which are also known as the *Bellman equations*:

$$p(t,s) = \arg\max_p \sum_{d=0}^{\infty} \mathbb{P}(D(p,t)=d) \cdot (p\min(d,s) + V(t+1, \min(s+d, S))),$$
$$\text{for } T \geq t \geq 0, S \geq s \geq s,$$

where

$$V(t,s) = \max_p \sum_{d=0}^{\infty} \mathbb{P}(D(p,t)=d) \cdot (p\min(d,s) + V(t+1, \min(s+d, S)))$$

is the value function for t time left till the performance and s seats still available. In these formulas we have that $\mathbb{P}(D(p,t)=d)$ is the probability that someone buys d tickets, $p\min(d,s)$ is the reward function and V is the reward function at time $t+1$. These formulas are given in a recursive way and furthermore we have a recursion in backward time.
The function $D(p,t)$ is the demand for a performance for price p and at time t. We assume that the probability that the demand for a performance $D(p,t)$ is Poisson distributed with parameter $\lambda(p,t) = \hat{P}(p) \cdot f(t) \cdot S$, that is:

$$\mathbb{P}(D(p,t)=d) = \frac{\lambda(p,t)^d}{d!} \cdot e^{-\lambda(p,t)}.$$

So we will have a price matrix p

$$p = \begin{pmatrix} p(0,0) & \cdots & p(0,S) \\ \vdots & \ddots & \vdots \\ p(T,0) & \cdots & p(T,S) \end{pmatrix}$$

that tells us what price to offer to the customer for each time t and number of seats s left to sell.

Dynamic pricing model

To summarise the model we will give the pseudo code in Algorithm 1.

Results

This model can give very large price matrices if we want an advice for performances in June in the Grote Zaal. We change the input S from the number of seats to percentages and we will make jumps of 10%. We will now have price matrices with at most 10 columns.

After running the model we will obtain price matrices for every approximation we made in this chapter.

Algorithm 1 Dynamic Pricing

Choose S the number of seats
Choose T the number of weeks before a performance.
Choose an orientation price P.
Let $\tilde{P}$ be the maximal price possible: $\tilde{P} = 1.3 \cdot P$.
Choose the right approximation of the KM curve $P(p)$.
Let $f(t) = \frac{a}{t+b}$.
Let $\lambda(p,t) = P(p) \cdot f(t) \cdot S$.
for $t = T$ **to** 0 **do**
  for $s = S$ **to** 0 **do**
    $\mathbb{P}(D(p,t) = d) = \frac{\lambda(p,t)^d}{d!} \cdot e^{-\lambda(p,t)}$.
    $V(t,s) = \max_{\substack{p \\ 10 \le p \le \tilde{P} \\ p \in \mathbb{N} \cup 0.5\mathbb{N}}} \sum_{d=0}^{\infty} \mathbb{P}(D(p,t) = d) \cdot (p \min(d,s) + V(t+1, \min(s+d, S))).$
    $p(t,s) = \arg\max \sum_{d=0}^{\infty} \mathbb{P}(D(p,t) = d) \cdot (p \min(d,ss) + V(t+1, \min(s+d, S))).$
  end for
end for

What is striking is that all matrices have the same form. There is a large block which has the same price and only changes for the last seats to be sold:

$$p = \left(\begin{array}{c|c|c} p(0,0) & \cdots & p(0,S) \\ p(1,0) & \cdots & \vdots \\ \vdots & \ddots & \vdots \\ \hline p(T,0) & \cdots & p(T,S) \end{array}\right)$$

The price in the large block is equal to the breaking point of the KM curve and the price for the last seats increases in time. The price in the last week or in the last two weeks is P, the highest price possible and it is gradually reached.

Table 5.1a and Table 5.1b show the price matrices for sold out Cabaret performances in Klein Bellevue and for not sold out Drama performances in the Grote Zaal. These results and the other results of the model show that the model is indeed built up from one large block and other smaller parts. The Tables show us that the model always advices to ask the breaking point price no matter what the orientation price of Theater Bellevue is. Since the KM curves, and thus the model, are based on all data of Cabaret performances of the last three theatre seasons it is only an average and does not take into account the number of performers on stage.
Suppose that:

- N is the maximal number of seats available and n is the number of seats sold,
- p is the orientation price and p' is the price at the breaking point,
- P_i is the price asked for the last 10% of the tickets in week i before the performance ($P_i \le P$, where $P = 1.3 \cdot p$).

and let R be the revenue collected.
If we follow the prices advised by the model and $p' < \frac{0.87}{0.90} p$, the revenue of a sold out performance $R = 0.9Np' + 0.1NP_i$ is always smaller than the maximal revenue ($R_p = Np$) when we are not using

Table 5.1: Price matrices for two different KM curves.

18.50	14.–	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50
18.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50
17.–	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50
17.–	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50
16.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50
14.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50	13.50

(a) Price matrix p for Sold out Cabaret performances in Klein Bellevue, for 6 weeks before the performance with an orientation price of € 14.50

23.–	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50
23.–	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50
21.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50
21.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50
18.–	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50
18.–	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50	16.50

(b) Price matrix p for not sold out Drama performances in the Grote Zaal, starting from 6 weeks before the performance with an orientation price of € 18.–

the model and we use the orientation price p:

$$\begin{aligned} 0.9Np' + 0.1NP_i &\leq 0.9Np' + 0.13Np \\ &< 0.9N \cdot \frac{0.87}{0.9} p + 0.13Np \\ &= Np = R_p. \end{aligned}$$

Example
Suppose that the orientation price of a performance is € 14.–. If the price at the breaking point is smaller than $\frac{0.87}{0.9}$·€ 14.– = € 13.53, the model will not yield more money than when all tickets are sold for the orientation price.

So since the current model only shows the breaking point price, the revenue will not become higher when implementing the model than when we will use the orientation prices. But if we adapt the model and ask the orientation price instead of the breaking point price if the orientation price is higher, we will see the differences and the revenue of the model will be:

$$R = 0.9Np + 0.1NP_i \geq 0.9Np' + 0.1NP_i.$$

Example
Suppose there is a Cabaret performance in Klein Bellevue and all 115 seats are avalaible and the performance sells out. The orientation price will be € 14.50 and 30% of the tickets are sold in the last week. Then according to the model and the results in Table 5.1a:

- $0.8 \cdot 115$ tickets are sold for € 13.50
- $0.1 \cdot 115$ tickets are sold for € 14.– and
- $0.1 \cdot 115$ tickets are sold for € 18.50

This yields a revenue of € 1615.75. If all tickets were sold for the orientation price the performance would have yielded € 1667.50, which is € 51.75 (3.2%) extra compared to the dynamic pricing model. But if we adapt the model and change the price of the large block from the breaking point price to the orientation price

- $0.9 \cdot 115$ tickets are sold for € 14.50 and
- $0.1 \cdot 115$ tickets are sold for €18.50

and the total revenue would have been € 1713.50. This is an extra € 97.75 (6.0%) compared to the original dynamic pricing model and an extra € 46.– (2.8%) compared to the orientation prices.

5.3 Discussion

The model for dynamic pricing is characterised by the information of the last three theatre seasons and gives information about what price to ask for a ticket at a certain time and for a certain amount of tickets left. We have already changed the model from giving information about each single tickets to a model that gives information about occupation degrees in larger steps.

The model in its present form cannot be implemented in the ticket system right now. The software (the ticket system and the website) is not advanced enough to implement the model. Changes in the prices will be done manually during the testing period (and probably also after this testing period, until the software has become more advanced), which will take a lot of time and the number of price steps per performance will differ between one and approximately six.

The changes in prices during the first testing phases of the implementation of the model have to be done manually. This will cause a lot of work and some problems. If a performance passes the 90%-level in the weekend, then the price will be changed on Monday and it is possible that extra revenue is lost. Performances in, for example, the Paloni Zaal have a maximal capacity of approximately 50 seats, then if 10% of the tickets are sold for a higher price and the critical level is passed in the weekend it is very likely that such a performance will sell out without collecting extra revenue. Since 5 seats are not very much and the extra revenue will not be collected. For this reason we adapted and simplified the model another time and together with Theater Bellevue we came to the following model:

- **The starting price.** The starting price is the original price, indicated by inter alia, the number of performers on the stage, the hall, the length of the performance, etc.
- **The first increase of prices.** If 80% of the tickets are sold a new, higher price is introduced, which is approximately 15% higher than the original price.
- **The second increase of prices.** If 90% of the tickets are sold an even higher price (the maximal price, which is approximately 30% higher than the original price) is introduced.

If a performance sells out, the revenue R will be

$$R = 0.8Np + 0.1N \cdot 1.15p + 0.1N \cdot 1.30p = 1.045Np.$$

This model is the most optimal model so far, and so it is better than the previous model. I have however the idea that if the reservation dates are registered and the pre-sale curve is adapted to the new data that the matrix based model will be more optimal than this model, since using, for example, linear approximations of the pre-sale curve resulted in matrices which did not contain large blocks and had many more price changes.

The models introduced in this chapter are only a guiding principle and are certainly not binding. In Chapter 6 and Chapter 7 we will discuss the implementation of the model further.

6 | Evaluation of the model

The pricing model is tested on selected performances in the remaining months of the theatre season to evaluate the characteristics of the model. Some performances are scheduled after the internship and will not be evaluated in this thesis so I will only show the set up of the investigation, while we will discuss the results of the model on other performances.
The model will be evaluated on the following performances:

1. *Café Lehmitz* by Theatergroep Carver
2. *Electropis* by Henry van Loon
3. *Revolte* by De Gebroeders Fretz
4. *Funzone* by mugmetdegoudentand
5. *Motregenvariaties* by Bellevue Lunchtheater and Robert Alberdingk Thijm
6. *Hiroshima mon amour* by Johanna ter Steege
7. *237 redenen voor seks (reprise)* by Bellevue Lunchtheater and Orkater
8. *Mijn slappe komedie voor vier personen, een handjevol personeel en een tafel die niet vrijkomt* by Magne van den Berg

Theater Bellevue has been communicating their new strategy to the customers since the start of 2013. Customers are told at the ticket booth and on the internet that prices are starting at a certain price and can increase depending on the popularity of a performance.

The model as described in Chapter 5 cannot be used for technical reasons which are discussed at the end of this same chapter. Therefore a simpler version has been chosen and the changes in the occupation rates are discovered manually as are the changes in prices. This will happen only on working days, so if the occupation rates pass the critical values in the weekends, changes will be made on Mondays and there is a chance that the extra revenue will be missed.

Table 6.1: The prices used for testing dynamic pricing.

	Full Price	Discount Price
Starting Price	€ 21.–	€ 18.50
80%	€ 24.–	€ 21.50
90%	€ 28.–	€ 25.50

(a) Café Lehmitz – Theatergroep Carver

	Full Price	Discount Price
Starting Price	€ 15.–	€ 13.50
80%	€ 17.50	€ 16.–
90%	€ 20.–	€ 18.50

(b) Electropis – Henry van Loon

	Full Price	Discount Price
Starting Price	€ 14.50	€ 13.–
80%	€ 16.–	€ 14.50
90%	€ 18.50	€ 17.–

(c) Revolte – De Gebroeders Fretz

6.1 Performances and results

An outline of the characteristics of the performances (genre, dates, hall, reasons for applying dynamic pricing, etc.) will be given for each performance. The results of applying dynamic pricing are discussed if it has already been applied to a performance and the results will be discussed per performance.

6.1.1 Café Lehmitz – Theatergroep Carver

Café Lehmitz has been performed earlier by Theatergroep Carver in 1991. The performance is now scheduled as the goodbye tour of Theatergroep Carver and has had many good reviews and a completely sold out run was expected, since it was the last possibility to see the performance and the actors performing together.
Café Lehmitz is a pantomime performance but its genre is Drama and it was scheduled from January 2 till and including January 27 in the Grote Zaal of Theater Bellevue.
The prices as shown in Table 6.1a were used for the test.

Results

The performances of *Café Lehmitz* were completely sold out, so this price test had the ideal conditions. 4977 tickets have been sold for 20 performances. On average 258.85 tickets have been sold per performances. The whole production has yielded € 103,525.10. If all tickets were sold for the normal prices the revenue would be € 94,615.14. Implementing dynamic pricing yielded an extra **€ 8913.–** which is a 9.4% extra compared to the revenue that could be expected with the normal prices.
So dynamic pricing has been very succesful for these performances.
However, there have been questions from customers at the ticket booth and there have been a couple of complaints, but after a short explaination of the situation most customers understood the idea and understood the reasons.

Another -not really fair- point is that in fact 30% of the tickets have been sold for a higher price. At the start of the sales period approximately 220 seats were available. So 22 tickets would be sold for € 24.– and 22 tickets would be sold for € 28.– and an extra revenue of € 220.– was expected per performance. But it is possible to place a zeroth row in front of the other rows and the capacity can be extended to maximal 254 seats. These extra seats are directly sold for € 24.– and after approximately 5 seats, the

remaining 25 seats are sold for € 28.– and the extra revenue increases by another € 180.–. The total extra revenue per performance is now approximately € 400.–. If the capacity was set directly at 254 only 50 tickets were sold for higher prices and the extra revenue would be € 250.–.

The question is what to do with this situation. The capacity was extended because the demand was clearly higher than expected. But would it not be better and more fair to set the capacity at the maximum from the start. For this performance it would have worked and it would have been fair to do so, but other performances never reach this maximal capacity and sell at most 220 seats.
A strategy and general idea have to be made about this problem.

6.1.2 Electropis – Henry van Loon

Electropis is the second solo performance of comedian Henry van Loon. He has performed earlier in Theater Bellevue in combination with other comedians and musicians and is known by the public of Theater Bellevue.
Electropis was scheduled from January 15 till and including January 19 in Klein Bellevue. The genre of the performance is Cabaret. Table 6.1b shows the prices that were used for the test.

Results
The performances of *Electropis* were not sold out. 515 seats were sold (89.6%) and € 6878.30 has been yielded. If the 515 seats would have been sold for the normal price, then the performances would have yielded € 6575.80. So implementing dynamic prising yielded an extra revenue of € 302.50, which is an increase of 4.6%. However, there were complaints at the ticket booth about the price and especially about the 90% full price. People think that € 20.– is a lot of money for a cabaret performance and these people did not buy tickets. So if all 575 available seats had been sold with the same ratio full price and discount prices, the performances could have yielded € 7341.40, which is an extra 7.6%. So more money could have been yielded if the performance had sold out without implementing dynamic pricing. Still we cannot predict if the performance would have sold out if no dynamic pricing had been implemented.

6.1.3 Revolte – De Gebroeders Fretz

The performance *Revolte* is a Cabaret performance about the speeches of Obama and making Johan Fretz the first coloured minister president of The Netherlands with spin doctor Marcel Harteveld. The performance has been getting a lot of media attention and a sold out run is expected. *Revolte* was scheduled from January 29 till and including February 2 in Klein Bellevue.
The prices in Table 6.1c were used for the test.

Results
The performances of Revolte were not sold out as well. 500 tickets have been sold in total over five performances, which gives us an occupation degree of 95%. The performances have yielded a revenue of € 6774.57 and if these tickets had been sold for the normal price, the performances would have yielded € 6636.57. The implementing of dynamic pricing yielded an extra revenue of € 138.–, which is an increase of 2.1%.
A blockade of 10 seats is set for each performance so 105 seats could have been sold in total per performance. If all 105 seats per performance were sold in the same proportion of seats that is the case now, than the five performances could have yielded a total of € 6953.06. This is € 178.49 extra with respect to the revenue yielded.
Although the performances did not sell out, there have been no complaints at the ticket booth about the implementation of dynamic pricing and the increased prices. This confirms the idea that came up during the testing period of Electropis that prices below € 20.– are no problem for the customers.

6.1.4 Funzone – mugmetdegoudentand

mugmetdegoudentand and Theater Bellevue are closely tied together. *Funzone* is the third mugmetdegoudentand production that is scheduled in Theater Bellevue this theatre season. The first two production sold out and this is expected for this performance as well.
Funzone is scheduled from February 28 till and including March 16 in the Grote Zaal. The genre is Drama, although it is often difficult to give a specific genre to performances of mugmetdegoudentand.

6.1.5 Motregenvariaties – Bellevue Lunchtheater and Robert Alberdingk Thijm

Motregenvariaties is a production of Bellevue Lunchtheater and is written by Robert Alberdingk Thijm. The performance is played by the well known actors Olga Zuiderhoek and Ria Eimers and it directed by Johan Simons. Theater Bellevue has noticed that lunch performances with well known actors are almost certain to sell out.
Motregenvariaties is a Drama performances and it is scheduled from March 19 till and including April 14 in the Paloni Zaal.

6.1.6 Hiroshima mon amour – Johanna ter Steege

Hiroshima mon amour was a hit when shown on theatre festival *Oeral* on Terschelling. Reviews were good and a national tour is scheduled this theatre season. The performance is scheduled from April 1 till and including April 3 in the Grote Zaal as a Drama performance.

6.1.7 237 redenen voor seks (reprise) – Bellevue Lunchtheater and Orkater

237 redenen voor seks has been developed for the Bellevue Lunch productions in 2011 and because of its success it is touring through the country and again playing in Theater Bellevue. The performance is scheduled from April 12 till and including April 16 in the Grote Zaal as a Drama performance.

6.1.8 Mijn slappe komedie voor vier personen, een handjevol personeel en een tafel die niet vrijkomt – Magne van den Berg

The play -as suggested in the title- is a comedy for four actors. Magne van den Berg has written many other plays and comedies and she won a price for one of them. The director and actors are also known for what they have achieved so far. The performances are scheduled from May 21 till and including June 9 in the Grote Zaal.

6.2 Observations

The results of applying dynamic pricing are promising. Especially the results of the performance Café Lehmitz. The extra revenue of almost 10% that was yielded during the testing period is a huge succes and the implementation went quite good. The results of the implementing dynamic pricing to the other performances are not as good as the results of Café Lehmitz, but still promising. The other performances (Electropis and Revolte) were not sold out, so it hard to compare the results since the performances would have yielded more revenue if they had sold out for the normal price. But we cannot predict if a performance would have sold out completely and if the revenue would have been higher.

7 | Tool

A tool has been made of the model described in the last part of Chapter 5 and the results of the evaluation of the model in Chapter 6 have been taken into account.

The following information is needed to calculate the prices:

- **Date**
  The day of the week and the month of the performance can be found using the date and this can be used for some recommendations
- **Genre and/or hall**
  If genres are schedulend in more than one hall and if there was enough data available, the genre and hall can be chosen.
- **Orientation price**
  The orientation price is used if the orientation price is larger than the price at the breaking point.
- **Limiting price**
  During the evaluation we noticed that there is a psychological level for customers and they do not want to pay more than this level for a ticket. Since this level depends on many different variables, the theatre can set the level at a certain if they think that there is a maximum amount of money that people want to pay.

The interface of the tool can be seen in Figure 7.1. The tool produces the prices to ask for the normal price, the 80% and the 90% price. These prices depend on the breaking price, the orientation price and if a limiting price is given, also on the limiting price.
Furthmore the tool will give advice about the days and months too. These recommendations are based on the results of the analysis of the historical data in Chapter 3.

The tool can be used to find the prices to ask for performances, but the users should always take into account that these prices and recommendations are only guiding and certainly not binding.

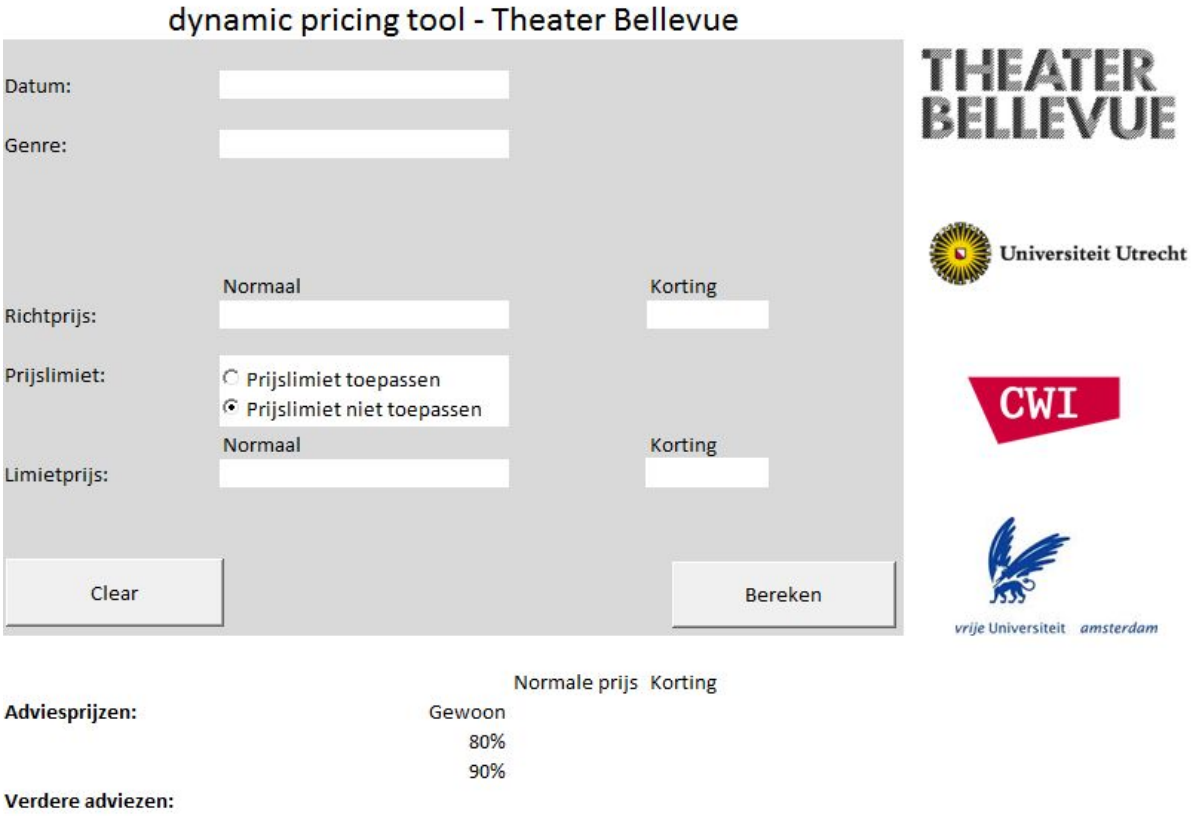


Figure 7.1: The interface of the dynamic pricing tool

8 | Conclusion

Eventjes dit, bij de haringsla
En de Raad voor de Kunst had de cholera
— Annie M.G. Schmidt
Ik hoef alleen maar even zo te doen
From the musical *En nu naar bed*

Although this may seem a good solution to avoid further advices that cut the subsidies even more, it is better to investigate the possibilities to increase the revenue.

8.1 Conclusion

This Master thesis describes the research I did for Theater Bellevue. I tried to answer the question if it is possible to increase the revenue of Theater Bellevue by implementing dynamic pricing.
Therefore I first did a small literature analysis and found out that it is indeed theoretical possible to implement Dynamic Pricing in Theater Bellevue. The only thing left to do was to investigate if Dynamic Pricing was practically possible.

The analysis of the historical data learned us that there are many differences between the halls. Each hall has it own characteristics. The distributions of the number of tickets sold differ per day of the week and per day of the month, the distributions of the ticket prices show us that there have not been major price changes in the last five seasons. The analysis can be summarised by a couple of recommendations at the end of this chapter.

After the analysis we used Kaplan-Meier estimators to estimate the demand and especially the probability that someone will buy a ticket for a certain price. When we plotted the estimators and compared the resulting curves, we discovered that many curves are statistically inequivalent while the ticket types belonging to these curves are sold for the same price. We used approximations of the Kaplan-Meier curves together with the pre-sale curve in our model. The model gives the optimal price to ask at a given time with a given number of seats left.

The problem is that it is impossible to implement the model in its current form. The website can only show one price for each performance of a production. So the prices on the website cannot vary per day or per performance. The ticketing system lacks the possibilities of implementing an automatic Dynamic Pricing system and all price changes should be done manually. To make it possible to implement some form of Dynamic Pricing we changed the model to a model that consists of only two steps. If the occupation degree of a performance passes the 80% boundary the price increases and if the 90% is passed prices are increased once more.

This model was tested on three performances and will be tested on another five performances. The results of the tests differ. Dynamic pricing has been very successful for the performances of Café Lehmitz by Theatergroep Carver. However, questions raised about the implementation since it has not been really fair for the customers. The results of the test on Electropis by Henry van Loon disappoint. The

performances were not sold out and although dynamic pricing increased the revenue of the seats that were sold, more revenue could have been yielded if the performance had sold out without dynamic pricing. The results of the performance Revolte by De Gebroeders Fretz are mixed as well. Implementing dynamic pricing increased the revenue but only by a small amount of money.
These experiences are taken into account by the development of the dynamic pricing tool.

Theater Bellevue is developing a new website and investigating the possibilities of a new ticketing system. If the website and the ticketing system will be new it may be possible to implement dynamic pricing and that price changes will change automatically.

So it is possible to implement dynamic pricing in Theater Bellevue and it can be used to gain extra revenue, but we must take into account that this model is only guiding and certainly not binding. Performances are not the same as flights, and the audience of the performances differs every time. The advices should not be implemented without any thought, but the advised prices and the advices should be adapted to each situation.

8.2 Recommendations

The analysis of the historical data and the other research that was done lead to the following set of recommendations for other pricing strategies:

- **Document the reservation date**
  When the reservation dates are known, one can get a better and thorough idea of the pre-sale ticket curves and the reliability of the model would increase.

- **Encourage on-line ticket sales**
  Customers who buy their tickets on-line buy their tickets earlier than customers who buy their tickets at the ticket booth. When the tickets are bought earlier there is more and earlier certainty about the occupation degree of performances.

- **Make Last minute tickets at the ticket booth more expensive than online**
  If customers come to the ticket booth on the day of the performance, they really want to see the performance and they will probably not care about an extra 10%, while customers buying their tickets on-line on the day of performance are not sure yet and could use an extra stimulant to see the performance.

- **Make separate strategies for each hall**
  It was already known and it is now also confirmed in this chapter, that there are many differences between the halls. The optimal strategy for Theater Bellevue in total consists of three sub-strategies, one for each hall.

- **Apply price differentiation between days**
  In the Grote Zaal performances on Friday and Saturday sell more tickets than performances on other days. To stimulate customers to visit other days as well and to yield more revenue, prices on Friday and Saturday could increase while prices on Tuesday could be lowered. This principle can be applied to performances in Klein Bellevue as well. Performances in the Paloni Zaal on Friday and Sunday could be priced higher to obtain more revenue.

- **Apply price differentiation between months**
  Performances in the Grote Zaal in January, February and March have a much higher occupation degree than performances in the other months, so these performances could be priced higher. The popular months for performances in Klein Bellevue are December, January February and March and these months could be priced higher as well. January, September and December are

the best months for performances in the Paloni Zaal and these months could be priced higher than performances in the other months.

- **Create more ranks**
  It is possible for popular performances in the Grote Zaal to create an extra rank in the center. These seats can have an extra service (for example they can enter the hall earlier) and can be sold for a higher price.

- **Create more pricing categories**
  We saw in Chapter 4 that the demand of different sales types is not the same. It is a possibility to split the discount price into more pricing categories. In this way we can focus more on the customer and make concessions for each customer.

8.3 Further research

My time at Theater Bellevue is finished, but there are possibilities for further research.
The results of the price tests on the other performances should be investigated in the future and furthermore it would be nice to update the model in a couple of years, if reservation data is available too and a better approximation of the pre-sale curve can be made. Implementing dynamic pricing is a continuous process as we learned in the introduction and it is not finished yet. Another possibility is a research that is more technical and it could be about the Markov Decission Process and the implementation of the model in the system of Theater Bellevue.
All in all, it seems that implementing dynamic pricing leads to promising results that can help Theater Bellevue to yield extra revenue.

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A | Estimation of demand – tables

The tables in this appendix contain the results of the Chi-squared tests and the corresponding p-values that were done to compare the Kaplan-Meier curves in section 4.2. The rows and columns in the table show the type of curves (the properties of these types are described in the corresponding section) and the intersection of a row and a column gives the value of the Chi-squared test statistic that compares these two curves and the p-value. Both principles are explained in Chapter 4.

Table A.1: Drama in the Grote Zaal

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 1.85338$ $p = 0.1733$	—	—	$X^2 = 0.33752$ $p = 0.5612$	$X^2 = 7.92026$ $p = 0.0048$	$X^2 = 3.82837$ $p = 0.0503$
INTERRED	—	—	—	—	$X^2 = 0.37011$ $p = 0.5429$	$X^2 = 5.0092$ $p = 0.0253$	$X^2 = 0.82562$ $p = 0.3635$
INTERVOL	—	—	—	$X^2 = 1.83182$ $p = 0.1759$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 4.49667$ $p = 0.0339$	$X^2 = 1.82099$ $p = 0.1771$
SENIOREN	—	—	—	—	—	—	$X^2 = 1.07268$ $p = 0.3003$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 1.83441$ $p = 0.1756$	—	—	$X^2 = 0.00945$ $p = 0.9225$	$X^2 = 1.10272$ $p = 0.2936$	$X^2 = 0.04790$ $p = 0.8267$
INTERRED	—	—	—	—	$X^2 = 3.04725$ $p = 0.0808$	$X^2 = 0.23120$ $p = 0.6306$	$X^2 = 2.20180$ $p = 0.1278$
INTERVOL	—	—	—	$X^2 = 11.64918$ $p = 0.0006$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 1.85940$ $p = 0.1726$	$X^2 = 0.43802$ $p = 0.5080$
SENIOREN	—	—	—	—	—	—	$X^2 = 1.44886$ $p = 0.2287$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

Table A.2: Drama in Klein Bellevue

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.01426$ $p = 0.9049$	—	—	$X^2 = 1.06247$ $p = 0.3026$	$X^2 = 0.24444$ $p = 0.6210$	$X^2 = 0.00561$ $p = 0.9402$
INTERRED	—	—	—	—	$X^2 = 0.79639$ $p = 0.3721$	$X^2 = 1.13066$ $p = 0.2876$	$X^2 = 0.00817$ $p = 0.9279$
INTERVOL	—	—	—	$X^2 = 0.00674$ $p = 0.9345$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 1.41445$ $p = 0.2343$	$X^2 = 0.27100$ $p = 0.6026$
SENIOREN	—	—	—	—	—	—	$X^2 = 1.56419$ $p = 0.2110$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.05913$ $p = 0.8078$	—	—	$X^2 = 1.94945$ $p = 0.1626$	$X^2 = 0.03243$ $p = 0.8570$	$X^2 = 0.00494$ $p = 0.9439$
INTERRED	—	—	—	—	$X^2 = 6.56547$ $p = 0.0103$	$X^2 = 0.01094$ $p = 0.9166$	$X^2 = 0.05823$ $p = 0.8093$
INTERVOL	—	—	—	$X^2 = 0.26305$ $p = 0.6080$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 7.26911$ $p = 0.0070$	$X^2 = 4.23968$ $p = 0.0394$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.01937$ $p = 0.8893$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

Table A.3: Drama in the Paloni Zaal

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.41827$ $p = 0.5178$	—	—	$X^2 = 1.05694$ $p = 0.3039$	$X^2 = 3.22376$ $p = 0.0725$	$X^2 = 1.91512$ $p = 0.1663$
INTERRED	—	—	—	—	$X^2 = 0.86887$ $p = 0.3512$	$X^2 = 7.61268$ $p = 0.0057$	$X^2 = 2.67325$ $p = 0.1020$
INTERVOL	—	—	—	$X^2 = 10.70348$ $p = 0.0010$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0.06564$ $p = 0.7977$	$X^2 = 0.00652$ $p = 0.9356$
SENIOREN	—	—	—	—	—	—	$X^2 = 1.02262$ $p = 0.3118$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 4.84294$ $p = 0.0277$	—	—	$X^2 = 1.06992$ $p = 0.0008$	$X^2 = 0.80702$ $p = 0.3690$	$X^2 = 1.65565$ $p = 0.1981$
INTERRED	—	—	—	—	$X^2 = 7.59514$ $p = 0.0058$	$X^2 = 12.52774$ $p = 0.0004$	$X^2 = 4.16388$ $p = 0.0412$
INTERVOL	—	—	—	$X^2 = 41.95498$ $p = 0$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 25.31161$ $p = 0$	$X^2 = 16.68471$ $p = 0$
SENIOREN	—	—	—	—	—	—	$X^2 = 1.39923$ $p = 0.2368$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

Table A.4: Cabaret in Klein Bellevue

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 3.05595$ $p = 0.0804$	—	—	$X^2 = 0.34461$ $p = 0.5571$	$X^2 = 2.08578$ $p = 0.1486$	$X^2 = 1.02117$ $p = 0.3122$
INTERRED	—	—	—	—	$X^2 = 2.27803$ $p = 0.1312$	$X^2 = 0.02616$ $p = 0.8715$	$X^2 = 0.24062$ $p = 0.6237$
INTERVOL	—	—	—	$X^2 = 23.59545$ $p = 0$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 1.72259$ $p = 0.1893$	$X^2 = 1.64712$ $p = 0.1993$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.24781$ $p = 0.6186$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 1.94101$ $p = 0.1635$	—	—	$X^2 = 0.06887$ $p = 0.7929$	$X^2 = 1.13143$ $p = 0.2874$	$X^2 = 1.71263$ $p = 0.1906$
INTERRED	—	—	—	—	$X^2 = 2.08232$ $p = 0.1490$	$X^2 = 0.43850$ $p = 0.5078$	$X^2 = 0.01431$ $p = 0.9047$
INTERVOL	—	—	—	$X^2 = 2.90474$ $p = 0.0883$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 1.08019$ $p = 0.2986$	$X^2 = 1.38177$ $p = 0.2397$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.18392$ $p = 0.6680$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

Table A.5: Muppet Theatre

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.24652$ $p = 0.6195$	—	—	$X^2 = 0.71687$ $p = 0.3971$	$X^2 = 0.48620$ $p = 0.4856$	$X^2 = 0.00449$ $p = 0.9465$
INTERRED	—	—	—	—	$X^2 = 0.24494$ $p = 0.6206$	$X^2 = 2.13276$ $p = 0.1441$	$X^2 = 0.30434$ $p = 0.5811$
INTERVOL	—	—	—	$X^2 = 0.45786$ $p = 0.4986$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 3.00457$ $p = 0.0830$	$X^2 = 0.84501$ $p = 0.1993$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.67153$ $p = 0.4125$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.31955$ $p = 0.5718$	—	—	$X^2 = 3.53025$ $p = 0.0602$	$X^2 = 1.52354$ $p = 0.2170$	$X^2 = 0.37102$ $p = 0.5424$
INTERRED	—	—	—	—	$X^2 = 3.50993$ $p = 0.0610$	$X^2 = 1.74489$ $p = 0.1865$	$X^2 = 0.01974$ $p = 0.8882$
INTERVOL	—	—	—	$X^2 = 0.65850$ $p = 0.4170$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0.27300$ $p = 0.6013$	$X^2 = 2.93356$ $p = 0.0867$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.98452$ $p = 0.3210$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

Table A.6: Musical Theatre

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 2.48180$ $p = 0.1151$	—	—	$X^2 = 0.13910$ $p = 0.7091$	$X^2 = 0.21032$ $p = 0.6465$	$X^2 = 0.56427$ $p = 0.4525$
INTERRED	—	—	—	—	$X^2 = 0.85505$ $p = 0.3551$	$X^2 = 2.04922$ $p = 0.1522$	$X^2 = 0.82878$ $p = 0.3626$
INTERVOL	—	—	—	$X^2 = 1.94084$ $p = 0.1635$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0.01950$ $p = 0.8889$	$X^2 = 0.02412$ $p = 0.8765$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.15486$ $p = 0.6939$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 1.44755$ $p = 0.2289$	—	—	$X^2 = 0.11718$ $p = 0.7321$	$X^2 = 2.07205$ $p = 0.1500$	$X^2 = 0.64993$ $p = 0.4201$
INTERRED	—	—	—	—	$X^2 = 0.39728$ $p = 0.5284$	$X^2 = 0.00942$ $p = 0.9226$	$X^2 = 0.22800$ $p = 0.6330$
INTERVOL	—	—	—	$X^2 = 2.23462$ $p = 0.1349$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0.59441$ $p = 0.4407$	$X^2 = 0.08650$ $p = 0.7686$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.56596$ $p = 0.4518$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

Table A.7: Music

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.00053$ $p = 0.9816$	—	—	$X^2 = 0.22541$ $p = 0.6349$	$X^2 = 0.21200$ $p = 0.6452$	$X^2 = 0.18217$ $p = 0.6695$
INTERRED	—	—	—	—	$X^2 = 0.53850$ $p = 0.4630$	$X^2 = 1.05442$ $p = 0.3044$	$X^2 = 0.55446$ $p = 0.4565$
INTERVOL	—	—	—	$X^2 = 0.22732$ $p = 0.6335$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0.77028$ $p = 0.3801$	$X^2 = 0.91047$ $p = 0.3399$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.01904$ $p = 0.8902$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.02611$ $p = 0.8716$	—	—	$X^2 = 0.38083$ $p = 0.5371$	$X^2 = 0.00283$ $p = 0.9575$	$X^2 = 0$ $p = 1$
INTERRED	—	—	—	—	$X^2 = 2.63603$ $p = 0.1044$	$X^2 = 0.12439$ $p = 0.7243$	$X^2 = 0.14299$ $p = 0.7053$
INTERVOL	—	—	—	$X^2 = 0.66978$ $p = 0.4028$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 2.70366$ $p = 0.1001$	$X^2 = 1.84547$ $p = 0.1743$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.01299$ $p = 0.9092$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

Table A.8: Dance

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0$ $p = 1$	—	—	$X^2 = 0.00059$ $p = 0.9806$	$X^2 = 0.01658$ $p = 0.8975$	$X^2 = 0.06688$ $p = 0.7959$
INTERRED	—	—	—	—	$X^2 = 0.03471$ $p = 0.8522$	$X^2 = 0.00134$ $p = 0.9707$	$X^2 = 0.02788$ $p = 0.8673$
INTERVOL	—	—	—	$X^2 = 2.74359$ $p = 0.0976$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0.07710$ $p = 0.7812$	$X^2 = 0.15553$ $p = 0.6933$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.00182$ $p = 0.9659$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.11005$ $p = 0.7400$	—	—	$X^2 = 0.01090$ $p = 0.9168$	$X^2 = 0.20560$ $p = 0.6502$	$X^2 = 0.00001$ $p = 0.9974$
INTERRED	—	—	—	—	$X^2 = 0.12506$ $p = 0.7236$	$X^2 = 0.11923$ $p = 0.7298$	$X^2 = 0.02766$ $p = 0.8679$
INTERVOL	—	—	—	$X^2 = 3.93159$ $p = 0.0473$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0.27009$ $p = 0.6032$	$X^2 = 0.00064$ $p = 0.9798$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.13355$ $p = 0.7147$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

Table A.9: Miscellaneous

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.28643$ $p = 0.5925$	—	—	$X^2 = 46935$ $p = 0.4932$	$X^2 = 0.00907$ $p = 0.9241$	$X^2 = 0$ $p = 1$
INTERRED	—	—	—	—	$X^2 = 0.07363$ $p = 0.7861$	$X^2 = 0.90769$ $p = 0.3407$	$X^2 = 0.30936$ $p = 0.5780$
INTERVOL	—	—	—	$X^2 = 1.88796$ $p = 0.1694$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0.56256$ $p = 0.4532$	$X^2 = 0.39351$ $p = 0.5304$
SENIOREN	—	—	—	—	—	—	$X^2 = 0.00723$ $p = 0.9322$
STADSPAS	—	—	—	—	—	—	—

(a) not sold out

	CJP	INTERRED	INTERVOl	NORMAAL	REDUCTIE	SENIOREN	STADSPAS
CJP	—	$X^2 = 0.00545$ $p = 0.9411$	—	—	$X^2 = 0$ $p = 1$	$X^2 = 0$ $p = 1$	$X^2 = 0$ $p = 1$
INTERRED	—	—	—	—	$X^2 = 0.01402$ $p = 0.9057$	$X^2 = 0.02678$ $p = 0.8700$	$X^2 = 0.02044$ $p = 0.8863$
INTERVOL	—	—	—	$X^2 = 4.25259$ $p = 0.0391$	—	—	—
NORMAAL	—	—	—	—	—	—	—
REDUCTIE	—	—	—	—	—	$X^2 = 0$ $p = 1$	$X^2 = 0$ $p = 1$
SENIOREN	—	—	—	—	—	—	$X^2 = 0$ $p = 1$
STADSPAS	—	—	—	—	—	—	—

(b) sold out

B | Approximations of the Kaplan-Meier curves

This appendix gives the approximations of the Kaplan-Meier curves that are calculated in Chapter 5.

Table B.1: The approximations of the KM curves of the not sold out performances

Genre	Hall	Full Price Curve	Discount Price Curve
Drama (Toneel)	Grote Zaal	$\begin{cases} 1 & p = 0 \\ 0.25 & \text{for } 0 < p \leq 16 \\ 0.0059p^2 - 0.2663p + 2.9956 & \text{for } 16 \leq p \leq 22.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.23 & \text{for } 0 < p \leq 13 \\ 0.0047p^2 - 0.1878p + 1.8776 & \text{for } 13 \leq p \leq 20 \end{cases}$
	Klein Bellevue	$\begin{cases} 1 & p = 0 \\ 0.30 & \text{for } 0 < p \leq 13.5 \\ 0.009549p^2 - 0.34948p + 3.277734 & \text{for } 13.5 \leq p \leq 18.3 \end{cases}$	
	Paloni Zaal	$\begin{cases} 1 & p = 0 \\ 0.40 & \text{for } 0 < p \leq 12 \\ 0.020625p^2 - 0.66p + 5.35 & \text{for } 12 \leq p \leq 16 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.33 & \text{for } 0 < p \leq 10.8 \\ 1.32 - 0.09167p & \text{for } 10.8 \leq p \leq 14.4 \end{cases}$
Cabaret	Klein Bellevue	$\begin{cases} 1 & p = 0 \\ 0.3 & \text{for } 0 < p \leq 13.5 \\ 91387e^{-0.929x} & \text{for } 13.5 < p \leq 17.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.3 & \text{for } 0 < p < 12 \\ 0.0109p^2 - 0.3714p + 3.1521 & \text{for } 12 \leq p \leq 15.5 \end{cases}$
Muppet Theatre		$\begin{cases} 1 & p = 0 \\ 0.25 & \text{for } 0 < p \leq 13 \\ -0.0069p^2 + 0.1807p - 0.9355 & \text{for } 13 < p < 18.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.26 & \text{for } 0 < p < 12.5 \\ 1.0779 - 0.064p & \text{for } 12.5 < p < 16 \end{cases}$
Musical Theatre		$\begin{cases} 1 & p = 0 \\ 0.36 & \text{for } 0 < p < 15.5 \\ 4069.2e^{-0.608p} & \text{for } 15.5 \leq p < 22.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.33 & \text{for } 0 < p \leq 13 \\ 217105e^{-1.02p} & \text{for } 13 < p < 20 \end{cases}$
Music		$\begin{cases} 1 & p = 0 \\ 0.23 & \text{for } 0 < p \leq 13.5 \\ 0.0052p^2 - 0.2168p + 2.2155 & \text{for } 13.5 < p < 18 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.23 & \text{for } 0 < p < 12 \\ -0.0127p^2 + 0.2677p - 1.171 & \text{for } 12 \leq p \leq 14.5 \end{cases}$
Dance		$\begin{cases} 1 & p = 0 \\ 0.2 & \text{for } 0 < p \leq 16.5 \\ 852.16e^{-0.52p} & \text{for } 16.5 < p \leq 23.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.2 & \text{for } 0 < p \leq 14 \\ 3340.8e^{-0.7p} & \text{for } 14 < p \leq 20.5 \end{cases}$
Miscellaneous		$\begin{cases} 1 & p = 0 \\ 0.14 & \text{for } 0 < p \leq 10 \\ 1.2518e^{-0.222p} & \text{for } 10 < p \leq 18.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.13 & \text{for } 0 < p < 9.5 \\ 0.2534 - 0.0151p & \text{for } 9.5 < p \leq 16.5 \end{cases}$

Table B.2: The approximations of the KM curves of the sold out performances

Genre	Hall	Full Price Curve	Discount Price Curve
Drama (Toneel)	Grote Zaal	$\begin{cases} 1 & p = 0 \\ 0.50 & \text{for } 0 < p \leq 16 \\ 256841.7 \cdot p^{-4.74263} & \text{for } 16 \leq p \leq 26 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.48 & \text{for } 0 < p \leq 14 \\ 0.023704p^2 - 0.87704p + 8.112593 & \text{for } 14 \leq p \leq 18.5 \end{cases}$
	Paloni Zaal	$\begin{cases} 1 & p = 0 \\ 0.72 & \text{for } 0 < p \leq 12.5 \\ 2.272373 - 0.12398p & \text{for } 12.5 \leq p \leq 17.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.71 & \text{for } 0 < p \leq 10.5 \\ 0.0044p^2 - 0.2542p + 2.9183 & \text{for } 10.5 < p \leq 16 \end{cases}$
Cabaret	Klein Bellevue	$\begin{cases} 1 & p = 0 \\ 0.8 & \text{for } 0 < p \leq 13.5 \\ 3135.8e^{-0.617p} & \text{for } 13.5 < p \leq 17.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.8 & \text{for } 0 < p \leq 12 \\ 0.0361p^2 - 1.2007p + 10.002 & \text{for } 12 < p \leq 16.5 \end{cases}$
Musical Theatre		$\begin{cases} 1 & p = 0 \\ 0.26 & \text{for } 0 < p \leq 15.5 \\ 4251.2e^{-0.608p} & \text{for } 15.5 < p < 22.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.25 & \text{for } 0 < p < 13.5 \\ 1.2266 - 0.0731p & \text{for } 13.5 \leq p < 17.5 \end{cases}$
Dance		$\begin{cases} 1 & p = 0 \\ 0.5 & \text{for } 0 < p \leq 16.5 \\ 5831.3e^{-0.608p} & \text{for } 16.5 < p \leq 22.5 \end{cases}$	$\begin{cases} 1 & p = 0 \\ 0.49 & \text{for } 0 < p \leq 14 \\ 611360e^{*1.027p} & \text{for } 14 < p \leq 14 \end{cases}$