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Master Thesis

Dynamical Electroweak Symmetry Breaking with an Antisymmetric Tensor Field

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"If the individual were no longer compelled to prove himself on the market, as a free economic subject, the disappearance of this kind of freedom would be one of the greatest achievements of civilization. The technological processes of mechanization and standardization might release individual energy into a yet uncharted realm of freedom beyond necessity. The very structure of human existence would be altered; the individual would be liberated from the work world's imposing upon him alien needs and alien possibilities. The individual would be free to exert autonomy over a life that would be his own."

Herbert Marcuse

"There is no happiness without knowledge. But knowledge of happiness is unhappy; for knowing ourselves happy is knowing ourselves passing through happiness, and having to, immediatly at once, leave it behind. To know is to kill, in happiness as in everything. Not to know, though, is not to exist."

Fernando Pessoa

UTRECHT UNIVERSITY

Abstract

Faculty of Science
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Dynamical Electroweak Symmetry Breaking with an Antisymmetric Tensor Field

by Dario PERRICONE

The fundamental scalar Higgs models presents triviality, unnaturalness and vacuum stability issues, which make them unappealing from a theoretical point of view. In this work we study a model with an antisymmetric tensor field coupled to fermions via a "B-Yukawa" term which, in analogy with Technicolor models, can provide a dynamical breaking of electro-weak symmetry through the formation of a fermion-antifermion condensate, giving masses to gauge bosons and fermions. We introduce a Lagrangian for the antisymmetric field which is instability free and compute a covariant propagator for it. Then, we evaluate the relevant Feynman diagrams for the calculation of the β function for the "B-Yukawa" coupling, whose value must be negative for the formation of the fermion-antifermion condensate to be allowed. This value, however, has been found to be gauge dependent and further work is needed to see if this issue can be overcome somehow.

Acknowledgements

As is the case for every work, there are many people which, maybe even without knowing, have given fundamental contribution to the making of this project and to which I have to express my gratitude. First, I would like to thank my supervisor, for helping me in learning and understanding theories of big interest, for patiently guiding me during this process. Then my thought goes to my family, always supportive and thoughtful, filling with affection the physical distance which has kept us apart during these last years. A special thanks goes to my dearest friends Angela, Silvana and Aldo for sharing with me youth unconsciousness, deepest thoughts and laughs on a Volkswagen van on his way through Europe to the roots of friendship. Thanks also to Alekampo and Peppe Sortino, my favorites black and white photographers, dearest companions of reflections and excursions, for their constant and tireless help in trying to find the true meaning of things. Thank you Ivano, Chiara, Fabio and Gianrocco for keeping alive the memories of past years of friendship, for the thousands of little daily life favors which acquire a priceless value when living in a foreign country. Thank you Agnese, wise and humble rock expert, for your suggestions and your sinless smile. Thank you Peppe Angilella, for having a thought for everyone, always monitoring that we are on the right path. Finally, a thanks is due to my guitar, together with my brother Alberto who introduced me to guitar playing, for letting endless rainy days pass by and for letting me have a new powerful tool to measure the depths of feeling.

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Abbreviations

QCD	**Q**uantum **C**hromo **D**ynamics
QED	**Q**uantum **E**lectro **D**ynamics
LHC	**L**arge **H**adron **C**ollider
SM	**S**tandard **M**odel
BEH	**B**rout **E**nglert **H**iggs
EWSB	**E**lectro **W**eak **S**ymmetry **B**reaking
TC	**T**echni **C**olor
ETC	**E**xtended **T**echni **C**olor
QFT	**Q**uantum **F**ield **T**heory
FCNC	**F**lavor **C**hanging **N**eutral **C**urrents

Dedicated to Riccardino

Chapter 1

Introduction

Electroweak symmetry breaking and Higgs mechanism are two of the key ingredients in the description of the world of subatomic particles physics given in the context of the Standard Model.

By introducing a scalar field which behaves as a complex doublet under $SU(2)$ and develops a vacuum expectation value due to its mexican-hat shaped potential, electroweak symmetry $SU(2) \otimes U(1)$ gets broken and fermions coupled to this field through a Yukawa term acquire a mass. Also, as was proved by Glashow-Weinberg and Salam in 1967, three out of the four gauge bosons become massive as a result of interactions with the Higgs field, while the photon stays massless. The energy scale at which this happens is $v_{weak} \sim 100$ GeV.

However, it has been argued that this kind of models with a fundamental Higgs suffer from a series of shortcomings which make them unappealing and unnatural. The most important of these, perhaps, is the fact that these models lack a dynamical explanation of the spontaneous symmetry breaking and so they do not answer to the question on why this breaking happens exactly at the scale v_{weak}.

This, together with others motivations, has led in the last decades of the past century to the formulation of several alternative ways of reproducing the outcomes of the Higgs mechanism without introducing any scalar field. One of these, Technicolor, is of particular interest because tries to give a dynamical explanation of electroweak symmetry breaking in analogy with what happens in QCD for the breaking of the chiral symmetry $SU(2)_L \otimes SU(2)_R$, around $\Lambda_{QCD} \sim 100$ MeV. There, the breaking of the symmetry is just a consequence of the strong fermions dynamics, which is responsible for the formation of a composite fermion-antifermion bound state $\langle \bar{\Psi}\Psi \rangle$, which in turns breaks the chiral symmetry down to $SU(2)_V$ of isospin, without the need of any scalar particle.

So the idea of Technicolor is to postulate the existence of a new strong, asymptotically free interaction, i.e. with a coupling characterized by a negative beta function, which

binds pairs of new technifermions-antitechnifermions into a condensate whose behavior mimics that of the Higgs field.
Anyhow, also technicolor models have their drawbacks, such as a problem in the explanation of the correct value of the top quark mass.
There is another model that has been proposed in a paper by Wetterich [1] in 2006 which has not been investigated so much in literature, which attempts to reproduce Higgs mechanism through the introduction of an antisymmetric tensor field coupled to fermions with chiral couplings.
The electroweak symmetry breaking again arises as a consequence of the asymptotic freedom of these couplings and of the emergence of a fermion-antifermion condensate.
We studied the properties of a related model developed in a precedent thesis work [2] in which such model with an antisymmetric tensor has been modified for the scope of eliminating some classical instabilities. The antisymmetric tensor here couples to fermions via a "B-Yukawa" term and the formation of $\langle \bar{\Psi}\Psi \rangle$ is possible providing that the coupling has a negative beta function. The condensate will play the role of the Higgs and the resulting effective theory will break the symmetry via an effective potential which resembles the one of the nonlinear sigma model. It must be said, however, that these models after the discovery of an Higgs boson with a mass of 126 GeV at LHC [3], if not completely ruled out, must be reconsidered in order to account for this scalar particle.
Even after the discovery, many questions remain open about this Higgs particle. It is not yet conclusively known, to this date, if such particle is fundamental or composite, or if it is a scalar or a pseudoscalar, or if it is a spin 0 or spin 2 particle. Through measurements of the coupling property, spin and parity, the new particle has so far been almost consistent with the Higgs boson predicted as a key boson responsible for the origin of mass in the standard model. One discrepancy from the SM Higgs has been, however, reported in the diphoton channel $H \to \gamma\gamma$ where the observed signal event is about two times larger than the SM Higgs prediction [4]. This would imply that the observed scalar boson is a SM-Higgs impostor concerning the underlying theory beyond the SM. Attempts to see if this impostor could be a pseudo Goldstone boson, the technidilaton predicted by walking Technicolor, are still in progress [5]. So there is still much mystery around the scalar particle discovered at LHC, and possible deviations from the fundamental Higgs scalar are worth to be investigated.
The work is organized as follows: in chapter 2 we give a brief overview of the key aspects of the Standard Model and the Higgs mechanism, then we move in chapter 3 to a description of the basic ideas which have brought to the formulation of Technicolor, extended Technicolor and walking Technicolor. In chapter 4 the antisymmetric tensor field is then introduced and quantized in a covariant fashion. We thus proceed to the definition of the Feynman rules of the theory in chapter 5 with which we will compute the value of the beta function for the Yukawa coupling, which will present some problems

because explicitly depends on gauge parameters. In chapter 6, finally, we present the conclusions and a comment of the results obtained. Further work is needed to see if this problem can be cured somehow or if the theory is just not viable.

Chapter 2

The Standard Model

In our understandings of the microscopic world of elementary particles and their interactions, the Standard Model (SM), ref. [6–8] is, to this date, the most precise and complete mathematical tool developed in physics. Such model in fact, has been able to account for many experimental results obtained in the last few decades, with an astonishing degree of precision, as for instance the measure of the electron gyromagnetic ratio g [9] . The model consists of 12 elementary fermion particles with half integer spin, six quarks (up, down, charm, strange, top, bottom) and six leptons (electron, muon, tau, electron neutrino, muon neutrino, tau neutrino), divided into three generations, each of which has its own antiparticle.
This classification is based on how different particles interact with other particles, i.e. by what *charges* they carry. Each fundamental force has its own charge, and particles with the same kind of charge may interact with each other via the exchange of a boson particle.
The three neutrinos only carry weak isospin, so their motion is influenced only by the weak nuclear force. The remaining leptons even carry electric charge, thus interacting also electromagnetically. Quarks instead, besides weak isospin and electric charge, carry a *color* charge and, hence they interact via the strong interaction among themselves, whereas both electromagnetically and weakly with other fermions.
The particles responsible for force mediation are the gauge bosons. To each force are associated one or more bosons, which are said to be the force carriers. The number of such bosons depends on the gauge symmetry of the Lagrangian that describes the interacting fields. More precisely, the boson number is exactly the number of generators of the invariant symmetry group. So, for example, the electromagnetic force has only one boson, the photon, since QED has a $U(1)$ symmetry, while the weak force is mediated by three massive bosons ,$W^{\pm}$ and Z^0, corresponding to the three generators of the $SU(2)$ symmetry. The strong force , is carried by 8 gluons, which themselves are color

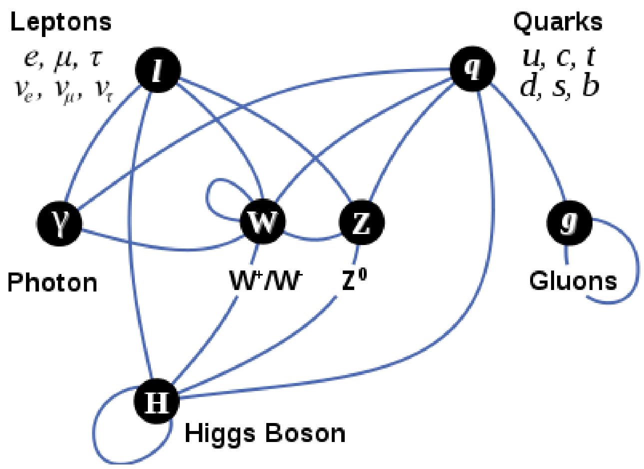


FIGURE 2.1: Schematic map of Standard Model interactions

charged, descending from the $SU(3)$ symmetry of QCD. A peculiar feature of this force is that it gives rise to what is called color *confinement*, a phenomenon that prevents quarks from being observed in isolation and forces them to always manifest in bound, color-neutral states. The reason of this behavior relies in the fact that the binding strong force between quarks increases with distance and tends to zero when quarks get close to each other, i.e. the theory has a property known as *asymptotic freedom*. This property plays, as we will see, a crucial role in determining some of the most important features of QCD.

From what we have said, we can already argue that the SM presents an internal symmetry $U(1) \times SU(2) \times SU(3)$, where $U(1) \times SU(2)$ is the electro-weak part, and $SU(3)$ the color part.

The starting point of this field theory is to describe particles wave functions as fields. A spin-$\frac{1}{2}$ fermion is then described by a four component spinor field,

$$\Psi_\alpha(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} = \begin{pmatrix} \phi(x) \\ \chi(x) \end{pmatrix} \tag{2.1}$$

where each component is a function of the space-time coordinate x^μ. If the two-component spinors ϕ and χ are independent, Ψ is a four-component complex spinor describing a fermion and its antiparticle, and is called a Dirac fermion. If one imposes a reality condition $\chi = \pm i\sigma_2\phi*$, then Ψ describes a single spin-$\frac{1}{2}$ particle and is called a Majorana fermion. One can also define a Dirac conjugate as

$$\bar{\Psi} \equiv \Psi^\dagger\gamma^0. \tag{2.2}$$

Under a Lorentz transformation a spinor behaves as

$$\Psi \to \Psi' = \exp(\frac{i}{4}\theta_{\mu\nu}\sigma^{\mu\nu})\Psi \tag{2.3}$$

where

$$\sigma^{\mu\nu} = -\frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) = -i\gamma^{[\mu}\gamma^{\nu]} \quad \text{and} \quad \theta_{\mu\nu} = -\theta_{\nu\mu}. \tag{2.4}$$

These $\theta_{\mu\nu}$ are anti-symmetric, real parameters that characterize spatial rotations for $\mu, \nu = 1, 2, 3$, and the Lorentz boosts for μ or $\nu = 0$. With these information it is possible to write down a Lorentz-invariant Dirac Lagrangian:

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi \tag{2.5}$$

where the symbol $\partial\!\!\!/$ stands for a 4×4 matrix defined by

$$(\partial\!\!\!/)_{\alpha\beta} = (\gamma^\mu)_{\alpha\beta}\frac{\partial}{\partial x^\mu}. \tag{2.6}$$

At this point, interactions with other particles are introduced through a clever mechanism, i.e. *gauge invariance*. This is implemented by demanding the Lagrangian to be invariant under spinor transformations of the type

$$\Psi'(x) = e^{iq\xi(x)}\Psi(x) \tag{2.7}$$

which is a phase transformation that generates the group $U(1)$.
After a brief look at (2.5), one can see that the Lagrangian is invariant under a global transformation (i.e. $\xi(x) = \xi$), but not under a local one. Indeed

$$\partial_\mu\Psi(x) \to (\partial_\mu\Psi(x))' = e^{iq\xi(x)}(\partial_\mu\Psi(x) + iq\partial_\mu\xi(x)\Psi(x)). \tag{2.8}$$

To achieve invariance, another field is introduced, called the gauge field, in such a way that, defining a covariant derivative as

$$D_\mu\Psi(x) = (\partial_\mu + iqA_\mu(x))\Psi(x) \tag{2.9}$$

and requiring the new field to transform as

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu\xi \tag{2.10}$$

we get rid of the extra term in (2.8) induced by the transformations at neighboring space-time points, and get

$$D_\mu\Psi(x) \to (D_\mu\Psi(x))' = e^{iq\xi(x)}(D_\mu\Psi(x)). \tag{2.11}$$

If we now replace the normal derivative with this covariant derivative in our original Lagrangian, we will see that it no longer describes free fermions, but rather interactions between fermions and the gauge field:

$$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi = i\bar{\Psi}\not{\partial}\Psi - m\bar{\Psi}\Psi - qA_\mu\bar{\Psi}\gamma^\mu\Psi. \tag{2.12}$$

It is conventional to assume that A_μ describes some new and independent degrees of freedom of the system. Actually, it is possible to construct a gauge invariant Lagrangian for the gauge field itself, moving from the observation that the commutator of two covariant derivatives is still a covariant object:

$$[D_\mu, D_\nu]\Psi = -iqF_{\mu\nu}\Psi \quad \text{where} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.13}$$

This observation leads to the Lagrangian for the gauge field of the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{2.14}$$

This Lagrangian can now be combined with (2.12)

$$\mathcal{L} = \mathcal{L}_{A_\mu} + \mathcal{L}_\Psi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\not{\partial}\Psi - m\bar{\Psi}\Psi - qA_\mu\bar{\Psi}\gamma^\mu\Psi \tag{2.15}$$

so that we have obtained an interacting theory of a vector field and a fermion field invariant under the combined local gauge transformations (2.7) and (2.10). Such theory is Quantum Electro-Dynamics, where the role of the photon is interpreted by the gauge field A_μ, while $F_{\mu\nu}$ is the electromagnetic field strength.

What we have done so far is to start with a representation of matter particles in terms of spinors, then derive a Lagrangian from which is possible to construct their equations of motion, and finally add interactions with bosons imposing a local phase transformation invariance. We can repeat this procedure to describe also the weak interaction. However, in this case, we will have to impose an invariance under a different group of transformation.

In general, groups of transformations that are considered in gauge theories are represented by matrices that can be parametrized in terms of a finite number of parameters, i.e. they are *Lie groups*. Under a generic group transformation the field behaves as

$$\Psi(x) \to \Psi'(x) = U\Psi(x) \tag{2.16}$$

where U is a matrix that satisfy the multiplication rule of the Lie group and can be written as $U = \exp(\xi^a t_a)$, with t_a the generators of the group. When considering local transformation, the derivative of the field will become

$$\partial_\mu \Psi(x) \to (\partial_\mu \Psi(x))' = U(x)\partial_\mu \Psi(x) + (\partial_\mu U(x))\Psi(x). \tag{2.17}$$

Again, we can see that this quantity does not transform covariantly, in the sense that it does not transform according to a representation of the group at the same space-time point, due to the presence of the second term in the last equation. In analogy with what we have done in the previous case, we get rid of this term defining a new, covariant derivative:

$$D_\mu \Psi \equiv \partial_\mu \Psi - gW_\mu \Psi \tag{2.18}$$

where W_μ is a matrix of the type generated by an infinitesimal gauge transformation. This means that W_μ can be decomposed into the generators t_a,

$$W_\mu = gW_\mu^a t_a. \tag{2.19}$$

If we now let W_μ transform as

$$W_\mu \to W'_\mu = UW_\mu U^{-1} + (\partial_\mu U)U^{-1} \tag{2.20}$$

we obtain that the derivative we previously defined is indeed covariant:

$$D_\mu \Psi(x) \to (D_\mu \Psi(x))' = U(x)D_\mu \Psi(x). \tag{2.21}$$

Now we are ready to define in a similar way as we did for QED the field strength tensor, but we have to keep in mind that, since this time two group transformations do not commute, i.e. the group is *nonabelian*, we will have a commutator of two fields in the field strength expression:

$$[D_\mu, D_\nu]\Psi = -g(\partial_\mu W_\nu - \partial_\nu W_\mu - g[W_\mu, W_\nu])\Psi = -gG_{\mu\nu}\Psi. \tag{2.22}$$

The presence of this commutator term implies interactions between gauge bosons. From (2.19) it follows that also the field strength takes values in the Lie algebra of the group

$$G_{\mu\nu} = G_{\mu\nu}^a t_a = (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a - gf_{bc}^a W_\mu^b W_\nu^c)t_a \tag{2.23}$$

where f_{bc}^a are the structure constant defined by the relation between the group generators

$$[t_b, t_c] = f_{bc}^a t_a. \tag{2.24}$$

Now the total Lagrangian will look like

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + i\bar{\Psi}\partial\!\!\!/\Psi - m\bar{\Psi}\Psi - ig\bar{\Psi}\gamma^{\mu}W_{\mu}\Psi \tag{2.25}$$

With this more abstract picture we can implement more complex interactions just by requiring the fermion Lagrangian to be invariant under a suitable group of transformations, which in the case of the weak interaction is $SU(2)$, while in the case of Quantum Chromo-Dynamics is $SU(3)$. This means that the gauge group of the whole SM, as we have already anticipated, is the cross product $U(1)_Y \otimes SU(2)_{I_w} \otimes SU(3)_c$, where the subscripts stand for the conserved quantum numbers associated with each group, namely hypercharge Y for $U(1)$, weak isospin I_w for $SU(2)$, and color c for $SU(3)$.

2.1 Mass generation of gauge bosons and fermions

From what we have seen so far, it is clear that a mass term for a gauge boson in the Lagrangian cannot be allowed since it would explicitly break the gauge symmetry, i.e. it would be not invariant under a gauge transformation. However, in nature, it appears that the bosons mediators of the weak force , $W^{\pm}$ and Z^0, are massive, while the photon and gluons are massless. In other words, the symmetry $SU(2) \times U(1)$ is said to be *spontaneously broken*. In order to avoid this issue, and to permit to introduce mass terms for bosons and fermions, a clever and elegant mechanism has been developed by Brout, Englert and Higgs, which we will briefly discuss in the next section. This mechanism has led in the subsequent years to the development by Glashow, Weinberg and Salam, of a unified theory of the Electro-Weak interactions, capable of explaining why $W^{\pm}$ and Z^0 bosons have a mass, whereas the photon remains massless.

2.1.1 The Brout-Englert-Higgs Mechanism

The key idea of the Brout-Englert-Higgs mechanism is to add to the standard expression of the Lagrangian a new complex scalar field, the Higgs field $\phi(x)$, which will have a nonvanishing vacuum expectation value $\langle\phi\rangle = v$, due to the mexican hat shaped potential

$$V(|\phi|) = -\mu^2|\phi|^2 + \lambda|\phi|^4. \tag{2.26}$$

In the case of the $SU(2)$ symmetry, this scalar field behaves as a doublet under the

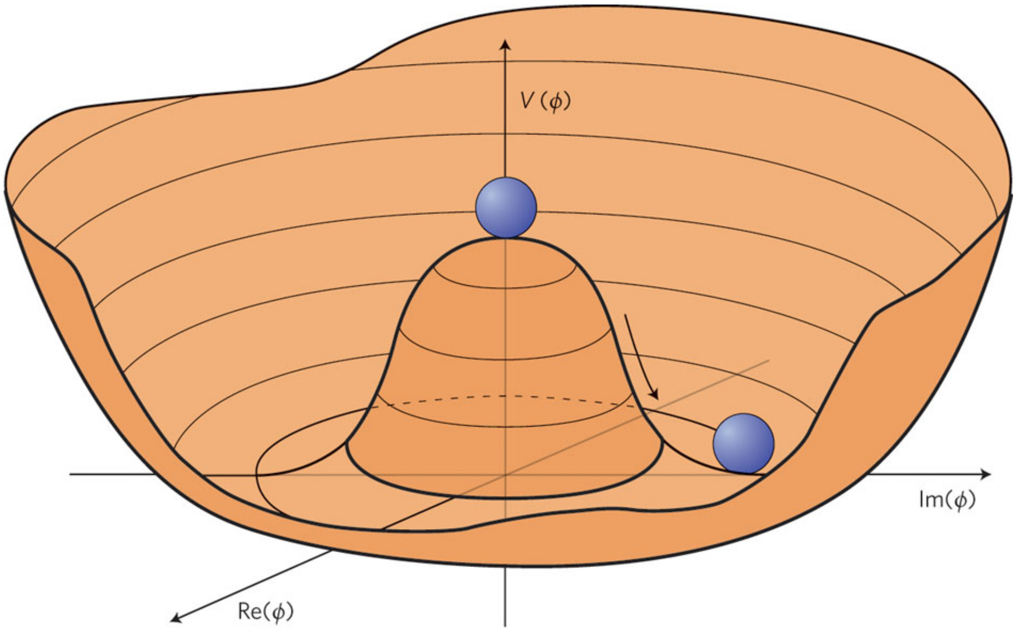


FIGURE 2.2: Form of the Higgs potential

gauge transformation, and it can generally be decomposed into the form

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} = \frac{1}{\sqrt{2}}\Phi(x)\begin{pmatrix} 0 \\ \rho(x) \end{pmatrix} \tag{2.27}$$

where $\Phi(x)$ is an x-dependent $SU(2)$ matrix, and $\rho(x)$ is a real field that represents the $SU(2)$ invariant length of the doublet field. Under $SU(2)$ the field $\Phi(x)$ transforms as

$$\Phi(x) \rightarrow U(x)\Phi(x). \tag{2.28}$$

If we now plug in this decomposition in the total lagrangian and define new gauge fields which are related to the old ones via

$$\hat{W}_\mu(x) = \Phi^{-1}(x)W_\mu(x)\Phi(x) + g^{-1}[\partial_\mu\Phi^{-1}(x)]\Phi(x), \tag{2.29}$$

we have constructed a theory with explicitly gauge-invariant gauge fields, in which the covariant derivative for the Higgs field will be

$$D_\mu\phi(x) = \frac{1}{\sqrt{2}}\Phi(x)\left[\partial_\mu - \frac{1}{2}ig\hat{W}^a_\mu(x)\tau_a\right]\begin{pmatrix} 0 \\ \rho(x) \end{pmatrix}. \tag{2.30}$$

So now, these new gauge fields *can* have a mass term and, as we can show, they do. The total Lagrangian in unitary gauge, i.e. setting $\Phi = I$ by means of an appropriate local gauge transformation, looks like

$$\mathcal{L} = \frac{1}{4}(\partial_\mu \hat{W}^a_\nu - \partial_\nu \hat{W}_\mu)^2 - g\varepsilon_{abc}\hat{W}^{\mu a}\hat{W}^{\nu b}\partial_\mu \hat{W}^c_\nu - \frac{1}{4}g^2\varepsilon_{abc}\varepsilon_{ade}\hat{W}^{\mu b}\hat{W}^d_\mu \hat{W}^{\nu c}\hat{W}^e_\nu$$
$$- \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}\mu^2\rho^2 - \frac{1}{4}\lambda\rho^4 - \frac{1}{8}g^2\rho^2(\hat{W}^a_\mu)^2. \tag{2.31}$$

As we can see from the quadratic terms, expanding the ρ field around the minima of its potential $\rho = v = \sqrt{\frac{\mu^2}{\lambda}}$, both the Higgs particle associated with the field ρ and the gauge bosons acquire a mass, respectively given by:

$$m^2_\rho = 2\lambda v^2, \quad M^2_W = \frac{1}{4}g^2 v^2. \tag{2.32}$$

2.1.2 The Glashow-Weinberg-Salam Theory of Weak Interactions

We are now ready to write down the spontaneously broken gauge theory that gives the experimentally correct description of the weak interaction. The symmetry group considered is the $SU(2) \times U(1)$ part of the whole symmetry group of the SM. The four gauge fields, three for $SU(2)$ and one for $U(1)$, will be denoted respectively by W^a_μ and B_μ. Initially all gauge fields are massless and have no direct interactions. After the introduction of the Higgs field and, consequently, the emergence of a mass term, it turns out that precisely one linear combination of these gauge fields remains massless, and this will be used to describe the photon.

To see this we notice that the complete gauge transformation acts on the Higgs field as

$$\phi(x) \to \phi'(x) = e^{i\alpha^a\tau^a}e^{i\beta/2}\phi(x). \tag{2.33}$$

Then we assume that the potential is such that it acquires a minimum for $\phi \neq 0$. In this case it is possible to decompose ϕ according to (2.27), which in the unitary gauge is equivalent to put

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho(x) \end{pmatrix}. \tag{2.34}$$

It is clear then from this form that a gauge transformation with

$$\alpha^1 = \alpha^2 = 0 \qquad \text{and} \qquad \alpha^3 = \beta \tag{2.35}$$

leaves the scalar field invariant. Thus, the theory will contain one massless gauge boson, corresponding to this particular combination of generators. The remaining three gauge fields instead, will acquire a mass through the usual BEH mechanism. To work out quantitatively the mass spectrum we focus on the covariant derivative of ϕ:

$$D_\mu \phi = (\partial_\mu - igW_\mu^a \tau^a - i\frac{g'}{2} B_\mu)\phi. \tag{2.36}$$

The gauge boson mass terms come from the square of this expression, evaluated at the scalar field vacuum expectation value $\langle \phi \rangle = v$. The quadratic terms are:

$$\begin{aligned}\Delta \mathcal{L} =& \frac{1}{2}(0 \quad v)\Big(gW_\mu^a \tau^a + \frac{1}{2}g' B_\mu\Big)\Big(gW^{\mu b}\tau^b + \frac{1}{2}g' B^\mu\Big)\begin{pmatrix}0\\ v\end{pmatrix} = \\ =& \frac{1}{2}\frac{v^2}{4}[g^2(W_\mu^1)^2 + g^2(W_\mu^2)^2 + (-gW_\mu^3 + g' B_\mu)^2].\end{aligned} \tag{2.37}$$

Defining

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \quad , \quad Z_\mu^0 = \frac{1}{\sqrt{g^2 + g'^2}}(gW_\mu^3 - g' B_\mu) \quad , \quad A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g' W_\mu^3 + gB_\mu) \tag{2.38}$$

it's easy to see that these fields get the masses $m_W = \frac{gv}{2}$, $m_Z = \sqrt{g^2 + g'^2}\frac{v}{2}$ and $m_A = 0$. For a fermion field belonging to a general $SU(2)$ representation, with $U(1)$ charge Y, the covariant derivative takes the form:

$$D_\mu = \partial_\mu - igW_\mu^a - ig' Y B_\mu \tag{2.39}$$

which in terms of the fields defined in (2.38) is

$$\begin{aligned}D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{1}{\sqrt{g^2 + g'^2}}Z_\mu^0(g^2 T^3 - g'^2 Y) \\ - i\frac{gg'}{\sqrt{g^2 + g'^2}}A_\mu(T^3 + Y)\end{aligned} \tag{2.40}$$

where $T^\pm = (T^1 \pm iT^2) = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$. We stress the fact that the field A_μ couples to the gauge generator $(T^3 + Y)$, which is exactly the one generating the symmetry operation (2.35). Since this field is interpreted to be the electromagnetic vector potential, it is straightforward to define the electric charge e as

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} \tag{2.41}$$

and the electric charge quantum number

$$Q = T^3 + Y. \tag{2.42}$$

One can further simplify the expression for the fermion covariant derivative by introducing a new parameter, the *weak mixing angle* θ_w, which define the change of basis from

(W^3, B) to (Z^0, A):

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix} \tag{2.43}$$

that implies

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}}, \qquad \sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}. \tag{2.44}$$

The covariant derivative can be now rewritten as

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{\cos\theta_w} Z_\mu^0 (T^3 - \sin^2\theta_w Q) - ieA_\mu Q \tag{2.45}$$

where we can finally recognize the usual interaction term for the photon in QED. Also, it is worth noting that the masses of the gauge fields with this new notation satisfy:

$$m_W = m_Z \cos\theta_w \tag{2.46}$$

2.1.3 Fermion mass terms

Let us now look more closely to the fermion content of the theory. It is common to decompose a fermion field into chiral components using the projections operators $P_\pm = \frac{1}{2}(I \pm \gamma^5)$ that satisfy:

$$P_\pm^2 = P_\pm, \qquad P_+ P_- = P_- P_+ = 0. \tag{2.47}$$

Thus that is possible to write

$$\Psi = P_+\Psi + P_-\Psi = \Psi_R + \Psi_L \tag{2.48}$$

The important feature of this decomposition is that Ψ_R and Ψ_L can now independently be assigned to representations of the gauge group. We can exploit this result to ensure that only left-handed components of the fermions fields couple to the W bosons, as it is suggested by experimental evidence. The left-handed fermion fields are assigned to a doublet representation of $SU(2)$, and their right-handed counterparts are $SU(2)$ singlets, i.e.

$$E_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L; \qquad e_R; \qquad \nu_R; \tag{2.49}$$

This fact amounts to choose the value $T^3 = 0$ for right-handed fields, and a value $T^3 = \pm\frac{1}{2}$ for left-handed ones. Notice that, once a value for the generator T^3 is chosen, it automatically implies a value for the hypercharge Y from (2.42). These assignments can be seen to reproduce the correct electric charge values.

However, the drawback of this choice is that it forbids the presence of fermion mass terms. This is the case because such terms would have the form

$$m_\Psi(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L) \tag{2.50}$$

and therefore, since the fields Ψ_L and Ψ_R belong to different $SU(2)$ representations and have different $U(1)$ charges, would be not gauge invariant. The only way for the fermions to acquire masses is then via a Yukawa coupling to the scalar doublet ϕ. For instance, for the electron we can add to the Lagrangian a term like:

$$\Delta\mathcal{L} = -\lambda_e \bar{E}_L \cdot \phi e_R + \text{h.c.} \tag{2.51}$$

To obtain the size of the masses we just have to replace ϕ by its vacuum expectation value :

$$\Delta\mathcal{L} = -\frac{1}{\sqrt{2}}\lambda_e v \bar{e}_L e_R + \text{h.c.} + \cdots \tag{2.52}$$

to get

$$m_e = \frac{1}{\sqrt{2}}\lambda_e v \tag{2.53}$$

In a completely analogous fashion it is possible to implement mass terms also for quark fields, which will depend on similar constants λ_d, λ_u.
A further possibility would be to consider a *Majorana* mass. This comes from defining a four component Majorana spinor Ψ_M, which in the chiral representation is

$$\Psi_M = \begin{pmatrix} \psi_L \\ i\sigma^2\psi_L^* \end{pmatrix}, \tag{2.54}$$

and which satisfies

$$(i\partial\!\!\!/ - m)\Psi_M = 0. \tag{2.55}$$

However,since left-handed and right-handed Majorana spinors are not independent, this Majorana equation is not invariant under global $U(1)$ symmetries. This means that a spin 1/2 particle which carries a $U(1)$ conserved charge cannot have a Majorana mass. So the only possible candidate which could have a Majorana mass is the neutrino. A Dirac mass for the neutrino would imply that, together with the left-handed neutrino, there exists also a right-handed neutrino, which combines with the left-handed one to produce the Dirac mass. However, these hypothetical right-handed neutrinos are not seen in weak interactions, and therefore, if they exist, they must be sterile, which means that they do not participate in weak interactions, or at least they participate much more weakly than the left-handed neutrinos. The other possibility is that neutrinos are described by purely left-handed fields and have Majorana masses. In this case the lepton number symmetry is violated. Experiments on neutrino-less double beta decay aim at detecting these violations.

Chapter 3

A Hint About The Technicolor Model

This chapter is intended to give a short but, if possible, complete introduction to the Technicolor model. We will mainly refer to concepts and formulas derived in [6, 10–18].

3.1 The Higgs Sector

We have seen how, in the context of the SM, the $W^{\pm}$ and Z^0 bosons may acquire a mass via the BEH mechanism with a single scalar field. However, it is worth asking whether the same result could be obtained through a more complicated mechanism. In principle, the breaking of $SU(2) \times U(1)$ might be the result of the dynamics of a complicated new set of particles and interactions, known as the *Higgs sector*.
There are a few constraints on this new sector, imposed by experiments. First, it must generate the masses of quarks and leptons. Second, it must generate the masses of the $W^{\pm}$ and Z^0 bosons as well. And last, it must reproduce in a natural way the relation between bosons masses (2.46), which is satisfied experimentally to better than 1% accuracy. It is possible to show that this relation follows from the much more general assumption of an unbroken global $SU(2)$ symmetry of the Higgs sector, often called *custodial* $SU(2)$ symmetry. For the case of a single scalar field, the custodial symmetry arises in the following way: if we write the field ϕ in terms of its four real components, its Lagrangian has $O(4)$ global symmetry. The vacuum expectation value of ϕ breaks this symmetry down to $O(3)$, which has a universal covering group, $SU(2)$. However, there are many other quantum field theories that break $SU(2)$ spontaneously while leaving another global $SU(2)$ symmetry unbroken. One example is given by QCD

with massless flavors. The Lagrangian in this case reads:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}W^0_{\mu\nu}W^{0\mu\nu} \\ &+ \sum_{i=1}^{n_G}\Big(\bar{q}_{i\alpha L}i\gamma_\mu D^\mu_{\alpha\beta}q_{i\beta L} + \bar{u}_{i\alpha R}i\gamma_\mu D^\mu_{\alpha\beta}u_{i\beta R} + \bar{d}_{i\alpha R}i\gamma_\mu D^\mu_{\alpha\beta}d_{i\beta R} \\ &+ \bar{L}_{iL}i\gamma_\mu D^\mu L_{iL} + \bar{l}_{iR}i\gamma_\mu D^\mu l_{iR}\Big). \end{aligned} \tag{3.1}$$

where there are n_G generations of quarks and leptons, the $SU(3)$ colors for the quark are labeled by $\alpha = 1,\,2,\,3$, and the electroweak gauge bosons are W^a with $a = 1,2,3$ for $SU(2)_{EW}$ and W^0 for $U(1)_{EW}$. It's easy to see that possesses a global $SU(2n_G)_L \otimes SU(2n_G)_R$ chiral symmetry, i.e. it is invariant for transformations (in the case of $n_G = 1$)

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \to U_L \begin{pmatrix} u \\ d \end{pmatrix}_L, \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R \to U_R \begin{pmatrix} u \\ d \end{pmatrix}_R. \tag{3.2}$$

The chiral nature of quark and lepton transformation laws under the electroweak gauge group forbid bare mass terms for these fermions. Let us ignore for the moment the small electroweak couplings of quarks. When the running QCD coupling constant becomes large, the strong interactions bind quark anti-quark pairs into a composite 0^+ field $\bar{\Psi}\Psi$. This can be understood thinking that, when massless quarks and antiquarks have strong attractive interactions, the energy cost of creating an extra quark-antiquark pair is small. Thus we expect that the vacuum of QCD will contain a condensate of quark-antiquark pairs. These fermion pairs must have zero total momentum and angular

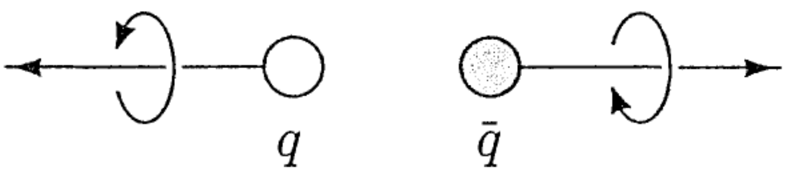


FIGURE 3.1: A quark-antiquark pair with zero total momentum and angular momentum

momentum. Thus, they must contain net chiral charge, pairing left-handed quarks with the antiparticles of right-handed quarks. This fact develops a non-zero vacuum expectation value for the scalar operator

$$\langle 0|\bar{\Psi}\Psi|0\rangle = \langle 0|\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L|0\rangle \neq 0 \tag{3.3}$$

This vacuum expectation value signals the spontaneous breaking of the full symmetry group (3.2) down to the subgroup of vector symmetries with $U_L = U_R$. In other

words, the original chiral symmetry $SU(2n_G)_L \otimes SU(2n_G)_R$ breaks down to its diagonal subgroup $SU(2n_G)_V$ of isospin. The resulting $4n_G^2 - 1$ Nambu-Goldstone bosons (the pions) will then be coupled to the appropriately defined axial-vector currents with strength $f_\pi = 93$ MeV .
This also allows the quarks to acquire effective masses as they move through the vacuum. In fact, if we follow these lines a step further and switch off the Higgs mechanism of the electroweak interactions, then we would have unbroken electroweak gauge fields coupled to identically massless quarks and leptons. However, it is apparent that the QCD-driven condensate $\langle \bar{\Psi}\Psi \rangle \neq 0$ will then spontaneously break the electroweak interactions at a scale of order Λ_{QCD}.

This statement will be clear when we restore the electroweak interactions. The quark parts of the $SU(2) \otimes U(1)$ currents couple to a normalized linear combination of these Goldstone bosons with strength $\sqrt{n_G} f_\pi$. These massless states appear as poles in the polarization tensors, $\Pi^{ab}_{\mu\nu}(q)$ of the electroweak gauge bosons. Near $q^2 = 0$ these take the form

$$\Pi^{ab}_{\mu\nu}(q) = (q_\mu q_\nu - q^2 \eta_{\mu\nu}) \left(\frac{g_a g_b n_G f_\pi^2}{4q^2} \right) + \text{nonpole terms.} \tag{3.4}$$

Here a, b = 0, 1, 2, 3; $g_0 = g'$ and $g_{1,2,3} = -g$. At this stage it appears that the electroweak symmetry $SU(2) \otimes U(1)$ has broken down to $U(1)_{EM}$ and the bosons $W^\pm$ and Z^0 as defined in (2.38) have acquired mass

$$m_W = \frac{1}{2} g \sqrt{n_G} f_\pi, \qquad m_Z = \frac{1}{2} \sqrt{g^2 + g'^2} \sqrt{n_G} f_\pi \tag{3.5}$$

while the photon stays massless. The three Goldstone bosons coupling to the electroweak currents now appear in the physical spectrum only as the longitudinal components of the $W^\pm$ and Z^0. This is the *dynamical* Higgs mechanism.
Unfortunately, such picture is phenomenologically unacceptable since it yields, for $n_G = 3$, wrong estimates of the gauge bosons masses

$$m_W \cong 53\,\text{MeV}, \qquad m_Z \cong 60\,\text{MeV} \tag{3.6}$$

in contrast to observed experimental values

$$m_W = 80.22 \pm 0.26\,\text{GeV}, \qquad m_Z = 91.173 \pm 0.020\,\text{GeV} \tag{3.7}$$

Because $f_\pi \approx 93$ MeV is so small compared to $v_{weak} \sim 175$ GeV, the familiar hadronic strong interactions cannot be the source of EWSB in nature. However, it is clear that EWSB could well involve a new strong dynamics similar to QCD, with a higher-energy-scale, $\sim v_{weak}$, with chiral symmetry breaking, and pions that become the longitudinal

$W^{\pm}$ and Z^0 modes. This kind of hypothetical new dynamics, known as *Technicolor*, was proposed in 1979 by Weinberg and Susskind.

3.2 Technicolor

Before introducing the model it is worth investigating some issues of the elementary Higgs models which have led to the quest for alternative solution to the EWSB problem. The first thing is that such models are not dynamical and there is no explanation of why EWSB occurs and why it has the scale v_{weak}.
Secondly, they are unnatural in the sense that the Higgs boson's mass is subject to large additive renormalizations, i.e. radiative corrections generally induce a mass even if the mass is ab initio set to zero. This makes fundamental scalars unappealing and unnatural.

A further problem is *triviality*. The self coupling $\lambda(M)$ of the minimal one-doublet Higgs boson runs with the energy scale M ,

$$M\frac{d}{dM} = \beta_\lambda, \tag{3.8}$$

and at one loop order this β_λ is given by [19]

$$\beta_\lambda = \frac{1}{(4\pi)^2}\Big[24\lambda^2 - 6y_t^4 + \frac{3}{8}\Big(2g^4 + (g^2+g'^2)^2\Big) + (-9g^2 - 3g'^2 + 12y_t^2)\lambda\Big], \tag{3.9}$$

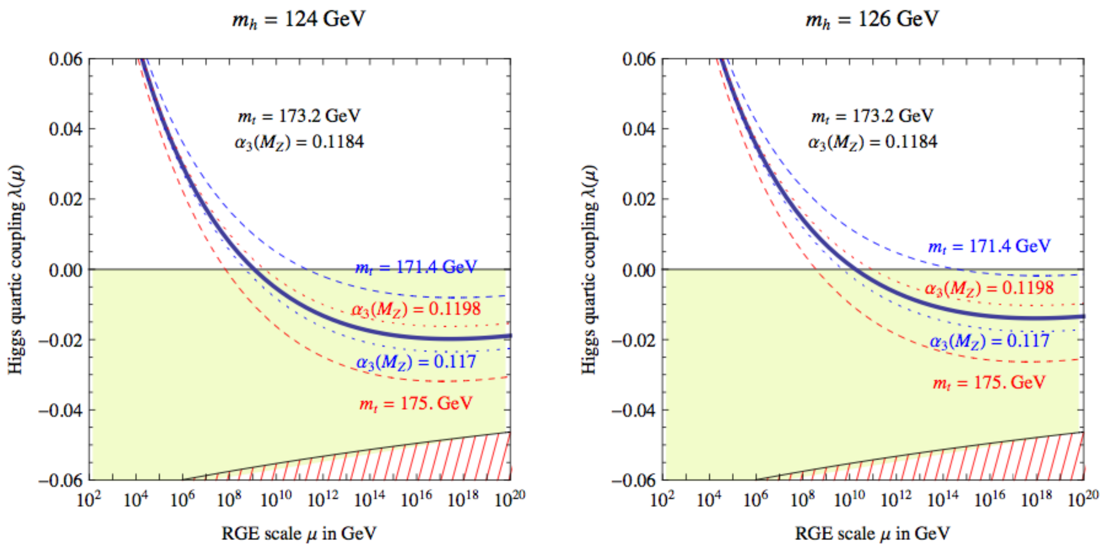


FIGURE 3.2: RG evolution of the Higgs self coupling, for different Higgs masses for the central value of the top mass m_t and α_S, as well as for $\pm 2\sigma$ variations of m_t (dashed lines) and α_S (dotted lines).

where the term $24\lambda^2$ comes from the Higgs self-interaction's contribution, $-6y_t^4$ from the top quarks loop, $\frac{3}{8}\Big(2g^4 + (g^2 + g'^2)^2\Big)$ from the gauge boson loop and the last term from Higgs field renormalization. If we neglect for a moment fermion and gauge bosons terms in (3.9), $\lambda(M)$ is determined by

$$\lambda(M) \cong \frac{\lambda(\Lambda)}{1 + (24/16\pi^2)\lambda(\Lambda)\log(\Lambda/M)}, \tag{3.10}$$

where Λ is the cutoff. This implies that $\lambda \to 0$ for all M as $\Lambda \to \infty$. So elementary-Higgs Lagrangians can be regarded as effective theories, valid until some energy cutoff Λ_∞ after which new physics sets in.
If we now investigate the fermion's effect, just considering the only term in β_λ

$$\beta_\lambda = \frac{1}{(4\pi)^2}[-6y_f^4]. \tag{3.11}$$

One can solve $\lambda(M)$ analitically when neglecting the running for the y_f to get

$$\lambda(M) = \lambda(\Lambda) - 6y_f^4 \log\frac{\lambda}{\Lambda}. \tag{3.12}$$

For a complete and consistent investigation, one should solve all the coupled RG equa-

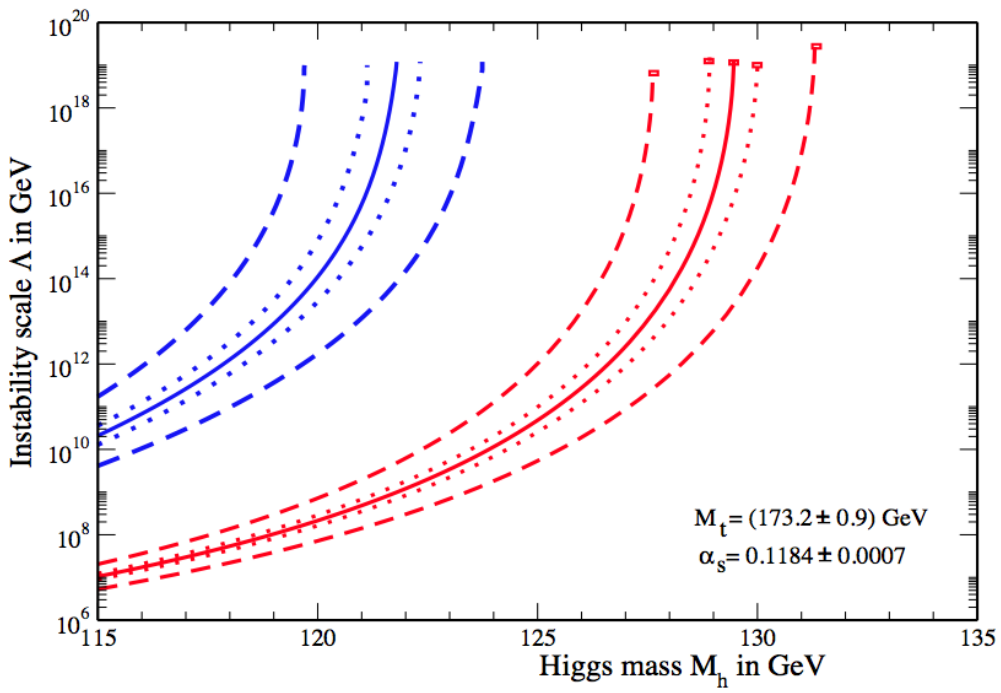


Figure 3.3: The scale at which the SM Higgs potential becomes negative as a function of the Higgs mass for the central value of m_t and α_S (plain red), as well as for $\pm 2\sigma$ variations of m_t (dashed red) and α_S (dotted red). The blue lines on the left are the metastability bounds (plain blue: central values of m_t and α_S; dashed blue: $\pm 2\sigma$ variations of m_t).

tions, but just for showing the physical effect of y_f this simplification is enough. It is clear that at some energy scale, λ can become negative. This implies that the potential is unbounded and the electroweak vacuum may be unstable due to quantum tunneling. This problem is known as *vacuum instability* problem, and the scale at which $\lambda(M)$ crosses zero depends on the mass of the Higgs boson . Another problem is the *hierarchy* problem. This problem arises when the standard model is embedded into some larger structure involving a mass scale M much larger than the electroweak scale characterised by the v.e.v. v. For example M might be identified with a scale of grand unification with $M \approx 1016$ GeV. In such a framework the Higgs sector responsible for breaking the larger gauge symmetry at the scale M and the Higgs sector responsible for breaking electroweak symmetry at the scale v cannot be kept distinct, and communicate through one-loop radiative corrections. The hierarchy of mass scales can then only be maintained at the one-loop level by fine- tuning the basic Higgs parameters of the theory to an accuracy of about 24 decimal places in this example. Such fine-tuning arises because of the quadratic nature of the scalar divergences. Furthermore the fine-tuning must be re-done at every order of perturbation theory.

Finally, elementary Higgs models have shed no light on the problem of flavor symmetry and its breaking, since Yukawa couplings of Higgs bosons to fermions are arbitrary free parameters, put in by hand.

In response to these shortcomings of the SM, the dynamical approach to electroweak and flavor symmetry breaking (Technicolor, TC) emerged in analogy with the dynamical Higgs mechanism described in the previous section. The basic idea is to assume that there exists a new, asymptotically free, gauge interaction, the technicolor interaction, with gauge group G_{TC}, and gauge coupling α_{TC} that becomes strong around $\Lambda_{TC} \sim 500$ GeV. A new set of technicolor interacting particles, N_D doublets of left and right handed *technifermions* $T_{iL,R} = (U_i, D_i)_{L,R}$, are also introduced and assigned to equivalent complex irreducible representations of G_{TC}. Namely, T_L are assigned to electroweak $SU(2)$ as doublets and the T_R as singlets. These technifermions are then massless and have the chiral flavor group

$$G_\chi = SU(2N_D)_L \otimes SU(2N_D)_R \supset SU(2)_L \otimes SU(2)_R. \tag{3.13}$$

Basically Technicolor is just a scaled-up version of QCD, so when α_{TC} becomes strong, technifermion condensates form in analogy with (3.3):

$$\langle 0|\bar{U}_{iL}U_{jR}|0\rangle = \langle 0|\bar{D}_{iL}D_{jR}|0\rangle = -\delta_{ij}\Delta_T. \tag{3.14}$$

The chiral symmetry breaks down to $S_\chi = SU(2N_D) \supset SU(2)_V$ and $4N_D^2 - 1$ massless Goldstone bosons appear, with decay constant F_{π_T}. Some linear combination of three of these will be eaten by the $W^{\pm}$, Z^0 bosons, and become their longitudinal components. Their masses will read

$$m_W = \frac{1}{2} g \sqrt{N_D} F_{\pi_T}, \qquad m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\sqrt{N_D} F_{\pi_T} = m_W / \cos\theta_W \tag{3.15}$$

In order to achieve the correct experimental magnitudes for these masses, we just have to identify $F_\pi = \sqrt{N_D} F_{\pi_T}$ with the vacuum expectation value of the Higgs field, i.e $F_\pi = v = 246$ GeV.

In this way we solved the problems that affected the Higgs mechanism. In particular the triviality problem has now vanished, since asymptotically free theories are nontrivial. A minus sign in the denominator of the analog of eq. (3.10) for $\alpha_{TC}(\mu)$ prevents one from concluding that it tends to zero for all μ as the cutoff is taken to infinity.
However, this theory is unacceptable because still lacks an explanation for quark and lepton flavor symmetries and their breaking. The quarks and leptons in this context remain massless. Tackling this issue is the motivation for an *extended* Technicolor model.

3.2.1 The Extended Technicolor Model

We have seen that TC leaves too much chiral symmetry, and, as a consequence, quarks and leptons have no hard masses. The general strategy of the extended Technicolor model (ETC) to avoid this problem is simple: introduce new interactions that break the unwanted symmetries. To do so, one embeds the TC gauge group G_{TC} into a larger ETC gauge group G_{ETC} which is broken somehow at a scale $\Lambda_{ETC} \gg \Lambda_{TC}$ down to G_{TC}, i.e.

$$G_{ETC} \to G_{TC} \otimes SU(3) \otimes \cdots . \tag{3.16}$$

After the breaking, there are heavy ETC gauge bosons corresponding to the broken ETC generators, with mass $M_{ETC} \sim g_{ETC}\Lambda_{ETC}$, where g_{ETC} is just a renormalized ETC gauge coupling. Such bosons can in general couple fermions to technifermions, which allows the generation of masses for quarks and leptons via a radiative process in which a fermion turns into a technifermion and back into a fermion, as is illustrated in figure. The typical mass is of order

$$m_f \backsimeq \frac{g^2_{ETC}\langle \bar{T}T\rangle_{ETC}}{M^2_{ETC}}, \tag{3.17}$$

(a) (b)

FIGURE 3.4: (a) Coupling between boson, fermion and technifermion. (b) Radiative process giving mass to fermions.

where the condensate $\langle \bar{T}T \rangle_{ETC}$ is related to the one renormalized at Λ_{TC} by the exponential scaling

$$\langle \bar{T}T \rangle_{ETC} = \langle \bar{T}T \rangle_{TC} \exp\left(\int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma_m(\mu) \right), \tag{3.18}$$

where

$$\gamma_m(\mu) \approx \frac{3C_2(R)}{2\pi} \alpha_{TC}(\mu) \tag{3.19}$$

is the anomalous dimension of the operator $\bar{T}T$ and $C_2(R)$ is the quadratic Casimir of the technifermion G_{TC} representation R. These are the fundamental ingredients of ETC. With the use of these equations one can estimate most quantities of phenomenological interest, such as Λ_{ETC} and the typical mass of technipions:

$$\Lambda_{ETC} \equiv \frac{M_{ETC}}{g_{ETC}} \simeq \sqrt{\frac{4\pi F_\pi^3}{m_f N_D^{3/2}}} \simeq 14 \sqrt{\frac{1\,\text{GeV}}{m_f N_D^{3/2}}}\,\text{TeV}. \tag{3.20}$$

$$M_\pi \simeq \frac{\langle \bar{T}T \rangle_{TC}}{\sqrt{2}\Lambda_{ETC} F_T} \simeq \frac{40}{N_D^{1/4}}\,\text{GeV}. \tag{3.21}$$

Despite some success in solving the problem of generating fermions masses in the original TC model, ETC suffers a series of flaws, which are the presence of flavor-changing neutral current interactions, precision measurements of electroweak quantities and the large mass of the top quark.
At an energy scale under Λ_{ETC} the massive bosons corresponding to the broken symmetries generators will produce three kinds of interactions between fermions and technifermions:

$$\alpha_{ab} \frac{\bar{Q}T^a Q \bar{Q} T^b Q}{\Lambda_{ETC}^2} + \beta_{ab} \frac{\bar{Q}_L T^a Q_R \bar{\Psi}_R T^b \Psi_L}{\Lambda_{ETC}^2} + \gamma_{ab} \frac{\bar{\Psi}_L T^a \Psi_R \bar{\Psi}_R T^b \Psi_L}{\Lambda_{ETC}^2} \tag{3.22}$$

Here we can see that the α term induces four technifermions interactions, and can elevate the mass of some Goldstone bosons, fundamental to reproduce some experimental results. The β term essentially gives masses to the ordinary quarks and leptons. The γ term, on the other hand, produces four fermion contact interactions which leads to flavor changing neutral currents and lepton number violation. Because ETC must couple differently to fermions of identical Standard Model gauge charges in order to provide the observed range of fermion masses, flavor-changing neutral current interactions amongst quarks and leptons generally result. Processes like:

$$\frac{(\bar{s}\gamma^5 d)(\bar{s}\gamma^5 d)}{\Lambda_{ETC}^2} + \frac{(\bar{\mu}\gamma^5 e)(\bar{e}\gamma^5 e)}{\Lambda_{ETC}^2} \tag{3.23}$$

are induced, and give new contributions to experimentally well constrained quantities, e.g. the $K_L K_S$ mass difference.
Another issue are the precision electroweak measurements. The SM has been tested with a high degree of accuracy, and its parameters are known very precisely, so that can be used to limit new physics above a scale of 100 GeV. The quantities most sensitive to the presence of new physics, the so called oblique correction functions S, T and U, are defined in terms of the correlation functions of electroweak currents:

$$\int d^4x\, e^{-iq\cdot x}\langle 0|T(j_i^\mu(x)j_j^\nu(0))|0\rangle = i\eta^{\mu\nu}\Pi_{ij}(q^2) + q^\mu q^\nu \,\text{terms}. \tag{3.24}$$

The S parameters is a measure of the splitting between m_W and m_Z induced by weak isospin conserving effects. The T parameter is given by the relation $1 + \alpha T = m_W^2/m_Z^2 \cos^2\theta_W$. The U parameter measures weak isospin breaking in the W and Z mass splitting. The most troubling fact is that, scaling the value from QCD, $S \simeq 1$, approximately four standard deviations away from experimental values.
Finally, to obtain the correct value for the top quark mass of $m_t = 175$ GeV one should have $\Lambda_{ETC} \approx 1.0\,\text{TeV}/N_D^{3/4}$, a value very close to Λ_{TC} itself. TC gets strong and ETC broken at the same energy and the representation of broken ETC as contacts operators is wrong and mass estimates are questionable. It would be possible to raise ETC scale, but then the problem of fine tuning of ETC coupling g_{ETC} would arise.

3.2.2 Walking Technicolor

One way to get rid of the FCNC and precision electroweak measurements of ETC is the formulation of a *walking* Technicolor. Normal Technicolor is nothing but a scaled-up version of QCD, a fact that is the root of both these problems. This assumption indeed implies that asymptotic freedom sets in quickly above Λ_{TC} and $\langle \bar{T}T\rangle_{ETC} \simeq \langle \bar{T}T\rangle_{TC}$, which in turn implies that fermions and technipions mass estimates are some order of magnitudes away from the correct values. Also, the QCD scaling means that the technihadron spectrum is just a magnified version of that of QCD and that the value of the S parameter is too large.

Therefore, it may be possible to cure at the same time both these difficulties by introducing a new Technicolor theory whose gauge dynamics are not QCD-like. The simplest realization of this is a theory in which the gauge coupling $\alpha_{TC}(\mu)$ has no dramatic increases and instead, runs slowly, hence the name "walking", over the scale range $\Lambda_{TC} \lesssim \mu \lesssim \Lambda_{ETC}$.

Thus far the condesates $\langle \bar{T}T \rangle_{ETC}$ and $\langle \bar{T}T \rangle_{TC}$ have been approximately equal because we have assumed that the anomalous dimension $\gamma_m(\mu) \approx 3C_2(R)\alpha_{TC}(\mu)/2\pi \ll 1$ when $\mu > \Lambda_{TC}$. In the extreme walking limit in which $\alpha_{TC}(\mu)$ can be regarded as constant, it is possible to get a nonperturbative approximation:

$$\gamma_m(\mu) = 1 - \sqrt{1 - \alpha_{TC}(\mu)/\alpha^*_{TC}} \qquad \text{where} \qquad \alpha^*_{TC} = \frac{\pi}{3C_2(R)}. \tag{3.25}$$

In this context the chiral symmetry breaking scale Λ_{TC} is defined by the condition

$$\alpha_{TC}(\Lambda_{TC}) = \alpha^*_{TC} \Longleftrightarrow \gamma_m(\Lambda_{TC}) = 1. \tag{3.26}$$

Such large value of γ_m allows quarks masses to be enhanced in eq. (3.18) to give the correct values up to the charm. For the top quark this still does not work, but it can be accounted for in other models, e.g. TC2.

More recently, after the discovery by LHC of a Higgs boson with mass 125 GeV, all of the minimal technicolor models have been conclusively ruled out. There still are, however, attempts to build some new models in which a scalar doublet is present together with a strong new dynamics which contributes in part to the breaking of electroweak symmetry.

These theories include both a Higgs doublet ϕ and a technicolor sector. Typically, the ϕ squared mass is assumed positive at the weak scale; the ϕ field develops a vacuum expectation value due to a linear term in the Higgs potential that is induced when the technifermions condense. In this sense, technicolor would still be the trigger of electroweak symmetry breaking. Yukawa couplings between ϕ and the quarks and leptons lead to fermion masses in the usual way. More analysis on the properties of the Higgs particle are needed to conclusively prove these new models right or wrong.

Chapter 4

The Antisymmetric Tensor Field

Let us now focus on the antysimmetric tensor field $B_{\mu\nu}(x) = -B_{\nu\mu}(x)$. It is this field that, in our model, is supposed to interact with fermions and gauge bosons of the SM via a B-Yukawa type of term and, if proved to have an asyptotically free coupling, can lead to the formation of a top-antitop condensate $\langle \phi_t \rangle \neq 0$ [1, 20] which, in analogy with Technicolor, may replace the fundamental Higgs scalar of the SM, and produce the mass terms for fermions and gauge bosons in a dynamical way.
Classically, a massless antisymmetric tensor field is described by the free Lagrangian:

$$\mathcal{L}_B = -\frac{1}{12}(\partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu})(\partial^\mu B^{\nu\lambda} + \partial^\nu B^{\lambda\mu} + \partial^\lambda B^{\mu\nu}) \equiv -\frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda}, \quad (4.1)$$

and it can be seen that it possesses the gauge invariance

$$\delta B_{\mu\nu} = \partial_\mu \theta_\nu - \partial_\nu \theta_\mu. \quad (4.2)$$

A four dimensional antisymmetric tensor has six components, which we can label by writing $B_{\mu\nu}$ as

$$B_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix} \quad (4.3)$$

These six components can be decomposed into two inequivalent ((3,1) and (1,3)) irreducible representations of the Lorentz group [21]:

$$B^{\pm}_{\mu\nu} = \frac{1}{2} B_{\mu\nu} \pm \frac{i}{4} \epsilon_{\mu\nu}{}^{\rho\sigma} B_{\rho\sigma}, \quad (4.4)$$

with $\epsilon_{\mu\nu\rho\sigma}$ the totally antisymmetric tensor.
The equation of motion is easily computed and reads:

$$\partial_\mu \partial^{[\mu} B^{\nu\lambda]} = 0. \tag{4.5}$$

In order to write down a B-Yukawa type of interaction term with fermions, one should work with a complex $B_{\mu\nu}$ field which transform as a doublet under weak $SU(2)$ interactions, exactly as the Higgs, and carry the same hypercharge $Y = 1$.
The B-Yukawa type of interaction term we will consider is

$$\Delta\mathcal{L} = y B_{\mu\nu} \bar{\Psi}^\beta_L \sigma^{\mu\nu}_{\alpha\beta} \Psi^\alpha_R + \text{h.c.}, \tag{4.6}$$

with α, β spinor indices, which we, from now on, will suppress.
We can already see that such term is Lorentz invariant, but we can also check that it also is invariant under a parity transformation. To see this, we first look how the $B_{\mu\nu}$ alone transforms under parity:

$$B_{\mu\nu} \to B_{\mu\nu} \qquad \text{for}\, \mu, \nu \neq 0 \qquad \text{or} \qquad \mu = \nu = 0; \tag{4.7}$$

$$B_{\mu\nu} \to -B_{\mu\nu} \qquad \text{for}\, \mu \vee \nu = 0, \tag{4.8}$$

and then see how the $\bar{\Psi}\sigma^{\mu\nu}\Psi$ piece behaves under the same parity transformation:

$$\begin{aligned}
\bar{\Psi}(t,\vec{x})\sigma^{\mu\nu}\Psi(t,\vec{x}) \to & \eta^* \bar{\Psi}(t,-\vec{x})\gamma^0 \sigma^{\mu\nu}\gamma^0 \Psi(t,-\vec{x})\eta = \\
= & \bar{\Psi}(t,-\vec{x})\sigma^{\mu\nu}\Psi(t,-\vec{x}) \qquad \text{for} \qquad \mu,\nu \neq 0 \qquad \text{or} \qquad \mu = \nu = 0; \\
= & -\bar{\Psi}(t,-\vec{x})\sigma^{\mu\nu}\Psi(t,-\vec{x}) \qquad \text{for} \qquad \mu \vee \nu = 0.
\end{aligned}$$

Thus it is clear that the whole term $B_{\mu\nu}\bar{\Psi}\sigma^{\mu\nu}\Psi$ is parity invariant.
In principle, other possible B-Yukawa terms would include the dual tensor:

$$\tilde{B}_{\alpha\beta} = \epsilon_{\alpha\beta\mu\nu} B^{\mu\nu} = \begin{pmatrix} 0 & -B_1 & -B_2 & -B_3 \\ B_1 & 0 & -E_3 & E_2 \\ B_2 & E_3 & 0 & -E_1 \\ B_3 & -E_2 & E_1 & 0 \end{pmatrix} \tag{4.9}$$

but, since the Levi-Civita tensor is a pseudo-tensor, this object would have parity properties inverted with respect to the normal antisymmetric tensor, hence the B-Yukawa terms

$$y\tilde{B}_{\mu\nu}\bar{\Psi}_L \sigma^{\mu\nu}\Psi_R + y^* \tilde{B}^\dagger_{\mu\nu}\bar{\Psi}_R \sigma^{\mu\nu}\Psi_L \tag{4.10}$$

would not be parity even. This is the reason why we will disregard such terms in our

model.
The full Lagrangian at this stage will then read:

$$
\begin{aligned}
\mathcal{L} =&\mathcal{L}_\Psi + \mathcal{L}_W + \mathcal{L}_A + \mathcal{L}_B + \Delta\mathcal{L}_{int} = \\
=&i\sum_{n=1}^{3}[\bar{e}_L^n\gamma^\mu D_\mu e_L^n + \bar{q}_L^n\gamma^\mu D_\mu q_L^n + \bar{e}_R^n\gamma^\mu D_\mu e_R^n + \bar{u}_R^n\gamma^\mu D_\mu u_R^n + \bar{d}_R^n\gamma^\mu D_\mu d_R^n] \\
&-\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}H^\dagger_{\mu\nu\lambda}H^{\mu\nu\lambda} \\
&+\sum_{i,j=1}^{3}(y_u^{ij}\bar{q}_L^i\hat{B}_{\mu\nu}\sigma^{\mu\nu}u_R^j + y_d^{ij}\bar{q}_L^i B_{\mu\nu}\sigma^{\mu\nu}d_R^j + y_e^{ij}\bar{e}_L^i B_{\mu\nu}\sigma^{\mu\nu}e_R^j),
\end{aligned} \tag{4.11}
$$

where D_μ is the usual SM covariant derivative and $\hat{B}_{\mu\nu} = i\sigma^2 B_{\mu\nu}$.
From this Lagrangian it is possible to derive in the usual manner through variation with respect to the different fields the equation of motion for $\bar{\Psi}$,Ψ, A_μ,W_μ,$B_{\mu\nu}$. This work has already been done in [2] and here we can just quote the results. Varying with respect to $\bar{\Psi}$ yields:

$$i\overrightarrow{D}_\mu\gamma^\mu + yB_{\mu\nu}\sigma^{\mu\nu}\Psi + y\hat{B}_{\mu\nu}\sigma^{\mu\nu}\Psi = 0, \tag{4.12}$$

and a similar equation holds for $\bar{\Psi}$. The variation with respect to $B_{\mu\nu}$ instead leads to

$$\partial_\mu D^{[\mu}B^{\nu\rho]} + 6y\bar{\Psi}\sigma^{\nu\rho}\Psi - (igA_{[\mu} + \frac{ig'}{2}W_{[\mu^a}\tau_a)(igA[\mu + \frac{ig'}{2}W^{a[\mu}\tau_a)B^{\nu\rho]} = 0, \tag{4.13}$$

which can be split in two equations, one for the E field and one for the B:

$$\partial_0 D^{[j}E^{i]} - (igA_{[j} + \frac{ig'}{2}W^a_{[j}\tau_a)(igA^{[j} + \frac{ig'}{2}W^{a[j}\tau_a)E^{i]} = 0 \tag{4.14}$$

$$
\begin{aligned}
&\partial_0 D^0\epsilon^{ijm}B^m + \partial_k D^{[k}\epsilon^{ij]m}B^m - (igA_{[k} + \frac{ig'}{2}W^a_{[k}\tau_a)(igA^{[k} + \frac{ig'}{2}W^{a[k}\tau_a)\epsilon^{ij]m}B^m \\
&+(igA_0 + \frac{ig'}{2}W^a_0\tau_a)(igA^0 + \frac{ig'}{2}W^{a0}\tau_a)\epsilon^{ijm}B^m - 6y\bar{\Psi}\sigma^{ij}\Psi - i6y\sigma^2\bar{\Psi}\sigma^{ij}\Psi = 0.
\end{aligned} \tag{4.15}
$$

4.1 Kinetic Term

We will now focus on the kinetic term for the antisymmetric field. We can easily read it off the Lagrangian (4.11):

$$\mathcal{L}_B = -\frac{1}{12}H^\dagger_{\mu\nu\lambda}H^{\mu\nu\lambda} = -\frac{1}{12}(D^{[\mu}B^{\nu\lambda]})^\dagger D_{[\mu}B_{\nu\lambda]}. \tag{4.16}$$

We will work in what follows with a metric choice (-,+,+,+) and a notation for the antisymmetric tensor field components as

$$B_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix} \tag{4.17}$$

$$\begin{aligned} B_{0i} = E_i \quad B_{i0} = -E_i \quad B_{ij} = -\epsilon_{ijl} B^l \\ B^\dagger_{0i} = E_i \quad B^\dagger_{i0} = -E_i \quad B^\dagger_{ij} = \epsilon_{ijk} B_k \\ B^{0i} = -E_i \quad B^{i0} = E_i \quad B^{ij} = -\epsilon_{ijk} B_k. \end{aligned} \tag{4.18}$$

It is worth to stress the fact that the possibility of a kinetic term with non-antiymmetrized indeces is not forbidden, but it has brought in the literature [20], to the manifestation of classical instabilities, which could be corrected by some non-perturbative effect that generates a mass term for the antisymmetric field. For this reason we will keep the antiymmetrized version. Let us now write the kinetic term explicitly in terms of the B and E fields. We have, with a bit of algebra,

$$\begin{aligned} \frac{1}{12}(D^{[\mu} B^{\nu\lambda]})^\dagger D_{[\mu} B_{\nu\lambda]} =& (D_\mu B_{\nu\lambda})^\dagger (D^\mu B^{\nu\lambda} + D^\lambda B^{\mu\nu} + D^\nu B^{\lambda\mu}) = \\ =& (D_\mu B_{\nu\lambda})^\dagger D^\mu B^{\nu\lambda} + 2(D_\mu B_{\nu\lambda})^\dagger D^\nu B^{\lambda\mu} = \\ =& 2(D_i E_j D^i E_j - D_j E_i D^i E_j - D_0 B_i D^0 B^i - D_l B_l D^m B^m), \end{aligned} \tag{4.19}$$

which, making use of the identity:

$$(\mathbf{D} \times \mathbf{E})^2 = \epsilon_{ijl} D_j E_l \epsilon D_m E_n = D_j E_n D_j E_n - D_j E_l D_l E_j \tag{4.20}$$

boils down to

$$-\frac{1}{12} H^\dagger_{\mu\nu\rho} H^{\mu\nu\rho} = (\mathbf{D} \times \mathbf{E})^2 - (\mathbf{D} \cdot \mathbf{B})^2 + (D_0 \mathbf{B})^2. \tag{4.21}$$

With this expression we can compute the equation of motion directly for the E and B fields to get

$$D_l^2 E_j - D_l D_j E_l = -2y \bar{\Psi} \sigma^{0i} \Psi \tag{4.22}$$

$$D_0^2 B_i - D_i (\mathbf{D} \cdot \mathbf{B}) = y \epsilon^{ijl} \bar{\Psi} \sigma^{jl} \Psi, \tag{4.23}$$

from which we can conclude that the truly dynamical information is encoded in the B field, while the E field is not dynamical. In fact, one can rewrite the equation of motion for the free $\overrightarrow{E}$ and $\overrightarrow{B}$ fields as:

$$\partial^2\overrightarrow{B} + \partial_0\Big(\overrightarrow{\nabla} \times \overrightarrow{E}\Big) - \overrightarrow{\nabla} \times \overrightarrow{\nabla} \times \overrightarrow{B} = 0 \tag{4.24}$$

$$\nabla^2\overrightarrow{E} - \overrightarrow{\nabla}(\overrightarrow{\nabla} \cdot \overrightarrow{E}) - \partial_0(\overrightarrow{\nabla} \times \overrightarrow{B}) = 0, \tag{4.25}$$

which in terms of the longitudinal and transverse components can be reduced to

$$\partial^2 B^L = 0 \tag{4.26}$$

$$\nabla \times E^T - \partial_0 B^T = 0. \tag{4.27}$$

So we see that the longitudinal component E^L has decoupled from the theory and can be just set to zero, while the two transverse components can be combined into a gauge invariant quantity that satisfies

$$E^T_{g.i.} = 0. \tag{4.28}$$

This implies that the theory has only one dynamical degree of freedom, encoded in B^L.
A similar analysis can be done also in d dimensions. The e.o.m. in this case are

$$\partial^2 E^i - \partial_i(\nabla \cdot \overrightarrow{E}) + \partial_0^2 E^i - \partial_0\partial_j B^{ij} = 0 \tag{4.29}$$

$$\partial^2 B^{ij} - \partial_0\partial_i E^j + \partial_l\partial_i B^{jl} + \partial_0\partial_j E^i + \partial_l\partial^j B^{li} = 0, \tag{4.30}$$

where this time E^i is a $d-1$ vector and B^{ij} is a $d-1$ rank antisymmetric tensor. Writing the equations in terms of longitudinal components leads to the classical wave equation:

$$\partial^2(\nabla \cdot B)^L = 0 \tag{4.31}$$

and a condition between the two transverse components

$$\nabla^2 E^T - \partial_0(\nabla \cdot B)^T = 0, \tag{4.32}$$

which again can be brought into a gauge invariant quantity that may be set to zero. Then, in d dimensions, our antisymmetric tensor has $\frac{(d-2)(d-3)}{2}$ dynamical degrees of freedom. With this analysis we can work with dimensional regularization when computing Feynman diagrams in chapter 5.
Finally, we will check that our kinetic term is invariant under $SU(2) \times U(1)$ SM transformation. One can prove this by writing $H_{\mu\nu\lambda}$ as the dual of the derivative of a scalar:

$$H_{\mu\nu\rho} = \epsilon^{\mu\nu\rho\alpha}\nabla_\alpha\sigma, \tag{4.33}$$

so that

$$-\frac{1}{2(d-1)!}H^{\mu\nu\rho}H^{\dagger}_{\mu\nu\rho} = -\frac{1}{2(d-1)!}\epsilon^{\mu\nu\rho\alpha}\epsilon_{\mu\nu\rho\beta}D_\alpha\sigma(D^\beta\sigma)^\dagger = -\frac{1}{2}D_\alpha\sigma(D^\beta\sigma)^\dagger. \quad (4.34)$$

We can then transform back, and get

$$D_\alpha\sigma = \epsilon_{\alpha\mu\nu\rho}D^{[\mu}B^{\nu\rho]}. \quad (4.35)$$

Since we know how a scalar field transforms under a $SU(2)\times U(1)$ transformation ($\sigma \to \sigma' = \exp(-\frac{ig'}{2}\Lambda - \frac{ig}{2}\xi^a\tau_a)\sigma$), we then also know how the $B_{\mu\nu}$ transforms, and it is easy to prove

$$D'_\mu B'_{\nu\rho} = D_\mu B_{\nu\rho}, \quad (4.36)$$

that is, invariance under $SU(2)\times U(1)$.
Other accidental invariances of this term have been searched but not found. A covariant version of the symmetry of the free antisymmetric field (4.2), for instance, does not hold. Other attempts have been made to see if the interaction term between the antisymmetric tensor and the fermions could be generated from a symmetry of the fermion Lagrangian, as it is usually done in local gauge theories. A generic transformation on the fermions can be written as

$$\Psi \to \Psi' = S\Psi. \quad (4.37)$$

Such transformation S should belong to some Lie group. We could see how the generators of this group should look like in order to reproduce the correct interaction term by imposing:

$$\gamma^\mu T^\nu = \sigma^{\mu\nu}. \quad (4.38)$$

This leads to

$$T^\nu = 2\gamma^\nu - 2\eta^{\mu\nu}(\gamma^\mu)^{-1}. \quad (4.39)$$

However we can already argue that this is not a proper Lie group since the generators do not respect the associated Lie algebra:

$$[T^\nu, T^\rho] \neq i f^{\nu\rho}_\sigma T^\sigma. \quad (4.40)$$

We can conclude then that there are no other manifest symmetries in our Lagrangian.

4.2 Covariant Quantization

Clearly, our symmetric tensor is redundant, i.e. the free theory has spurious degrees of freedom that need to be fixed somehow. This can be seen from the invariance (4.2),

which is an analogous of the gauge invariance of the photon field $A_\mu \to A'_\mu = A_\mu + \partial_\mu \xi$. When it comes to quantization of the field, this redundancy can be overcome by using Dirac quantization [22], a generalization of canonical quantization to systems of fields with constraints. The price to pay is the lost of manifest Lorentz invariance. This procedure has been used in the preceding work [2], but it has brought to cumbersome calculations in coordinate space and therefore, has been abandoned in our approach.
Another solution to the problem at hand would be to introduce the missing (gauge) degrees of freedom via a *gauge fixing* term in the Lagrangian to keep track of manifest Lorentz invariance. This term would formally spoil the gauge invariance, but it would make the quadratic term invertible, so that it is possible to compute a covariant propagator for the $B_{\mu\nu}$ field.
The degrees of freedom introduced will not affect the interactions of the theory,and the effect of this procedure can still be separated from the true gauge invariant part of the theory, so that the physical consequences remain unchanged.
To proceed to the computation of the propagator the first thing to do is to find a good candidate for the gauge fixing term. Of course, this new term should not modify the physics of our model, which means that it should not modify the equation of motion of the dynamical degrees of freedom, which are encoded in B_L, the longitudinal components of the B field.
Bearing this in mind we tried to add a term like $\frac{2}{\xi}(\partial^\mu B_{\mu\nu})^\dagger(\partial_\rho B^{\rho\nu})$ and see what changes in the equation of motion for the fields B and E.
First let us compute this term in terms of the fields.

$$\begin{aligned}(\partial^\mu B_{\mu\nu})^\dagger(\partial_\rho B^{\rho\nu}) &= \partial^0 B^\dagger_{0i}\partial_0 B^{0i} + \partial^i B^\dagger_{i0}\partial_j B^{j0} + \partial^i B^\dagger_{ij}\partial_k B^{kj} \\ &= -\partial_0 E_i \partial_0 E_i - \partial^i E_i \partial_j E_j - \partial^i \epsilon_{ijl} B_l \partial_k \epsilon_{kjm} B_m \\ &= -(\partial_0 E_i)^2 + (\nabla \cdot \mathbf{E})^2 + (-\delta_{ik}\delta_{ml} + \delta_{il}\delta_{mk})\partial^i B_m \partial_k B_l \\ &= -(\partial_0 E_i)^2 + (\nabla \cdot \mathbf{E})^2 + (\nabla \times \mathbf{B})^2. \end{aligned} \tag{4.41}$$

Since this kind of term only adds a contribution to the equation of motion of the B field in the form of $(\nabla\times\mathbf{B})^2$, it does not affect the longitudinal component B_L, the dynamical degree of freedom, and its equation of motion.
We can explicitly check this by computing the modified e.o.m. to get

$$\partial^2 B^{\nu\rho} + \Big(1 - \frac{1}{\xi}\Big)[\partial_\mu \partial^\rho B^{\mu\nu} - \partial_\mu \partial^\nu B^{\mu\rho}] = 0, \tag{4.42}$$

and rewriting it as

$$\nabla^2 E_i - \frac{1}{\xi}\partial_0^2 E_i + \Big(1 - \frac{1}{\xi}\Big)[\partial_0(\nabla \times B) - \vec{\nabla}(\vec{\nabla} \cdot \vec{E})] = 0 \tag{4.43}$$

$$-\partial^2\overrightarrow{B} - \left(1-\frac{1}{\xi}\right)[\partial_0(\overrightarrow{\nabla}\times\overrightarrow{E}) + \overrightarrow{\nabla}\times(\overrightarrow{\nabla}\times\overrightarrow{B})] = 0, \tag{4.44}$$

which again yields

$$\partial^2 B^L = 0. \tag{4.45}$$

This proves that is a suitable gauge fixing term to add to the Lagrangian. Besides, this is the only type of term one can add, since all the other possible contractions of the derivative with the B field either give zero or are already present in the original Lagrangian. However, in the end of our calculations we want to send ξ to zero, in order to restore the correct number of degrees of freedom of the antisymmetric tensor field.
With this result we can then proceed to the calculation of the antisymmetric tensor field covariant propagator.

4.2.1 The Covariant Propagator

The following calculation that leads to an expression for the time-ordered (Feynman) covariant propagator has been done in analogy with what can be found in [23].
The first step is to look at the modified equation of motion. The get modified E.o.m. for the $B_{\mu\nu}$ field we just have to vary the modified quadratic part of the Lagrangian with respect to $B^\dagger_{\mu\nu}$:

$$\begin{aligned}
&\mathcal{L}^\xi = (\partial_\mu B_{\nu\rho})^\dagger(\partial^\mu B^{\nu\rho} + \partial^\rho B^{\mu\nu} + \partial^\nu B^{\rho\mu}) + \frac{2}{\xi}(\partial^\mu B_{\mu\nu})^\dagger(\partial_\rho B^{\rho\nu}) \\
&\frac{\delta\mathcal{L}^\xi}{\delta B^\dagger_{\nu\rho}} = \frac{\delta(H^{\dagger\mu\nu\rho})H_{\mu\nu\rho}}{\delta B^\dagger_{\nu\rho}} + \frac{1}{\xi}\frac{\delta}{\delta B^\dagger_{\nu\rho}}(\partial^\nu B^\dagger_{\nu\rho})\partial_\mu B^{\mu\rho} + \frac{1}{\xi}\frac{\delta}{\delta B^\dagger_{\nu\rho}}(\partial^\rho B^\dagger_{\rho\nu})\partial_\mu B^{\mu\nu} = \\
&= \partial^2 B^{\nu\rho} + \partial_\mu\partial^\rho B^{\mu\nu} - \partial_\mu\partial^\nu B^{\mu\rho} + \frac{1}{\xi}(\partial_\mu\partial^\nu B^{\mu\rho} - \partial_\mu\partial^\rho B^{\mu\nu}) = \\
&= \partial^2 B^{\nu\rho} + \left(1-\frac{1}{\xi}\right)[\partial_\mu\partial^\rho B^{\mu\nu} - \partial_\mu\partial^\nu B^{\mu\rho}] = 0.
\end{aligned} \tag{4.46}$$

Next thing we can compute is the canonical momentum

$$\Pi^{\nu\rho} = \frac{\delta\mathcal{L}}{\delta(\partial_0 B^\dagger_{\nu\rho})} = \partial^0 B^{\nu\rho} + \partial^\rho B^{0\nu} - \partial^\nu B^{0\rho} - \frac{1}{\xi}\delta^\nu_0\partial_\mu B^{\mu\rho} + \frac{1}{\xi}\delta^\rho_0\partial_\mu B^{\mu\nu}. \tag{4.47}$$

To calculate the propagator then we have to invert the operator $L^{\nu\rho}_{\alpha\beta}$ which is:

$$\left[\partial^2\eta^\nu_{[\alpha}\eta^\rho_{\beta]} + 2\left(1-\frac{1}{\xi}\right)\partial_{[\alpha}\partial^{[\rho}\eta^{\nu]}_{\beta]}\right]B^{\alpha\beta} = L^{\nu\rho}_{\alpha\beta}B^{\alpha\beta} = 0. \tag{4.48}$$

The time ordered propagator would then satisfy

$$L^{\nu\rho}_{\alpha\beta} i[_{\nu\rho}\Delta^{++}_{\gamma\delta}](x,x') =_{\alpha\beta} P_{\gamma\delta} \times i\hbar\delta^D(x-x'), \tag{4.49}$$

for some unknown tensor structure $_{\alpha\beta}P_{\gamma\delta}$. To find it, we make use of the canonical commutator:

$$[B_{\mu\nu}(\vec{x},t), \Pi^{\rho\sigma}(\vec{x}',t)] = i\hbar\delta^{[\rho}_{\mu}\delta^{\sigma]}_{\nu}\delta^{D-1}(\vec{x}-\vec{x}') \tag{4.50}$$

and of the fact that the time ordered propagator can be written as:

$$i_{\rho\sigma}\Delta^{++}_{\alpha\beta}(x,x') = \theta(t-t')i[_{\rho\sigma}\Delta^{+-}_{\alpha\beta}](x,x') + \theta(t'-t)i[_{\rho\sigma}\Delta^{-+}_{\alpha\beta}](x,x'), \tag{4.51}$$

where

$$i_{\rho\sigma}\Delta^{-+}_{\alpha\beta}(x,x') = \langle B_{\rho\sigma}(x)B_{\alpha\beta}(x')\rangle, \qquad i_{\rho\sigma}\Delta^{+-}_{\alpha\beta}(x,x') = \langle B_{\alpha\beta}(x')B_{\rho\sigma}(x)\rangle \tag{4.52}$$

are the two Wightman functions.

The Lagrangian implies that the Wigthman functions obey the equations:

$$L^{\rho\sigma}_{\mu\nu} i[_{\rho\sigma}\Delta^{+-}_{\alpha\beta}](x,x') = 0, \qquad L^{\rho\sigma}_{\mu\nu} i[_{\rho\sigma}\Delta^{-+}_{\alpha\beta}](x,x') = 0. \tag{4.53}$$

So, when the operator $L^{\rho\sigma}_{\mu\nu}$ acts on the whole time ordered propagator, one gets a non-vanishing contribution only when one time derivative hits a Whigtman function and one a θ function. The result is:

$$\begin{aligned}
&L^{\nu\rho}_{\alpha\beta}[\theta(t-t')i[_{\nu\rho}\Delta^{+-}_{\gamma\sigma}](x,x') + \theta(t'-t)i[_{\nu\rho}\Delta^{-+}_{\gamma\sigma}](x,x')] = \\
&L^{\nu\rho}_{\alpha\beta}[\theta(t-t')\langle B_{\rho\sigma}(x)B_{\alpha\beta}(x')\rangle + \theta(t'-t)\langle B_{\rho\sigma}(x)B_{\alpha\beta}(x')\rangle] = \\
&= -\delta(t-t')\left[-\eta^{\nu}_{[\alpha}\eta^{\rho}_{\beta]}\dot{B}_{\nu\rho}(x) + \left(1-\frac{1}{\xi}\right)\left(\delta^0_{[\alpha}\delta^{\rho}_0\eta^{\nu}_{\beta]}\dot{B}_{\nu\rho}(x) - \delta^0_{[\alpha}\delta^{\nu}_0\eta^{\rho}_{\beta]}\dot{B}_{\nu\rho}(x)\right), B_{\gamma\sigma}(x')\right] = \\
&= -\delta(t-t')\left[-\dot{B}_{\alpha\beta}(x) + \left(1-\frac{1}{\xi}\right)\left(\delta^0_{[\alpha}\dot{B}_{\beta]0}(x)\right), B_{\gamma\sigma}(x')\right] = \\
&= \delta(t-t')\left[-\Pi_{\alpha\beta}(x,t), B_{\gamma\sigma}(x',t)\right] = \delta(t-t')\times\eta_{\alpha[\gamma}\eta_{\sigma]\beta}i\hbar\delta^{D-1}(\vec{x}-\vec{x}').
\end{aligned} \tag{4.54}$$

From which we infer that:

$$_{\alpha\beta}P_{\gamma\sigma} = \eta_{\alpha[\gamma}\eta_{\sigma]\beta}. \tag{4.55}$$

The passage between the last lines can be obtained by realising that spatial derivatives do not contribute to the commutator.

Hence, the equation we need to solve for the Keldysh propagator is

$$L^{\rho\sigma}_{\mu\nu} i[_{\rho\sigma}\Delta^{ab}_{\alpha\beta}](x,x') = \eta_{\mu[\alpha}\eta_{\beta]\nu}(\sigma^3)^{ab}i\hbar\delta^D(x-x'), \tag{4.56}$$

where $a, b = \pm$.

If we now insert the Ansatz

$$[_{\rho\sigma}\Delta^{ab}_{\alpha\beta}](x,x') = \eta_{\rho[\alpha}\eta_{\beta]\sigma}A^{ab}(x,x') + \partial_{[\rho}\eta_{\sigma][\alpha}\partial_{\beta]}B^{ab}(x,x') \tag{4.57}$$

into equation (4.56) we get

$$\partial^2 A^{ab}(x,x') = i\hbar(\sigma^3)^{ab}\delta^D(x-x') \tag{4.58}$$

$$\left(-\frac{1}{\xi}\right)\partial^2 B^{ab}(x,x') - 2\left(1-\frac{1}{\xi}\right)A^{ab}(x,x') = 0. \tag{4.59}$$

Which are solved by

$$A^{ab}(x,x') = i\hbar\Delta^{ab}_0(x,x') \tag{4.60}$$

$$B^{ab}(x,x') = 2(1-\xi)\hbar\int d^D z i\Delta^{ac}_0(x,z)(\sigma^3)^{cd}\Delta^{db}_0(z,x'), \tag{4.61}$$

where

$$i\Delta^{ab}_0(x,x') = \frac{\Gamma(\frac{D-2}{2})}{4\pi^{D/2}}\frac{1}{(\Delta x^2_{ab})^{(D-2)/2}} \tag{4.62}$$

is the Keldysh propagator for a massless scalar field in Minkowski space.
Eventually, we can write the propagator for the antisymmetric tensor field as

$$\begin{aligned} i[_{\rho\sigma}\Delta^{ab}_{\alpha\beta}](x,x') =& \hbar\eta_{\rho[\alpha}\eta_{\beta]\sigma}i\Delta^{ab}_0(x,x') \\ &+ 2(1-\xi)\hbar\partial_{[\rho}\eta_{\sigma][\alpha}\partial_{\beta]}\int d^D z i\Delta^{ac}_0(x,z)(\sigma^3)^{cd}\Delta^{db}_0(z,x'). \end{aligned} \tag{4.63}$$

To obtain the expression of the propagator in momentum space we just have to plug into last equation the expression for the propagator for the scalar field in momentum space (see Appendix A). Doing this we have, for the Feynman propagator:

$$i[_{\rho\sigma}\tilde{\Delta}^{++}_{\alpha\beta}](k) = \hbar\eta_{\rho[\alpha}\eta_{\beta]\sigma}\frac{-i}{k_\mu k^\mu - i\epsilon} + 2(1-\xi)i\hbar\frac{k_{[\rho}\eta_{\sigma][\alpha}k_{\beta]}}{(k_\mu k^\mu - i\epsilon)^2}. \tag{4.64}$$

This is the expression we will use in what follows. We will keep manifest track of the gauge parameter ξ and let it go to 0 at the end of the calculations, in order to restore the exact (transverse) gauge $\partial_\mu B^{\mu\nu} = 0$ in which the antisymmetric field has only one degree of freedom and the propagator has the correct transversality properties.

Chapter 5

Computation of the Beta Function for the B-Yukawa Type Interaction

The key idea of this work is to provide the antisymmetric tensor field with a B-Yukawa coupling to the fermions, in order to have some condensate to play the role of the Higgs and give mass to fermions and gauge bosons. The only way for this to be possible is to have an asymptotically free coupling constant. So our task will be to check that this is indeed the case.

5.1 The Callan-Symanzik Equation

In this section we will refer to [6] to have an idea about the meaning of the beta function and how it is computed in quantum field theories.

Basically, the beta function of a coupling constant is a measure of the variation of the magnitude of the coupling as a function of the energy scale M at which the theory is renormalized. To better understand this concept we will use the example of the scalar field theory with a quartic interaction term $\lambda\phi^4$ in four dimensions.

To properly define this theory, i.e. to regulate divergences coming from some Feynman diagrams, it is customary to impose some *renormalization conditions* which will lead to the introduction of counterterms in the original Lagrangian. The function of these counterterms is only to cancel divergences and they will not affect the theory in any other way. For this example the renormalization conditions at a spacelike momentum p with $p^2 = -M^2$ are:

- The vanishing of the self energy diagram at $p^2 = -M^2$

- The vanishing of the derivative with respect to p^2 of the self energy diagram at $p^2 = -M^2$

- The vertex correction at $(p_1 + p_2)^2 = (p_1 + p_3)^2 = (p_1 + p_4)^2 = -M^2$ has to be equal to $-i\lambda$.

One can then rescale the bare field ϕ_0 by

$$\phi = Z^{-\frac{1}{2}}\phi_0, \tag{5.1}$$

where the factor Z comes from the bare two-point Green's function at the renormalization scale:

$$\langle\Omega|\phi_0(p)\phi_0(-p)|\Omega\rangle = \frac{iZ}{p^2} \quad \text{at} \quad p^2 = -M^2. \tag{5.2}$$

The Lagrangian can be rewritten, defining the counterterms $\delta_Z = Z - 1$ and $\delta_\lambda = \lambda_0 Z^2 - \lambda$, as

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\lambda}{4!}\phi^4 + \frac{1}{2}\delta_Z(\partial_\mu\phi)^2 - \frac{\delta_\lambda}{4!}\phi^4. \tag{5.3}$$

We can see that this theory appears to have new interactions with respect to the original one. This is just fictitious because all we have done has been to split each term of the starting Lagrangian in terms of the physical field and coupling. The counterterms can now be adjusted to provide the renormalization conditions above.

An important observation to make is that the energy scale M is completely arbitrary in the renormalization conditions. In fact, the bare Green's functions are not affected at all by this scale. What is influenced by M are the renormalized Green's functions, which are proportional to the bare ones: $G^n = Z^{-\frac{n}{2}}G_0^n$. Thus, as long as to each change in M there is a corresponding adjustment to Z, we are equally well describing the same bare theory.

When an infinitesimal change to M is performed, another one is induced on the renormalized n-point Green's function. Quantitatively, for a change

$$M \to M + \delta M; \qquad \lambda \to \lambda + \delta\lambda; \qquad \phi \to (1 + \delta\eta)\phi; \tag{5.4}$$

corresponds

$$\delta G^n = \frac{\partial G^n}{\partial M}\delta M + \frac{\partial G^n}{\partial \lambda}\delta\lambda = n\delta\eta G^n. \tag{5.5}$$

The rearrangement of this equation in terms of

$$\beta \equiv \frac{M}{\delta M}\delta\lambda; \qquad \gamma \equiv -\frac{M}{\delta M}\delta\eta, \tag{5.6}$$

is generally known as the Callan-Symanzik equation:

$$\Big[M\frac{\partial}{\partial M} + \beta(\lambda)\frac{\partial}{\partial\lambda} + n\gamma(\lambda)\Big]G^n(\{x_i\}; M, \lambda) = 0 \tag{5.7}$$

The parameters β and γ are independent of n. Besides, since G^n is the renormalized Green's function,i.e. it cannot depend on the cutoff Λ, neither can β and γ depend on the cutoff and hence, by dimensional analysis, on the scale M.
The two parameters are then only functions of the coupling λ, and they are related to the shifts in the coupling constant and field strength. The β function, in particular, is of fundamental importance because tells us how the coupling strength varies with the scale M and therefore, clarifies the conditions under which perturbation analysis is applicable.
To explicitly compute these functions one should implement a set of renormalized conditions, together with the introduction of counterterms, and then impose that the conveniently chosen renormalized Green's functions satisfy the Callan-Symanzik equation. The expressions for β and γ will of course depend on these counterterms and on the particular choice for the renormalization conditions. Luckily however, to one-loop order, their expressions will be unambiguous.
In a general theory with dimensionless coupling the M dependence of the Green's functions enters through the field strength and vertex counterterms, which are used to subtract the divergent logarithms. This means that the β and γ functions can be computed only by knowing the counterterms[1]. This will be more clear if we write the generic n-point Green function for the coupling g associated with an n-point vertex. To one loop order we have:

$$G^n = \Big(\prod_i \frac{i}{p^2}\Big)\Big[-ig - iB\log\frac{\Lambda^2}{-p^2} - i\delta_g + (-ig)\sum_i\Big(A_i\log\frac{\Lambda^2}{-p^2} - \delta_{Z_i}\Big)\Big] + \text{finite} \tag{5.8}$$

where δ_g is the counterterm for the vertex, δ_{Z_i} are the counterterms for the external legs, Λ is the cutoff and B and A_i are the coefficients of the divergent part of the relevant Feynman diagrams.
Imposing that this Green's function satisfies the Callan-Symanzik equation we find:

$$\beta(g) = M\frac{\partial}{\partial M}\Big(-\delta_g + \frac{1}{2}g\sum_i \delta_{Z_i}\Big). \tag{5.9}$$

[1]This statement is only true for flat background spaces. In the general case of curved backgrounds in fact, there will be extra terms which in principle could modify the expression of the Green function

But, since to cancel divergences one imposes $\delta_g = -B\log\frac{\Lambda^2}{M^2}+$finite; and $\delta_{Z_i} = A_i\log\frac{\Lambda^2}{M^2}$, we have that the β function, to lowest order, is just a combination of the coefficients of the divergent logarithms in the counterterms:

$$\beta(g) = -2B - g\sum_i A_i. \tag{5.10}$$

Such arguments can be applied to more complex theories like QED or theories with Yukawa couplings, which is the case that interests us.

The meaning of the β function

Our definition (5.6) of the β function can be rewritten in terms of the bare coupling λ_0 and the cutoff Λ as

$$\beta(\lambda) = M\frac{\partial}{\partial M}\lambda|_{\lambda_0,\Lambda} \tag{5.11}$$

which basically means that it is the rate of change of the renormalized coupling at the scale M needed to maintain a fixed bare coupling. It is possible to show that the *running* coupling constant $\bar{\lambda}(p,\lambda)$, that depends on the momentum p and equals the renormalized coupling at the reference scale M, i.e. $\bar{\lambda}(M,\lambda) = \lambda$, satisfy

$$\frac{d}{d\log(p/M)}\bar{\lambda}(p,\lambda) = \beta(\bar{\lambda}). \tag{5.12}$$

Thus the β function contains the basic information on the behavior of the running coupling as a function of momentum. Of particular importance is the sign of this function, since different signs describe very different theories.

If $\beta > 0$ the running coupling becomes smaller and smaller as $p \to 0$, which means that is possible to use perturbation theory to make predictions about the small momentum behavior of the theory. On the other hand, in the region $p \to \infty$ one can no longer use perturbation theory as a valid tool to study the theory. So Feynman diagrams are only useful to analyze macroscopic behavior.

When $\beta = 0$ the coupling does not flow with momentum and therefore remains equal to the bare coupling at all scales. There are no ultraviolet divergences and for this reason such theories are called finite quantum field theories.

For $\beta < 0$ the running coupling has an opposite behavior respect to the first case. Interactions become stronger at large distances (or $p \to 0$), while are weaker for large momenta (or short distances). These theories are called asymptotically free and the most important example is QCD, where quarks are confined in bound states due to the the strong interaction that grows with distance. In this case Feynman diagrams can only be relied on for computation on the short distance behavior. This is also the case that we want to prove for the B-Yukawa coupling introduced between fermions and the antisymmetric field. So our next task will be to compute the β function for this coupling and check that it has a negative sign.

5.2 Feynman rules

In order to proceed with calculations we first briefly present the Feynman rules of the theory in momentum space, needed for the computation of the various Feynman diagrams of interest. In chapter 4 we already computed what will be our covariant propagator for the antisymmetric field. We will use expression (4.64) in all of the following calculations. Also, to render notation simpler, we will omit the $i\varepsilon$ prescription and take it as understood.

The Gauge Fields Covariant Propagators

The standard procedure to find the expression for the propagator of a field is to look at the quadratic terms in the free Lagrangian and then try to invert it. In the case of gauge fields, however, this procedure encounters a problem in the fact that, due to gauge invariance and the presence of spurious degrees of freedom, this term is not invertible since it has a zero eigenvalue. Also in this case the solution lies in the introduction of gauge fixing terms to make these operators invertible. We will work with covariant propagators for both W_μ and A_μ fields, with their gauge parameters being, respectively, ξ_W and ξ_A.
For the A_μ field we will use an explicitly covariant propagator in analogy with what has been done for the antisymmetric tensor field. The formal expression is

$$ {}_{\mu}\Delta^{++}_{\nu}(k) = -i\hbar\frac{\eta_{\mu\nu}}{k_\mu k^\mu - i\varepsilon} + i\hbar(1-\xi_A)\frac{k_\mu k_\nu}{(k_\mu k^\mu - i\varepsilon)^2}. \tag{5.13} $$

The propagator for the W_μ is almost identical to this latter expression, except for the presence of the identity matrix in $SU(2)$ space:

$$ {}_{\mu,a}\Delta^{++}_{b,\nu}(k) = \left[-i\hbar\frac{\eta_{\mu\nu}}{k_\mu k^\mu - i\varepsilon} + i\hbar(1-\xi_W)\frac{k_\mu k_\nu}{(k_\mu k^\mu - i\varepsilon)^2}\right]\mathbf{I}_{ab} \tag{5.14} $$

The Fermion Propagator

Also for fermions, the propagator can be found by inverting the quadratic part of the free Lagrangian and this is done in any good QFT book. In our case we will look at massless fermions, since it is the scope of our model to give mass to them after the formation of some condensate. The action in momentum space is

$$S[\Psi, \bar{\Psi}] = \int d^4 p \bar{\Psi}_\alpha (i\not{p})_{\alpha\beta} \Psi_\beta(p). \tag{5.15}$$

The propagator is defined as the inverse of this integrand and it can be shown to be :

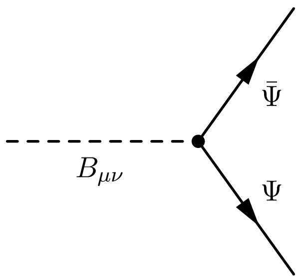
$$i_a S_b(k) = \frac{i p_\mu \gamma^\mu_{ab}}{p_\mu p^\mu - i\epsilon} \tag{5.16}$$

Vertices

To find the interaction vertices of our model we write the explicit interaction part of the Lagrangian:

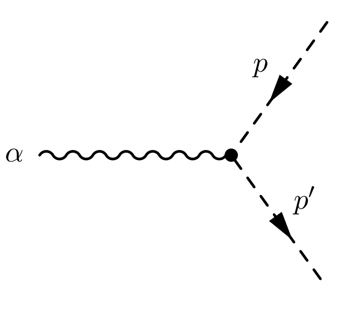
$$\begin{aligned}\mathcal{L}_{\text{int}} = & - yB_{\mu\nu}\bar{\Psi}\sigma^{\mu\nu}\Psi - \frac{1}{12}\Big(-ig\partial_{[\mu}B^\dagger_{\nu\rho]}A^{[\mu}B^{\nu\rho]} + igB^\dagger_{[\nu\rho}A_{\mu]}\partial^{[\mu}B^{\nu\rho]} + \\ & - ig'\partial_{[\mu}B^\dagger_{\nu\rho}W^{b[\mu}\tau^b B^{\nu\rho]} + ig'B^\dagger_{[\nu\rho}\tau^a W^a_{\mu]}\partial^{[\mu}B^{\nu\rho]} + \\ & + gg'B^\dagger_{[\nu\rho}A_{\mu]}W^{b[\mu}\tau^b B^{\nu\rho]} + gg'B^\dagger_{[\nu\rho}\tau^a W^a_{\mu]}A^{[\mu}B^{\nu\rho]} + \\ & + g^2 B^\dagger_{[\nu\rho}A_{\mu]}A^{[\mu}B^{\nu\rho]} + g^2 B^\dagger_{[\nu\rho}\tau^a W^a_{\mu]}W^{b[\mu}\tau^b B^{\nu\rho]}\Big) + \\ & + i\bar{\Psi}(-igA_\mu - \frac{ig'}{2}W^a_\mu\tau_a)\gamma^\mu\Psi. \end{aligned} \tag{5.17}$$

From this we can read off the relevant interaction vertices of the theory. The first term is the B-Yukawa type coupling which give rise to the vertex depicted in the following figure



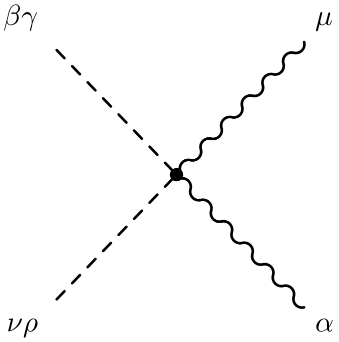
$$= -iy\sigma^{\mu\nu}. \tag{5.18}$$

Then there is the interaction vertex between two $B_{\mu\nu}$ and one A_μ which is



$$= -g\Big[(p+p')^\alpha \eta^{\nu[\beta}\eta^{\gamma]\rho} + 2p^{[\beta}\eta^{\gamma][\nu}\eta^{\rho]\alpha} + 2p'^{[\nu}\eta^{\rho][\beta}\eta^{\gamma]\alpha}\Big]. \tag{5.19}$$

And an analogous expression holds for the interaction between two $B_{\mu\nu}$ and one W_μ. Finally there is also a quartic interaction between two $B_{\mu\nu}$ and two A_μ/W_μ:



$$= -ig^2[\eta^{\mu\alpha}\eta^{\beta[\nu}\eta^{\rho]\gamma} + 2\eta^{\mu[\beta}\eta^{\gamma][\nu}\eta^{\rho]\alpha}]. \tag{5.20}$$

These rules are all we need to compute the Feynman diagrams and their divergences, which will lead us to the final evaluation of the β function. So now, without further ado, let us begin the computation.

5.3 Computation In Momentum Space

In what follows we will explicitly evaluate all Feynman diagrams of interest in the evaluation of the β function. These include three type of corrections to the B-Yukawa vertex: corrections to the external $B_{\mu\nu}$ leg, corrections to the vertex, and corrections to the external fermionic leg. We will perform calculations with a massive fermion propagator but let $m \to 0$ in the end.

5.3.1 Corrections to the $B_{\mu\nu}$ leg

The corrections $B_{\mu\nu}$ propagator at 1 loop are given by the following sum of diagrams:

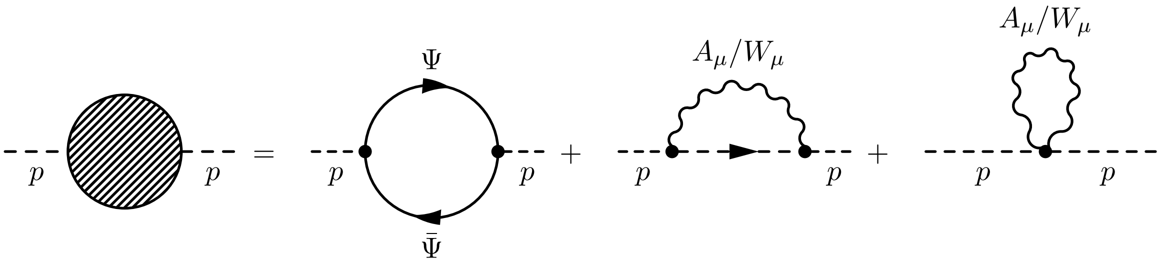


FIGURE 5.1: 1 loop corrections to the $B_{\mu\nu}$ propagator.

We begin the calculations for the beta function by computing the amplitude for the amputated diagram in figure 5.2, contributing to the corrections to the B field propagator coming from the interaction term with the fermions $yB_{\mu\nu}\bar{\Psi}\sigma^{\mu\nu}\Psi$:

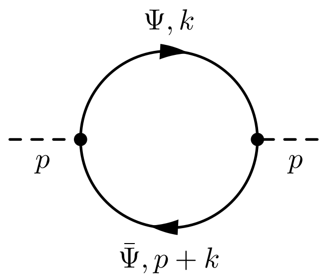


FIGURE 5.2: 1 loop vacuum polarization of $B_{\mu\nu}$ propagator

The amplitude is, in momentum space, following the Feynman rules defined before and recalling that the fermion propagator is

$$iS_{ab}(p) = \frac{i(p_\mu \gamma^\mu_{ab} + m)}{p^2 - m^2},$$

we get for the one-loop vacuum polarization diagram of $B_{\mu\nu}$

$$\begin{aligned}
\Pi^{\mu\nu\alpha\beta}(p) = (-iy)^2 \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\left[\sigma^{\mu\nu}\frac{i(\not{k}+m)}{k^2-m^2}\sigma^{\alpha\beta}\frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2}\right] = \\
= y^2 \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\left[\frac{\sigma^{\mu\nu}(\not{k}+m)\sigma^{\alpha\beta}(\not{p}+\not{k}+m)}{(k^2-m^2)((p+k)^2-m^2)}\right] = \\
= y^2 \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}[\sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{p} + \sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{k} + m^2\sigma^{\mu\nu}\sigma^{\alpha\beta}]}{(k^2-m^2)((p+k)^2-m^2)} \\
= y^2 \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}[\sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{k} - \frac{1}{4}\sigma^{\mu\nu}\not{p}\sigma^{\alpha\beta}\not{p} + m^2\sigma^{\mu\nu}\sigma^{\alpha\beta}]}{((k+p/2)^2-m^2)((k-p/2)^2-m^2)},
\end{aligned} \tag{5.21}$$

where for the third equality we used the fact that the trace of an odd number of γ is 0, while in the last we shifted the integration variable $k \to k - p/2$ and got rid of terms linear in k.

Now we focus on the trace and try to get terms with 4 γ's:

$$\mathrm{Tr}[\sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{k}] = \mathrm{Tr}[\gamma^\mu\gamma^\nu\not{k}\gamma^\alpha\gamma^\beta\not{k} - \gamma^\nu\gamma^\mu\not{k}\gamma^\alpha\gamma^\beta\not{k} - \gamma^\mu\gamma^\nu\not{k}\gamma^\beta\gamma^\alpha\not{k} + \gamma^\nu\gamma^\mu\not{k}\gamma^\beta\gamma^\alpha\not{k}] \tag{5.22}$$

$$\begin{aligned}
\mathrm{Tr}[\gamma^\mu\gamma^\nu\not{k}\gamma^\alpha\gamma^\beta\not{k}] = -\frac{1}{2}\mathrm{Tr}[\gamma^\mu\gamma^\nu\not{k}\gamma^\alpha\not{k}\gamma^\beta] + k^\beta\mathrm{Tr}[\gamma^\mu\gamma^\nu\not{k}\gamma^\alpha] - \frac{1}{2}\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\alpha\not{k}\gamma^\beta\not{k}] + k^\alpha\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\beta\not{k}] = \\
= k^2\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta] - k^\alpha\mathrm{Tr}[\gamma^\mu\gamma^\nu\not{k}\gamma^\beta] + k^\beta\mathrm{Tr}[\gamma^\mu\gamma^\nu\not{k}\gamma^\alpha] + k^\alpha\mathrm{Tr}[\not{k}\gamma^\mu\gamma^\nu\gamma^\beta] - k^\beta\mathrm{Tr}[\not{k}\gamma^\mu\gamma^\nu\gamma^\alpha] = \\
= 4k^2(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}) - 4k^\alpha(k^\beta\eta^{\mu\nu} - k^\mu\eta^{\nu\beta} + k^\nu\eta^{\mu\beta}) + \\
+ 4k^\beta(k^\alpha\eta^{\mu\nu} - k^\mu\eta^{\nu\alpha} + k^\nu\eta^{\mu\alpha}) + 4k^\alpha(k^\mu\eta^{\nu\beta} - k^\nu\eta^{\mu\beta} + k^\beta\eta^{\mu\nu}) + \\
- 4k^\beta(k^\mu\eta^{\nu\alpha} - k^\nu\eta^{\mu\alpha} + k^\alpha\eta^{\mu\nu}) = \\
= 4k^2(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}) + 8(-k^\beta k^\mu\eta^{\nu\alpha} + k^\beta k^\nu\eta^{\mu\alpha} + k^\mu k^\alpha\eta^{\nu\beta} - k^\nu k^\alpha\eta^{\mu\beta}).
\end{aligned}$$

Combining with the other terms of (5.22), which differs only for the change $(\mu \leftrightarrow \nu)$, $(\alpha \leftrightarrow \beta)$ and $(\mu \leftrightarrow \nu, \alpha \leftrightarrow \beta)$ we get:

$$\mathrm{Tr}[\sigma^{\mu\nu} \not{k} \sigma^{\alpha\beta} \not{k}] = 16k^2[\eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\alpha}\eta^{\nu\beta}] + 32[k^\beta k^\nu \eta^{\mu\alpha} - k^\beta k^\mu \eta^{\nu\alpha} + k^\mu k^\alpha \eta^{\nu\beta} - k^\nu k^\alpha \eta^{\mu\beta}]. \tag{5.23}$$

And, in a completely analogous fashion:

$$\mathrm{Tr}[\sigma^{\mu\nu} \not{p} \sigma^{\alpha\beta} \not{p}] = 16p^2[\eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\alpha}\eta^{\nu\beta}] + 32[p^\beta p^\nu \eta^{\mu\alpha} - p^\beta p^\mu \eta^{\nu\alpha} + p^\mu p^\alpha \eta^{\nu\beta} - p^\nu p^\alpha \eta^{\mu\beta}]. \tag{5.24}$$

And finally:

$$\mathrm{Tr}[\sigma^{\mu\nu}\sigma^{\alpha\beta}] = 16(\eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\alpha}\eta^{\nu\beta}). \tag{5.25}$$

Putting these terms altogether we have then:

$$\begin{aligned}\Pi^{\mu\nu\alpha\beta}(p) =& 16y^2 \int \frac{d^dk}{(2\pi)^d} \{[k^2 - p^2/4 + m^2][\eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\alpha}\eta^{\nu\beta}] + 2[k^\beta k^\nu \eta^{\mu\alpha} - k^\beta k^\mu \eta^{\nu\alpha} + \\ & k^\mu k^\alpha \eta^{\nu\beta} - k^\nu k^\alpha \eta^{\mu\beta}] + \frac{1}{2}\left[-p^\beta p^\nu \eta^{\mu\alpha} + p^\beta p^\mu \eta^{\nu\alpha} - p^\mu p^\alpha \eta^{\nu\beta} + p^\nu p^\alpha \eta^{\mu\beta}\right] \\ & \times \frac{1}{[(k+p/2)^2 - m^2][(k-p/2)^2 - m^2]}.\end{aligned} \tag{5.26}$$

We now consider one of the integrals of the type:

$$I_{\mu\nu} = \int d^dk f(k,p) k_\mu k_\nu, \tag{5.27}$$

where $f(k,p)$ is a Lorentz-invariant function. Since the answer must be a second rank Lorentz-invariant tensor it must be expressible in terms of $\eta_{\mu\nu}$ and $p_\mu p_\nu$. This observation implies

$$\int d^dk f(k,p) k_\mu k_\nu = a\eta_{\mu\nu} + b p_\mu p_\nu.$$

We can find a and b by solving the two algebraic equations formed by contracting with $\eta_{\mu\nu}$ and $p_\mu p_\nu$:

$$\begin{aligned}\int d^dk f(k,p) k^2 =& da + bp^2 \\ \int d^dk f(k,p) (k \cdot p)^2 =& ap^2 + bp^4,\end{aligned}$$

implying that

$$\Longrightarrow b = \int d^d k f(k,p) \frac{1}{d-1}\left[-k^2 + \frac{d(k\cdot p)^2}{p^2}\right]\frac{1}{p^2}$$
$$a = \int d^d k f(k,p) \frac{1}{d-1}\left[k^2 - \frac{(k\cdot p)^2}{p^2}\right].$$

So that we can make the replacement:

$$I_{\mu\nu} = \int d^d k f(k,p)\frac{1}{d-1}\left[\left(k^2 - \frac{(k\cdot p)^2}{p^2}\right)\eta_{\mu\nu} - \left(k^2 - d\frac{(k\cdot p)^2}{p^2}\right)\frac{p_\mu p_\nu}{p^2}\right]. \tag{5.28}$$

We can now use this result to collect all terms like $k^\mu k^\nu$ in (5.26):

$$\begin{aligned}
&32y^2 \int \frac{d^d k}{(2\pi)^d}\frac{k^\beta k^\nu \eta^{\mu\alpha} - k^\beta k^\mu \eta^{\nu\alpha} + k^\mu k^\alpha \eta^{\nu\beta} - k^\nu k^\alpha \eta^{\mu\beta}}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]} = \\
=&32y^2 \int \frac{d^d k}{(2\pi)^d}\frac{1}{d-1}\left[\left(k^2 - \frac{(k\cdot p)^2}{p^2}\right)\left(\eta^{\nu\beta}\eta^{\mu\alpha} - \eta^{\mu\beta}\eta^{\nu\alpha} + \eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\nu\alpha}\eta^{\mu\beta}\right)\right] + \\
&+ \frac{1}{d-1}\frac{1}{p^2}\left[\left(k^2 - d\frac{(k\cdot p)^2}{p^2}\right)(-p^\beta p^\nu \eta^{\mu\alpha} + p^\beta p^\mu \eta^{\nu\alpha} - p^\mu p^\alpha \eta^{\nu\beta} + p^\nu p^\alpha \eta^{\mu\beta})\right]\times \\
&\times \frac{1}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]} = \\
=&\frac{32y^2}{d-1}\int \frac{d^d k}{(2\pi)^d}[2\left(k^2 - \frac{(k\cdot p)^2}{p^2}\right)\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\beta}\eta^{\nu\alpha}\right) + \\
&+ \frac{1}{p^2}\left(k^2 - d\frac{(k\cdot p)^2}{p^2}\right)(-p^\beta p^\nu \eta^{\mu\alpha} + p^\beta p^\mu \eta^{\nu\alpha} - p^\mu p^\alpha \eta^{\nu\beta} + p^\nu p^\alpha \eta^{\mu\beta})]\times \\
&\times \frac{1}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}.
\end{aligned} \tag{5.29}$$

Plugging (5.29) into (5.26) we get:

$$\begin{aligned}
\Pi^{\mu\nu\alpha\beta}(p) =&16y^2 \int \frac{d^d k}{(2\pi)^d}\frac{k^2 - p^2/4 + m^2 + \frac{4}{d-1}\left(k^2 - \frac{(k\cdot p)^2}{p^2}\right)}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\beta}\eta^{\nu\alpha}) + \\
&+ 32y^2\frac{1}{p^2}\int \frac{d^d k}{(2\pi)^d}\frac{\frac{1}{d-1}\left(k^2 - d\frac{(k\cdot p)^2}{p^2}\right) - p^2/4}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}(-p^\beta p^\nu \eta^{\mu\alpha} + p^\beta p^\mu \eta^{\nu\alpha} + \\
&- p^\mu p^\alpha \eta^{\nu\beta} + p^\nu p^\alpha \eta^{\mu\beta}).
\end{aligned} \tag{5.30}$$

Now we focus on the first integral. As regards the numerator we have

$$k^2 - p^2/4 + m^2 + \frac{4k^2}{d-1} - \frac{4(k\cdot p)^2}{p^2(d-1)} = \frac{d+3}{d-1}k^2 - p^2/4 + m^2 - \frac{4(k\cdot p)^2}{p^2(d-1)}. \tag{5.31}$$

Looking at an integral of the type

$$\begin{aligned}
&\int \frac{d^dk}{(2\pi)^d} \frac{(k\cdot p)^2}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]} = \\
=&\int \frac{d^dk}{(2\pi)^d} \left[-1+\frac{1}{2}\frac{k^2+p^2/4-m^2}{(k+p/2)^2-m^2}+\frac{1}{2}\frac{k^2+p^2/4-m^2}{(k-p/2)^2-m^2}\right] = \\
=&\int \frac{d^dk}{(2\pi)^d} \left[-1+\frac{1}{2}\frac{(k-p/2)^2+p^2/4-m^2}{k^2-m^2}+\frac{1}{2}\frac{(k+p/2)^2+p^2/4-m^2}{k^2-m^2}\right] = \\
=&\int \frac{d^dk}{(2\pi)^d} \left[-1+\frac{k^2+p^2/2-m^2}{k^2-m^2}\right] = \int \frac{d^dk}{(2\pi)^d}\frac{p^2/2}{k^2-m^2},
\end{aligned} \tag{5.32}$$

where we shifted the integration variable separately for the last two terms in the second line. Whereas, for the other type we have:

$$\begin{aligned}
&\int \frac{d^dk}{(2\pi)^d} \frac{k^2}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]} = \\
=&\int \frac{d^dk}{(2\pi)^d} \frac{k^2+p^2/4\pm(k\cdot p)+m^2-p^2/4-m^2}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]} = \\
=&\int \frac{d^dk}{(2\pi)^d} \frac{(k\pm p/2)^2+m^2-p^2/4-m^2}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]} = \\
=&\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2} - \frac{p^2}{4}\int \frac{d^dk}{(2\pi)^d} \frac{\left(-\frac{4m^2}{p^2}+1\right)}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}.
\end{aligned} \tag{5.33}$$

Using these relations for (5.31) one gets

$$\begin{aligned}
&\frac{d+3}{d-1}\int \frac{d^dk}{(2\pi)^d} \frac{k^2}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]} = \\
=&\frac{d+3}{d-1}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2} - \frac{d+3}{d-1}\frac{p^2}{4}\int \frac{d^dk}{(2\pi)^d} \frac{\left(-\frac{4m^2}{p^2}+1\right)}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]},
\end{aligned} \tag{5.34}$$

and

$$-\frac{4}{p^2(d-1)}\int \frac{d^dk}{(2\pi)^d} \frac{(k\cdot p)^2}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]} = -\frac{2}{d-1}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}.$$

Recollecting all terms in (5.31):

$$\begin{aligned}
&\left(\frac{d+3}{d-1}-\frac{2}{d-1}\right)\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}+\\
&+\left(-p^2/4+m^2-\frac{d+3}{d-1}\left(-m^2+p^2/4\right)\right)\int\frac{d^dk}{(2\pi)^d}\frac{1}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}=\\
&=\frac{d+1}{d-1}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}+\\
&+\left(\frac{-4}{d-1}m^2-\frac{d+1}{d-1}\frac{p^2}{2}\right)\int\frac{d^dk}{(2\pi)^d}\frac{1}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}.
\end{aligned}\tag{5.35}$$

Focusing now on the second term of (5.30) we have that the numerator is

$$\frac{k^2}{d-1}-\frac{d}{d-1}\frac{(k\cdot p)^2}{p^2}-\frac{p^2}{4}.\tag{5.36}$$

Again we can rewrite these integrals as

$$\begin{aligned}
&\frac{1}{d-1}\int\frac{d^dk}{(2\pi)^d}\frac{k^2}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}=\\
&=\frac{1}{d-1}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}-\frac{p^2}{4(d-1)}\int\frac{d^dk}{(2\pi)^d}\frac{\left(\frac{4m^2}{p^2}+1\right)}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]},
\end{aligned}$$

and

$$-\frac{d}{d-1}\frac{1}{p^2}\int\frac{d^dk}{(2\pi)^d}\frac{(k\cdot p)^2}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}=-\frac{d}{2(d-1)}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}.$$

Which adds up to

$$\frac{1}{2}\frac{2-d}{d-1}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}-\frac{d+\frac{4m^2}{p^2}}{d-1}\int\frac{d^dk}{(2\pi)^d}\frac{1}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}.\tag{5.37}$$

Now the integrals

$$\mathrm{I}(d,\alpha)=\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-m^2)^\alpha}\tag{5.38}$$

and

$$\mathrm{I}(p^2,m^2,m^2)=\int\frac{d^dk}{(2\pi)^d}\frac{1}{[(k+p/2)^2-m^2][(k-p/2)^2-m^2]}\tag{5.39}$$

are well known and their result is [7]

$$\mathrm{I}(d,1)=i(4\pi)^{-d/2}\Gamma(1-d/2)(m^2)^{d/2-1}\tag{5.40}$$

$$\mathrm{I}(p^2,m^2,m^2)=i(4\pi)^{-d/2}\Gamma(2-d/2)\int_0^1\frac{dx}{(m^2+p^2x(1-x))^{2-d/2}},\tag{5.41}$$

which for $d \to 4$ take the form

$$\mathrm{I}(p^2, m^2, m^2) = \frac{1}{(4\pi)^2}\Big\{\frac{2}{\epsilon} - \gamma_E - \log[m^2 - x(1-x)p^2] + \log(4\pi) + O(\epsilon)\Big\}, \qquad (5.42)$$

where $\epsilon = 4 - d$. So the total contribution of the vacuum polarization diagram near $d = 4$ is finally given by

$$\Pi^{\mu\nu\alpha\beta}(p^2) = \frac{2iy^2\hbar^2}{\pi^2}\Bigg\{\Bigg[\frac{5}{3}\Gamma(1-d/2)m^2 - \frac{8m^2/p^2+5}{6}\Gamma(2-d/2)\int_0^1 \frac{dx}{(m^2+p^2x(1-x))^{2-d/2}}\Bigg]\eta^{\mu[\alpha}\eta^{\beta]\nu}$$
$$+\frac{4}{p^2}\Bigg[-\frac{1}{3}\Gamma(1-d/2)m^2 - \frac{4+4m^2/p^2}{3}\Gamma(2-d/2)\int_0^1 \frac{dx}{(m^2+p^2x(1-x))^{2-d/2}}\Bigg]p^{[\mu}\eta^{\nu][\alpha}p^{\beta]}\Bigg\} =$$
$$=\frac{2iy^2\hbar^2}{\pi^2}\Bigg\{\Bigg[\frac{5}{3}\Gamma(1-d/2)m^2 - \frac{8m^2/p^2+5}{6}\Big(\frac{2}{\epsilon} - \gamma_E - \log[m^2 - x(1-x)p^2] + \log(4\pi) + O(\epsilon)\Big)\Bigg]\eta^{\mu[\alpha}\eta^{\beta]\nu}+$$
$$+\frac{4}{p^2}\Bigg[-\frac{1}{3}\Gamma(1-d/2)m^2 - \frac{4+4m^2/p^2}{3}\Big(\frac{2}{\epsilon} - \gamma_E - \log[m^2 - x(1-x)p^2] + \log(4\pi) + O(\epsilon)\Big)\Bigg]p^{[\mu}\eta^{\nu][\alpha}p^{\beta]}\Bigg\}. \qquad (5.43)$$

In the case of massless fermions this expression reduces to

$$\Pi^{\mu\nu\alpha\beta}(p^2) = -\frac{2iy^2\hbar^2}{\pi^2}\Bigg\{\Bigg[\frac{5}{6}\Big(\frac{2}{\epsilon} - \gamma_E - \log[x(x-1)p^2] + \log(4\pi) + O(\epsilon)\Big)\Bigg]\eta^{\mu[\alpha}\eta^{\beta]\nu}+$$
$$+\frac{4}{p^2}\Bigg[\frac{4}{3}\Big(\frac{2}{\epsilon} - \gamma_E - \log[x(x-1)p^2] + \log(4\pi) + O(\epsilon)\Big)\Bigg]p^{[\mu}\eta^{\nu][\alpha}p^{\beta]}\Bigg\}. \qquad (5.44)$$

Alternatively, for massless fermions,we could have done the following,:

$$y^2\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2(p+k)^2}\mathrm{Tr}[\sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}(\not{p}+\not{k})] = y^2\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2(p+k)^2}\mathrm{Tr}[\sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{k} + \sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{p}]. \qquad (5.45)$$

Then introduce Feynman parameters as follows

$$\frac{1}{k^2(p+k)^2} = \int_0^1 dx\,dy\,\frac{1}{D^2}\delta(x+y-1), \qquad (5.46)$$

where $D = l^2 - \Delta$, with $l = k + xp$ and $\Delta = x(x-1)p^2$. After this we can change the integration variable $k \to l$ and substitute $k = l - xp$ in the trace to get

$$\mathrm{Tr}[\sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{k} + \sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{p}] = \mathrm{Tr}[\sigma^{\mu\nu}\not{l}\sigma^{\alpha\beta}\not{l} + x(x-1)\sigma^{\mu\nu}\not{p}\sigma^{\alpha\beta}\not{p}], \qquad (5.47)$$

where we dropped out linear terms in l.Then we can rewrite traces in order to reduce the number of γ matrices in them. For instance

$$\begin{aligned}\mathrm{Tr}[\sigma^{\mu\nu}\not{p}\sigma^{\alpha\beta}\not{p}] =&4\mathrm{Tr}[\gamma^{[\mu}\gamma^{\nu]}\not{p}\gamma^{[\alpha}\gamma^{\beta]}\not{p}] = -4\mathrm{Tr}[\gamma^{[\mu}\gamma^{\nu]}\not{p}\gamma^{[\alpha}\not{p}\gamma^{\beta]}] + 8\mathrm{Tr}[\gamma^{[\mu}\gamma^{\nu]}\not{p}\gamma^{[\alpha}p^{\beta]}] = \\ =&4p^2\mathrm{Tr}[\gamma^{[\mu}\gamma^{\nu]}\gamma^{[\alpha}\gamma^{\beta]}] - 8\mathrm{Tr}[\gamma^{[\mu}\gamma^{\nu]}p^{[\alpha}\gamma^{\beta]}] + 8\mathrm{Tr}[\gamma^{[\mu}\gamma^{\nu]}\gamma^{[\alpha}p^{\beta]}] = \\ =&32p^2\eta^{\mu[\beta}\eta^{\alpha]\nu} + 128p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}, \end{aligned} \tag{5.48}$$

so that our total trace will yield

$$\mathrm{Tr}[\sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{k}+\sigma^{\mu\nu}\not{k}\sigma^{\alpha\beta}\not{p}] = 32[\eta^{\mu[\beta}\eta^{\alpha]\nu}l^2+4l^{[\mu}\eta^{\nu][\beta}l^{\alpha]}+x(x-1)\eta^{\mu[\beta}\eta^{\alpha]\nu}p^2+4x(x-1)p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}]. \tag{5.49}$$

Then, substituting $l^\mu l^\nu \to l^2/d\,\eta^{\mu\nu}$, we get the final result

$$32\eta^{\mu[\beta}\eta^{\alpha]\nu}[l^2 + x(x-1)p^2 - 4\frac{l^2}{d}] + 128x(x-1)p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}. \tag{5.50}$$

Notice that in $d = 4$ limit there is only a logarithmic divergence. The final expression for the divergent part of the amplitude of this bubble diagram is then

$$\begin{aligned}32p^2\int_0^1 dx\, x(x-1)\int\frac{d^dl}{(2\pi)^d}\frac{1}{(l^2-\Delta)^2}\eta^{\mu[\beta}\eta^{\alpha]\nu} =& \\ 32ip^2\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}\int_0^1 dx\, x(x-1)\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}\eta^{\mu[\beta}\eta^{\alpha]\nu}&\end{aligned} \tag{5.51}$$

and

$$\begin{aligned}128\int_0^1 dx\, x(x-1)\int\frac{d^dl}{(2\pi)^d}\frac{1}{(l^2-\Delta)^2}p^{[\mu}\eta^{\nu][\beta}p^{\alpha]} =& \\ 128i\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}\int_0^1 dx\, x(x-1)\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}.&\end{aligned} \tag{5.52}$$

So the total divergent part of this diagram is

$$\Pi^{\mu\nu\alpha\beta}(p^2) = 32ip^2\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}\int_0^1 dx\, x(x-1)\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}\Big[p^2\eta^{\mu[\beta}\eta^{\alpha]\nu} + 4p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}\Big]. \tag{5.53}$$

Self Energy

Interaction with A_μ

Next thing we want to compute is a second order correction to the antisymmetric field propagator in figure. The expression for the self-energy diagram is:

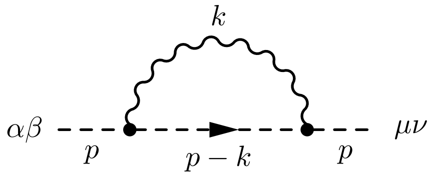


FIGURE 5.3: Self energy diagram for $B_{\mu\nu}$ propagator.

$$\Sigma^{\alpha\beta\mu\nu}(p) = -g^2\hbar^2 \int \frac{d^dk}{(2\pi)^d}\left[-\eta_{\gamma[\delta}\eta_{\lambda]\rho}\frac{1}{(p-k)^2} + 2(1-\xi_B)\frac{(p-k)_{[\gamma}\eta_{\delta][\lambda}(p-k)_{\rho]}}{[(p-k)^2]^2}\right]\times$$
$$\left[-\frac{\eta_{\tau\epsilon}}{k^2} + (1-\xi_A)\frac{k_\tau k_\epsilon}{(k^2)^2}\right]\times\left[(2p-k)^\tau\eta^{\alpha[\gamma}\eta^{\delta]\beta} + 2p^{[\gamma}\eta^{\delta][\alpha}\eta^{\beta]\tau} + 2(p-k)^{[\alpha}\eta^{\beta][\gamma}\eta^{\delta]\tau}\right]\times$$
$$\times\left[(2p-k)^\epsilon\eta^{\lambda[\mu}\eta^{\nu]\rho} + 2(p-k)^{[\mu}\eta^{\nu][\lambda}\eta^{\rho]\epsilon} + 2p^{[\lambda}\eta^{\rho][\mu}\eta^{\nu]\epsilon}\right] =$$
$$= -g^2\hbar^2\int\frac{d^dk}{(2\pi)^d}\left[-\eta_{\gamma[\delta}\eta_{\lambda]\rho}\frac{1}{(p-k)^2} + 2(1-\xi_B)\frac{(p-k)_{[\gamma}\eta_{\delta][\lambda}(p-k)_{\rho]}}{[(p-k)^2]^2}\right]\times$$
$$\times\left[-\frac{1}{k^2}\Big((2p-k)_\epsilon\eta^{\alpha[\gamma}\eta^{\delta]\beta} + 2p^{[\gamma}\eta^{\delta][\alpha}\eta^{\beta]}_\epsilon + 2(p-k)^{[\alpha}\eta^{\beta][\gamma}\eta^{\delta]}_\epsilon\Big)+\right.$$
$$\left.+\frac{1-\xi_A}{(k^2)^2}\Big(k\cdot(2p-k)k_\epsilon\eta^{\alpha[\gamma}\eta^{\delta]\beta} + 2p^{[\gamma}\eta^{\delta][\alpha}k^{\beta]}k_\epsilon + 2(p-k)^{[\alpha}\eta^{\beta][\gamma}k^{\delta]}k_\epsilon\Big)\right]\times$$
$$\times\left[(2p-k)^\epsilon\eta^{\lambda[\mu}\eta^{\nu]\rho} + 2(p-k)^{[\mu}\eta^{\nu][\lambda}\eta^{\rho]\epsilon} + 2p^{[\lambda}\eta^{\rho][\mu}\eta^{\nu]\epsilon}\right] =$$
$$= -g^2\hbar^2\int\frac{d^dk}{(2\pi)^d}\left[-\eta_{\gamma[\delta}\eta_{\lambda]\rho}\frac{1}{(p-k)^2} + 2(1-\xi_B)\frac{(p-k)_{[\gamma}\eta_{\delta][\lambda}(p-k)_{\rho]}}{[(p-k)^2]^2}\right]\times$$
$$\times\left\{-\frac{1}{k^2}\Big[(2p-k)^2\eta^{\alpha[\gamma}\eta^{\delta]\beta}\eta^{\lambda[\mu}\eta^{\nu]\rho} + 2(p-k)^{[\mu}\eta^{\nu][\lambda}(2p-k)^{\rho]}\eta^{\alpha[\gamma}\eta^{\delta]\beta}+\right.$$
$$2p^{[\lambda}\eta^{\rho][\mu}(2p-k)^{\nu]}\eta^{\alpha[\gamma}\eta^{\delta]\beta} + +2p^{[\gamma}\eta^{\delta][\alpha}(2p-k)^{\beta]}\eta^{\lambda[\mu}\eta^{\nu]\rho} + 4p^{[\gamma}\eta^{\delta][\alpha}\eta^{\beta][\rho}\eta^{\lambda][\nu}(p-k)^{\mu]}+$$
$$+4p^{[\gamma}\eta^{\delta][\alpha}\eta^{\beta][\nu}\eta^{\mu][\rho}p^{\lambda]} + 2(p-k)^{[\alpha}\eta^{\beta][\gamma}(2p-k)^{\delta]}\eta^{\lambda[\mu}\eta^{\nu]\rho}+$$
$$+4(p-k)^{[\alpha}\eta^{\beta][\gamma}\eta^{\delta][\rho}\eta^{\lambda][\nu}(p-k)^{\mu]} + 4(p-k)^{[\alpha}\eta^{\beta][\gamma}\eta^{\delta][\nu}\eta^{\mu][\rho}p^{\lambda]}\Big]+$$
$$+\frac{1-\xi_A}{(k^2)^2}\Big\{\Big[k\cdot(2p-k)\eta^{\alpha[\gamma}\eta^{\delta]\beta} + 2p^{[\gamma}\eta^{\delta][\alpha}k^{\beta]} + 2(p-k)^{[\alpha}\eta^{\beta][\gamma}k^{\delta]}\Big]\times$$
$$\left.\times\Big[k\cdot(2p-k)\eta^{\lambda[\mu}\eta^{\nu]\rho} + 2(p-k)^{[\mu}\eta^{\nu][\lambda}k^{\rho]} + 2p^{[\lambda}\eta^{\rho][\mu}k^{\nu]}\Big]\Big\}\right\} =$$
$$= -g^2\hbar^2\int\frac{d^dk}{(2\pi)^d}\left\{\frac{1}{k^2(p-k)^2}\Big[\frac{(2p-k)^2}{2}\eta^{\alpha[\nu}\eta^{\mu]\beta} + (p-k)^{[\mu}\eta^{\nu][\beta}(2p-k)^{\alpha]}\right.$$
$$+\,p^{[\beta}\eta^{\alpha][\mu}(2p-k)^{\nu]} + +p^{[\nu}\eta^{\mu][\alpha}(2p-k)^{\beta]} + (p-k)^{[\mu}\eta^{\nu][\alpha}p^{\beta]} + p^{[\alpha}\eta^{\beta][\nu}p^{\mu]}$$
$$-p^2\eta^{\mu[\alpha}\eta^{\beta]\nu} - (p-k)^{[\alpha}\eta^{\beta][\mu}(2p-k)^{\nu]} + +(d-1)(p-k)^{[\alpha}\eta^{\beta][\nu}(p-k)^{\mu]} + (p-k)^{[\alpha}\eta^{\beta][\mu}p^{\nu]}\Big]+$$
$$\left.+\frac{2(1-\xi_B)}{k^2(p-k)^4}\Big[A\Big] + \frac{1-\xi_A}{k^4(p-k)^2}\Big[B\Big] + \frac{2(1-\xi_B)(1-\xi_A)}{k^4(p-k)^4}\Big[C\Big].\right.$$

As regards the term proportional to $\frac{1}{k^2(p-k)^2}$ we can, as for the case of the vacuum polarization, symmetrize the denominator by shifting the integration variable as $k \to$

$k + p/2$. After rewriting the numerator, we can then get rid of terms linear in k:

$$\Big[\frac{1}{2}\big(\frac{3}{2}p - k\big)^2 + p^2\Big]\eta^{\alpha[\nu}\eta^{\mu]\beta} + \big(\frac{p}{2} - k\big)^{[\mu}\eta^{\nu][\beta}\big(\frac{3}{2}p - k\big)^{\alpha]} + p^{[\beta}\eta^{\alpha][\mu}\big(\frac{3}{2}p - k\big)^{\nu]}$$
$$+p^{[\nu}\eta^{\mu][\alpha}\big(\frac{3}{2}p - k\big)^{\beta]} + \big(\frac{p}{2} - k\big)^{[\mu}\eta^{\nu][\alpha}p^{\beta]} + p^{[\alpha}\eta^{\beta][\nu}p^{\mu]} + \cdots$$

At the end of this procedure we end up with

$$\Big[\frac{17}{8}p^2 + \frac{1}{2}k^2\Big]\eta^{\alpha[\nu}\eta^{\mu]\beta} + \frac{d+17}{4}p^{[\mu}\eta^{\nu][\beta}p^{\alpha]} + (1+d)k^{[\mu}\eta^{\nu][\beta}k^{\alpha]}. \tag{5.54}$$

Now we can replace all terms quadratic in k as in (5.28) and get

$$\int \frac{d^dk}{(2\pi)^d}\frac{k^{[\mu}\eta^{\nu][\beta}k^{\alpha]}}{(k+p/2)^2(k-p/2^2)} =$$
$$= \frac{1}{d-1}\int \frac{d^dk}{(2\pi)^d}\Big[\big(k^2 - \frac{(k\cdot p)^2}{p^2}\big)\eta^{\mu[\alpha}\eta^{\beta]\nu} + \frac{1}{p^2}\big(k^2 - d\frac{(k\cdot p)^2}{p^2}\big)p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}\Big]$$
$$\times \frac{1}{(k+p/2)^2(k-p/2^2)}.$$

So the total expression under the integral will now become

$$\Big[\frac{17}{8}p^2 + \frac{1}{2}k^2 - \frac{d+1}{(d-1)}k^2 + \frac{d+1}{(d-1)}\frac{(k\cdot p)^2}{p^2}\Big]\eta^{\alpha[\nu}\eta^{\mu]\beta} +$$
$$+\frac{1}{p^2}\Big[\frac{d+17}{4}p^2 + \frac{d+1}{(d-1)}k^2 - \frac{d+1}{(d-1)}\frac{(k\cdot p)^2}{p^2}\Big]p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}.$$

At this point we can express the terms proportional to $(k \cdot p)^2$ and k^2 in terms of p^2 as we did in the first section. This time however we will have in the denominator only k^2. To avoid that we introduce a parameter ζ which we will approach to zero at the end of the calculation. We can see that the integral

$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2 - \zeta^2} = \frac{\Gamma(1-d/2)}{(4\pi)^{d/2}}\Big(\frac{1}{\zeta^2}\Big)^{1-d/2} \tag{5.55}$$

vanishes for $d > 2$ in the limit $\zeta \to 0$. Since we are interested in the UV divergent part of this diagram near $d = 4$, we can disregard contributions coming from this type of integral. That said, we can do the algebra, remembering (5.32) and (5.33) and get

$$\Sigma^{\alpha\beta\mu\nu}(p^2) = -g^2\hbar^2\Bigg\{\Bigg[\frac{19d-11}{8(d-1)}\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k+p/2)^2(k-p/2)^2}\Bigg]\eta^{\mu[\alpha}\eta^{\beta]\nu} +$$
$$+\frac{1}{p^2}\Bigg[\frac{(d-1)(d+17)-d+1}{4(d-1)}\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k+p/2)^2(k-p/2)^2}\Bigg]p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}\Bigg\},$$

which near $d = 4$ is just

$$\Sigma^{\alpha\beta\mu\nu}(p^2) = -i\frac{g^2\hbar^2}{(4\pi)^2}\Bigg\{\frac{65}{24}\Big[\Gamma(2-d/2)\int_0^1 \frac{dx}{(p^2x(1-x))^{2-d/2}}\Big]\eta^{\mu[\alpha}\eta^{\beta]\nu}+$$
$$+\frac{1}{p^2}5\Big[\Gamma(2-d/2)\int_0^1 \frac{dx}{(p^2x(1-x))^{2-d/2}}\Big]p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}\Bigg\} =$$
$$= -i\frac{g^2\hbar^2}{(4\pi)^2}\Bigg\{\frac{65}{24}\Big[\frac{2}{\epsilon} - \gamma_E - \log[x(x-1)p^2] + \log(4\pi) + O(\epsilon)\Big]\eta^{\mu[\alpha}\eta^{\beta]\nu}+$$
$$+\frac{1}{p^2}5\Big[\frac{2}{\epsilon} - \gamma_E - \log[x(x-1)p^2] + \log(4\pi) + O(\epsilon)\Big]p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}\Bigg\}. \tag{5.56}$$

Alternatively we could have also done the following. Group the terms as

$$(d+6)p^{[\mu}\eta^{\nu][\beta}p^{\alpha]} - (d+2)[p^{[\mu}\eta^{\nu][\beta}k^{\alpha]} + k^{[\mu}\eta^{\nu][\beta}p^{\alpha]}] + (d+1)k^{[\mu}\eta^{\nu][\beta}k^{\alpha]}$$
$$+(3p^2 + \frac{k^2}{2} - 2p\cdot k)\eta^{\alpha[\nu}\eta^{\mu]\beta}$$

and then introduce Feynman parameters and shift $k = l + xp$ to get

$$p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}[d+6-2x(d+2)+x^2] + \eta^{\alpha[\nu}\eta^{\mu]\beta}[p^2(3-2x+\frac{x^2}{2}) - \frac{d+2}{d}l^2],$$

which gives the same result for $p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}$ part, while a different answer for the $\eta^{\alpha[\nu}\eta^{\mu]\beta}$ term ($\frac{13}{6}$ instead of $\frac{65}{24}$) As regards the gauge-dependent part of the expression we have, for the term proportional to $\frac{2(1-\xi_B)}{k^2(p-k)^4}$

$$\begin{aligned}
&\frac{2(1-\xi_B)}{k^2(p-k)^4}\Big[(2p-k)^2(p-k)^{[\alpha}\eta^{\beta][\mu}(p-k)^{\nu]} + (p-k)\cdot(2p-k)(p-k)^{[\mu}\eta^{\nu][\beta}(p-k)^{\alpha]} \\
&\quad +p^{[\mu}k^{\nu]}p^{[\alpha}k^{\beta]} - p\cdot(p-k)(p-k)^{[\alpha}\eta^{\beta][\mu}(2p-k)^{\nu]} \\
&\quad +p\cdot(p-k)(p-k)^{[\nu}\eta^{\mu][\alpha}(2p-k)^{\beta]} - p^{[\nu}k^{\mu]}p^{[\alpha}k^{\beta]} \\
&\quad +p\cdot(p-k)(p-k)^{[\beta}\eta^{\alpha][\nu}(p-k)^{\mu]}+ \\
&\quad +p\cdot(p-k)\big[(p-k)^{[\mu}\eta^{\nu][\beta}p^{\alpha]} + (p-k)^{[\alpha}\eta^{\beta][\nu}p^{\mu]} - p\cdot(p-k)\eta^{\mu[\alpha}\eta^{\beta]\nu}\big] \\
&\quad\quad - p^2(p-k)^{[\alpha}\eta^{\beta][\nu}(p-k)^{\mu]} + -(2p-k)\cdot(p-k)(p-k)^{[\alpha}\eta^{\beta][\mu}(p-k)^{\nu]}+ \\
&\quad\quad (p-k)^2(p-k)^{[\alpha}\eta^{\beta][\mu}(p-k)^{\nu]} + +p\cdot(p-k)(p-k)^{[\alpha}\eta^{\beta][\mu}(p-k)^{\nu]}\Big] = \\
&= \frac{2(1-\xi_B)}{k^2(p-k)^4}\Big\{(p-k)^{[\alpha}\eta^{\beta][\mu}(p-k)^{\nu]}\Big(3p^2\Big) + 2p^{[\mu}k^{\nu]}p^{[\alpha}k^{\beta]} \\
&\quad +p\cdot(p-k)\Big[(p-k)^{[\nu}\eta^{\mu][\alpha}(2p-k)^{\beta]} - (p-k)^{[\alpha}\eta^{\beta][\mu}(2p-k)^{\nu]} \\
&\quad +(p-k)^{[\mu}\eta^{\nu][\beta}p^{\alpha]} + (p-k)^{[\alpha}\eta^{\beta][\nu}p^{\mu]} - p\cdot(p-k)\eta^{\mu[\alpha}\eta^{\beta]\nu}\Big]\Big\} = \\
&= \frac{2(1-\xi_B)}{k^2(p-k)^4}\Big\{p^{[\alpha}\eta^{\beta][\mu}p^{\nu]}[-3p^2+6(p\cdot k)] + k^{[\alpha}\eta^{\beta][\mu}p^{\nu]}[p^2-4(p\cdot k)] \\
&\quad +p^{[\alpha}\eta^{\beta][\mu}k^{\nu]}[p^2-4(p\cdot k)] + k^{[\alpha}\eta^{\beta][\mu}k^{\nu]}[p^2+2(p\cdot k)] - [p^2-(p\cdot k)]^2\eta^{\mu[\alpha}\eta^{\beta]\nu} \\
&\quad +2p^{[\mu}k^{\nu]}p^{[\alpha}k^{\beta]}\Big\}.
\end{aligned}$$

For this term we will adopt a slightly different strategy as we did before and we will use the Feynman trick writing

$$\frac{1}{k^2(p-k)^2(p-k)^2} = \int_0^1 dx\,dy\,dz\;\delta(x+y+z-1)\frac{2}{D^3}, \tag{5.57}$$

where the denominator is just

$$D = l^2 + x(1-x)p^2 = l^2 - \Delta; \qquad l = k-(1-x)p.$$

Now, since the integral now only depends on l^2 we can drop all linear terms in l and make the substitution $l^\mu l^\nu \to \frac{1}{d}\eta^{\mu\nu}l^2$. Rearranging the numerator in terms of l we get, after some tedious algebra,

$$p^2[(4-x)\frac{l^2}{d} + x^2p^2]\eta^{\mu[\beta}\eta^{\alpha]\nu} + \Big[(x^3-2x^2-x-1)p^2 - \frac{2l^2}{d}(2x+3)\Big]p^{[\alpha}\eta^{\beta][\mu}p^{\nu]}. \tag{5.58}$$

At this point we can make use of the formulas, valid in Minkowski space,

$$\int\frac{d^dl}{(2\pi)^d}\frac{1}{(l^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}}\frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)}\Big(\frac{1}{\Delta}\Big)^{n-\frac{d}{2}} \tag{5.59}$$

$$\int \frac{d^dl}{(2\pi)^d}\frac{l^2}{(l^2-\Delta)^n} = \frac{(-1)^{n-1}i}{(4\pi)^{d/2}}\frac{d}{2}\frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)}\Big(\frac{1}{\Delta}\Big)^{n-\frac{d}{2}-1} \tag{5.60}$$

$$\int \frac{d^dl}{(2\pi)^d}\frac{(l^2)^2}{(l^2-\Delta)^n} = \frac{(-1)^{n}i}{(4\pi)^{d/2}}\Big(\frac{d}{2}\Big)\Big(\frac{d}{2}+1\Big)\frac{\Gamma(n-\frac{d}{2}-2)}{\Gamma(n)}\Big(\frac{1}{\Delta}\Big)^{n-\frac{d}{2}-2}. \tag{5.61}$$

We can already see from the first formula that, since in our case $n = 3$, terms with no powers of l^2 will give a finite contribution when $d \to 4$. But we do not need these terms when calculating the β function. The part proportional to $\eta^{\mu[\alpha}\eta^{\beta]\nu}$ is

$$\frac{2p^2}{d}\int_0^1 dx\,dy\,(4-x)\int\frac{d^dl}{(2\pi)^d}\frac{l^2}{(l^2-\Delta)^3} = i\frac{p^2}{4}\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}}\int_0^1 dx\,dy\,(4-x)\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}.$$

As regards the part proportional to $p^{[\alpha}\eta^{\beta][\mu}p^{\nu]}$ we get

$$-\frac{4}{d}\int_0^1 dx\,dy\,(2x+3)\int\frac{d^dl}{(2\pi)^d}\frac{l^2}{(l^2-\Delta)^3} = -i\frac{1}{2}\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}}\int_0^1 dx\,dy\,(2x+3)\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}.$$

Then we have a third piece proportional to $\frac{1-\xi_A}{k^4(p-k)^2}$ which is:

$$\begin{aligned}
&\frac{1-\xi_A}{k^4(p-k)^2}\Big[[k\cdot(2p-k)]^2\eta^{\alpha[\gamma}\eta^{\delta]\beta}\eta^{\lambda[\mu}\eta^{\nu]\rho}+2k\cdot(2p-k)\eta^{\alpha[\gamma}\eta^{\delta]\beta}(p-k)^{[\mu}\eta^{\nu][\lambda}k^{\rho]}+\\
&\quad+2k\cdot(2p-k)\eta^{\alpha[\gamma}\eta^{\delta]\beta}p^{[\lambda}\eta^{\rho][\mu}k^{\nu]}+2k\cdot(2p-k)\eta^{\lambda[\mu}\eta^{\nu]\rho}p^{[\gamma}\eta^{\delta][\alpha}k^{\beta]}+\\
&\quad+4p^{[\gamma}\eta^{\delta][\alpha}k^{\beta]}(p-k)^{[\mu}\eta^{\nu][\lambda}k^{\rho]}+4p^{[\gamma}\eta^{\delta][\alpha}k^{\beta]}p^{[\lambda}\eta^{\rho][\mu}k^{\nu]}\\
&\quad+2k\cdot(2p-k)\eta^{\lambda[\mu}\eta^{\nu]\rho}(p-k)^{[\alpha}\eta^{\beta][\gamma}k^{\delta]}+\\
&\quad+4(p-k)^{[\mu}\eta^{\nu][\lambda}k^{\rho]}(p-k)^{[\alpha}\eta^{\beta][\gamma}k^{\delta]}+4p^{[\lambda}\eta^{\rho][\mu}k^{\nu]}(p-k)^{[\alpha}\eta^{\beta][\gamma}k^{\delta]}\Big]\\
&\quad\times\Big[-\eta_{\gamma[\delta}\eta_{\lambda]\rho}\Big]=\\
&=\frac{1-\xi_A}{k^4(p-k)^2}\Big[\frac{[k\cdot(2p-k)]^2}{2}\eta^{\alpha[\mu}\eta^{\nu]\beta}+k\cdot(2p-k)(p-k)^{[\mu}\eta^{\nu][\alpha}k^{\beta]}\\
&\quad+k\cdot(2p-k)p^{[\alpha}\eta^{\beta][\mu}k^{\nu]}+k\cdot(2p-k)p^{[\mu}\eta^{\nu][\alpha}k^{\beta]}-(p\cdot k)(p-k)^{[\mu}\eta^{\nu][\alpha}k^{\beta]}\\
&\quad+p^2k^{[\nu}\eta^{\mu][\alpha}k^{\beta]}-p^{[\alpha}k^{\beta]}p^{[\mu}k^{\nu]}+k\cdot(2p-k)(p-k)^{[\alpha}\eta^{\beta][\mu}k^{\nu]}\\
&\quad+k^2(p-k)^{[\mu}\eta^{\nu][\beta}(p-k)^{\alpha]}-(p-k)^{[\mu}k^{\nu]}(p-k)^{[\alpha}k^{\beta]}+\\
&\quad-(p\cdot k)(p-k)^{[\alpha}\eta^{\beta][\mu}k^{\nu]}\Big]=\\
&=\frac{1-\xi_A}{k^4(p-k)^2}\Big[-k^2p^{[\mu}\eta^{\nu][\alpha}p^{\beta]}+(p^{[\mu}\eta^{\nu][\alpha}k^{\beta]}+k^{[\mu}\eta^{\nu][\alpha}p^{\beta]})(3(p\cdot k)-k^2)\\
&\quad+k^{[\mu}\eta^{\nu][\alpha}k^{\beta]}(k^2-p^2-2p\cdot k)+\\
&\quad+\frac{4(p\cdot k)^2+k^4-4k^2(p\cdot k)}{2}\eta^{\alpha[\mu}\eta^{\nu]\beta}-2p^{[\alpha}k^{\beta]}p^{[\mu}k^{\nu]}\Big].
\end{aligned}$$

Now again we rewrite the integral using Feynman trick as in (5.57). This time $D =$

$l^2 + x(1-x)p^2$ with $l = k - xp$. After rewriting all the numerator in terms of l we end up with

$$\eta^{\alpha[\mu}\eta^{\nu]\beta}\Big[\frac{d-2}{2d}l^4 + p^2 l^2\Big(\frac{(6d-2)x^2 + (4-4d)x + 6}{2d}\Big) + x^2p^4(4+x^2)\Big] +$$
$$p^{[\mu}\eta^{\nu][\alpha}p^{\beta]}\Big[l^2\Big(\frac{-(4+d)x^2 + (8-2d)x + 2 + 5d}{d}\Big) + x^2p^2(5-x^2)\Big]. \quad (5.62)$$

So the result for the part proportional to $\eta^{\alpha[\mu}\eta^{\nu]\beta}$ will be

$$\frac{2-d}{d}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\int\frac{d^dl}{(2\pi)^d}\frac{l^4}{(l^2-\Delta)^3} = \quad (5.63)$$
$$= i\frac{(d+2)(d-2)}{8}\frac{\Gamma(1-\frac{d}{2})}{(4\pi)^2}\int_0^1 dx\,dy\Big(\frac{1}{\Delta}\Big)^{1-\frac{d}{2}} \quad (5.64)$$

and the second contribution

$$\frac{p^2}{d}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)[(6d-2)x^2 + (4-4d)x + 6]\int\frac{d^dl}{(2\pi)^d}\frac{l^2}{(l^2-\Delta)^3} =$$
$$= i\frac{p^2}{4}\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}\int_0^1 dx\,dy\,[(6d-2)x^2 + (4-4d)x + 6]\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}. \quad (5.65)$$

For the part proportional to $p^{[\mu}\eta^{\nu][\alpha}p^{\beta]}$ we get:

$$\frac{2}{d}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)[-(4+d)x^2 + (8-2d)x + 2 + 5d]\int\frac{d^dl}{(2\pi)^d}\frac{l^2}{(l^2-\Delta)^3} =$$
$$= i\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}\int_0^1 dx\,dy\,[-(4+d)x^2 + (8-2d)x + 2 + 5d]\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}. \quad (5.66)$$

Finally, we have the last part proportional to $\frac{2(1-\xi_A)(1-\xi_B)}{k^4(p-k)^4}$:

$$
\begin{aligned}
&\frac{2(1-\xi_A)(1-\xi_B)}{k^4(p-k)^4}\Big[[k\cdot(2p-k)]^2\eta^{\alpha[\gamma}\eta^{\delta]\beta}\eta^{\lambda[\mu}\eta^{\nu]\rho}+2k\cdot(2p-k)\eta^{\alpha[\gamma}\eta^{\delta]\beta}(p-k)^{[\mu}\eta^{\nu][\lambda}k^{\rho]}+\\
&\qquad+2k\cdot(2p-k)\eta^{\alpha[\gamma}\eta^{\delta]\beta}p^{[\lambda}\eta^{\rho][\mu}k^{\nu]}+2k\cdot(2p-k)\eta^{\lambda[\mu}\eta^{\nu]\rho}p^{[\gamma}\eta^{\delta][\alpha}k^{\beta]}+\\
&\qquad+4p^{[\gamma}\eta^{\delta][\alpha}k^{\beta]}(p-k)^{[\mu}\eta^{\nu][\lambda}k^{\rho]}+4p^{[\gamma}\eta^{\delta][\alpha}k^{\beta]}p^{[\lambda}\eta^{\rho][\mu}k^{\nu]}+\\
&\qquad+2k\cdot(2p-k)\eta^{\lambda[\mu}\eta^{\nu]\rho}(p-k)^{[\alpha}\eta^{\beta][\gamma}k^{\delta]}+4(p-k)^{[\mu}\eta^{\nu][\lambda}k^{\rho]}(p-k)^{[\alpha}\eta^{\beta][\gamma}k^{\delta]}+\\
&\qquad+4p^{[\lambda}\eta^{\rho][\mu}k^{\nu]}(p-k)^{[\alpha}\eta^{\beta][\gamma}k^{\delta]}\Big]\times\Big[(p-k)_{[\gamma}\eta_{\delta][\lambda}(p-k)_{\rho]}\Big]=\\
&\frac{2(1-\xi_A)(1-\xi_B)}{k^4(p-k)^4}\Big\{[k\cdot(2p-k)]^2(p-k)^{[\alpha}\eta^{\beta][\mu}(p-k)^{\nu]}\\
&\qquad+[k\cdot(2p-k)][k\cdot(p-k)](p-k)^{[\alpha}\eta^{\beta][\nu}(p-k)^{\mu]}+\\
&\qquad+k\cdot(2p-k)(p-k)^{[\alpha}p^{\beta]}(p-k)^{[\mu}k^{\nu]}\\
&\qquad-[k\cdot(2p-k)][p\cdot(p-k)](p-k)^{[\alpha}\eta^{\beta][\mu}k^{\nu]}+\\
&\qquad+k\cdot(2p-k)(p-k)^{[\alpha}k^{\beta]}(p-k)^{[\mu}p^{\nu]}\\
&\qquad-[k\cdot(2p-k)][p\cdot(p-k)](p-k)^{[\mu}\eta^{\nu][\alpha}k^{\beta]}+\\
&\qquad+k\cdot(p-k)(p-k)^{[\mu}p^{\nu]}(p-k)^{[\beta}k^{\alpha]}\\
&\qquad-[k\cdot(p-k)][p\cdot(p-k)](p-k)^{[\nu}\eta^{\mu][\alpha}k^{\beta]}+\\
&\qquad+p\cdot(p-k)p^{[\alpha}k^{\beta]}(p-k)^{[\mu}k^{\nu]}-[p\cdot(p-k)]^2k^{[\beta}\eta^{\alpha][\mu}k^{\nu]}\\
&\qquad-p^2(p-k)^{[\mu}k^{\nu]}(p-k)^{[\alpha}k^{\beta]}+p\cdot(p-k)p^{[\mu}k^{\nu]}(p-k)^{[\alpha}k^{\beta]}\\
&\qquad-[k\cdot(2p-k)][k\cdot(p-k)](p-k)^{[\alpha}\eta^{\beta][\mu}(p-k)^{\nu]}\\
&\qquad+[k\cdot(p-k)]^2(p-k)^{[\mu}\eta^{\nu][\alpha}(p-k)^{\beta]}\\
&\qquad+[k\cdot(p-k)][p\cdot(p-k)](p-k)^{[\alpha}\eta^{\beta][\mu}k^{\nu]}+\\
&\qquad-k\cdot(p-k)(p-k)^{[\alpha}p^{\beta]}(p-k)^{[\mu}k^{\nu]}\Big\}
\end{aligned}
$$

Grouping the terms in a convenient manner, many factors simplify and the final expression reduces to:

$$
\begin{aligned}
&\frac{2(1-\xi_A)(1-\xi_B)}{k^4(p-k)^4}\Big[(p\cdot k)^2p^{[\alpha}\eta^{\beta][\mu}p^{\nu]}+p^4k^{[\alpha}\eta^{\beta][\mu}k^{\nu]}+p^2(p\cdot k)\big[p^{[\alpha}\eta^{\beta][\mu}k^{\nu]}+k^{[\alpha}\eta^{\beta][\mu}p^{\nu]}\big]+\\
&\qquad+p^2p^{[\mu}k^{\nu]}p^{[\alpha}k^{\beta]}\Big].
\end{aligned}
\tag{5.67}
$$

Introducing again some Feynman parameters to rewrite the denominator, we have

$$
\frac{1}{k^4(p-k)^4}=\int_0^1 dx\,dy\,dz\,dt\delta(x+y+z+t-1)\frac{6}{D^4},
\tag{5.68}
$$

with this time being simply $D = l^2$, where $l = k - (x+y)p$. The numerator written in terms of l will be

$$p^4 \frac{l^2}{d} \eta^{\mu[\beta} \eta^{\alpha]\nu} + 2\Big[p^2 \frac{l^2}{d} + 2(x+y)^2 p^4\Big] p^{[\alpha} \eta^{\beta][\mu} p^{\nu]} \tag{5.69}$$

and as we can see, given that the denominator goes as l^8, gives only finite contributes, which we can neglect for the evaluation of the beta function.

Tadpole diagram

Another contribution to the one loop order corrections to the two point Green function is given by the following diagram coming from the interaction term $-g^2 B^{\dagger}_{[\nu\rho} A_{\mu]} A^{[\mu} B^{\nu\rho]}$.

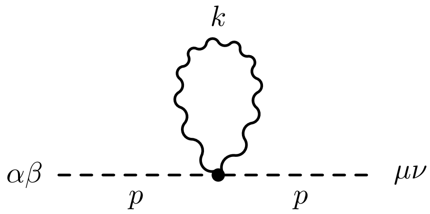


FIGURE 5.4: Tadpole diagram one loop correction to $B_{\mu\nu}$ propagator.

We recall that the Feynman rule in momentum space for this interaction is :

$$- ig^2 [\eta^{\mu\alpha} \eta^{\beta[\nu} \eta^{\rho]\gamma} + 2\eta^{\mu[\beta} \eta^{\gamma][\nu} \eta^{\rho]\alpha}] \tag{5.70}$$

So the expression for this diagram is just

$$g^2 \hbar \int \frac{d^d k}{(2\pi)^d} [\frac{\eta_{\tau\epsilon}}{k^2} - (1 - \xi_A) \frac{k_\tau k_\epsilon}{k^4}][\eta^{\tau\epsilon} \eta^{\alpha[\mu} \eta^{\nu]\beta} + 2\eta^{\tau[\alpha} \eta^{\beta][\mu} \eta^{\nu]\epsilon}] \tag{5.71}$$

However we can already see that the integrand is proportional to $\frac{1}{k^2}$ .Hence it will give zero contribution because of (5.55) . So the total contribution of this diagram is actually zero using dimensional regularization.

Interactions with W_μ

Regarding the Lagrangian one can already see that the interactions between the anti-symmetric $B_{\mu\nu}$ field and the W^a_μ fields are basically the same as those with the A_μ field, except that this time every term will be a 2×2 matrix. The vertex, in particular will look almost identical to the one with A_μ:

$$= -g'\Big[(p+p')^\alpha \eta^{\nu[\beta}\eta^{\gamma]\rho} + 2p^{[\beta}\eta^{\gamma][\nu}\eta^{\rho]\alpha} + 2p'^{[\nu}\eta^{\rho][\beta}\eta^{\gamma]\alpha}\Big]t^a \qquad (5.72)$$

Then, since the propagator for W^a_μ is the same as that of A_μ multiplied by a δ^{ab}, in the end everything will be proportional to $t^a t^a$, the Casimir operator that, in the two dimensional representation of SU(2), is just $t^a t^a = \frac{3}{4}\cdot\mathbf{1}$. This means that the calculations we have done in the previous section can be repeated identically also in this case and will lead to the same results for both diagrams, except for the substitution $\xi_A \to \xi_W$.

5.3.2 Corrections to the vertex

The full vertex to one loop order is the sum of

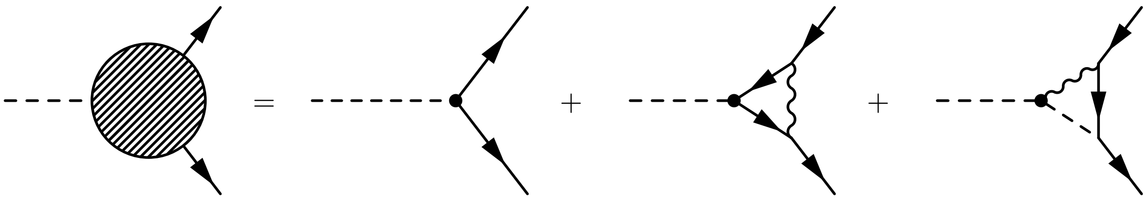


FIGURE 5.5: Full B-Yukawa vertex to one loop order.

We begin the computation with the first topology. This can occur both with the A_μ and W_μ fields. However we can compute the diagram for the A_μ case and then easily imply the result for the W_μ interaction.

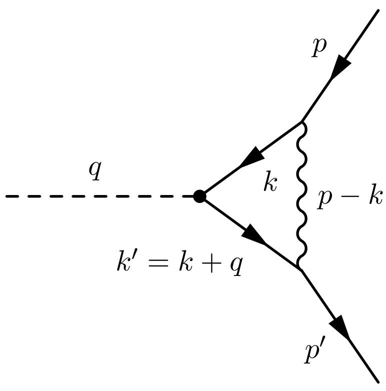


FIGURE 5.6: First type of one loop correction to the B-Yukawa vertex.

One can easily write the amplitude of this diagram using Feynman rules used so far, and the result is

$$(-iy)(ig)^2(-i\hbar)\int\frac{d^dk}{(2\pi)^d}\Big[\frac{\eta_{\mu\nu}}{(k-p)^2}-(1-\xi_A)\frac{(p-k)_\mu(p-k)_\nu}{(k-p)^4}\Big]\times$$
$$\times\bar{u}(p')\gamma^\mu\Big[\frac{i(\not{k}'+m)}{k'^2-m^2}\Big]\sigma^{\alpha\beta}\Big[\frac{i(\not{k}+m)}{k^2-m^2}\Big]\gamma^\nu u(p)= \tag{5.73}$$

$$=yg^2\hbar\int\frac{d^dk}{(2\pi)^d}\frac{2\bar{u}(p')[\not{k}'\not{k}\sigma^{\alpha\beta}-\sigma^{\alpha\beta}\not{k}\not{k}'-m\sigma^{\alpha\beta}\not{k}'-m\not{k}\sigma^{\alpha\beta}+)]u(p)}{(k-p)^2(k'^2-m^2)(k^2-m^2)} \tag{5.74}$$
$$+\frac{2\epsilon\bar{u}(p')[(\not{k}'\sigma^{\alpha\beta}\not{k}+m^2\sigma^{\alpha\beta}-m\not{k}'\sigma^{\alpha\beta}-m\sigma^{\alpha\beta}\not{k})]u(p)}{(k-p)^2(k'^2-m^2)(k^2-m^2)}$$
$$+\frac{1-\xi_A}{(k-p)^4(k'^2-m^2)(k^2-m^2)}\bar{u}(p')\Big[(\not{p}-\not{k})(\not{k}'+m)\sigma^{\alpha\beta}(\not{k}+m)(\not{p}-\not{k})\Big]u(p), \tag{5.75}$$

where for the first term there is a part proportional to $\epsilon=4-d$ coming from the modified contraction identities of γ's in d dimensions

$$\gamma^\mu\gamma^\nu\gamma_\mu = -(2-\epsilon)\gamma^\nu \tag{5.76}$$
$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4\eta^{\nu\rho}-\epsilon\gamma^\nu\gamma^\rho \tag{5.77}$$
$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma_\nu+\epsilon\gamma^\nu\gamma^\rho\gamma^\sigma \tag{5.78}$$
$$\gamma^\mu\gamma^\lambda\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = 2\gamma^\lambda\gamma^\sigma\gamma^\rho\gamma^\nu+2\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\lambda-\epsilon\gamma^\lambda\gamma^\nu\gamma^\rho\gamma^\sigma. \tag{5.79}$$

At this point we may focus on the first term and introduce as usual Feynman parameters to simplify the denominator. This time is $D=l^2-\Delta$, with $l=k+yq-zp$ and $\Delta=-xyq^2+(1-z)^2m^2$. Substituting $k=l-yq+zp$ and remembering that $k'=k+q$ we get, for the numerator of the gauge independent part and excluding terms of $O(\epsilon)$,

$$\big[l^2+y(y-1)q^2+z^2p^2+z\not{q}\not{p}-2yz(p\cdot q)\big]\sigma^{\alpha\beta}-\sigma^{\alpha\beta}\big[l^2+y(y-1)q^2+z^2p^2+z\not{p}\not{q}-2yz(p\cdot q)\big]+$$
$$-2m(-y\not{q}+z\not{p})\sigma^{\alpha\beta}-2m\sigma^{\alpha\beta}((1-y)\not{q}+z\not{p})=$$
$$=[z\not{q}\not{p}+2my\not{q}-2mz\not{p}]\sigma^{\alpha\beta}+\sigma^{\alpha\beta}[-z\not{p}\not{q}+2m(y-1)\not{q}-2mz\not{p}]=$$
$$=[2z\not{q}\not{p}-2z(p\cdot q)+2m(2y-1)\not{q}-4mz\not{p}]\sigma^{\alpha\beta}+8z[\gamma^{[\beta}q^{\alpha]}\not{p}+\gamma^{[\alpha}p^{\beta]}\not{q}+p^{[\alpha}q^{\beta]}]+$$
$$+16myq^{[\alpha}\gamma^{\beta]}-16mzp^{[\alpha}\gamma^{\beta]}.$$

We can already see that the term of order l^2 dropped out. This means that there will be no logarithmic divergent part and, ultimately, no contribution to the beta function. As regards the gauge-dependent part we have

$$-\bar{u}(p')[(\not{p}\not{k}'+m\not{p}-\not{k}\not{k}'-m\not{k})\sigma^{\alpha\beta}(\not{k}\not{p}-k^2+m\not{p}-m\not{k})]u(p),$$

Here again we introduce Feynman parameters to reduce the denominator as

$$\frac{1}{(k-p)^2(k-p)^2(k'^2-m^2)(k^2-m^2)} = \int_0^1 dx\,dy\,dz\,dt\,\delta(x+y+z+t-1)\frac{6}{D^4}, \quad (5.80)$$

where $D = l^2 - \Delta$ and $\Delta = -zq^2 + (z+t)(1-x+y)m^2$. After rewriting all in terms of $l = k - (x+y)p$, we can argue that the only relevant term that contributes to the evaluation of the beta function is the one of order l^4, since now the denominator goes as l^8. The only of such terms is just $l^4\sigma^{\alpha\beta}$ and the integral gives as result as $d \to 4$

$$\begin{aligned} 6(1-\xi_A)\int_0^1 dx\,dy\,dz\,dt\,\delta(x+y+z+t-1)\int\frac{d^dl}{(2\pi)^d}\frac{l^4}{(l^2-\Delta)^4} = \\ =6i(1-\xi_A)\frac{\Gamma\left(2-\frac{d}{2}\right)}{(4\pi)^2}\int_0^1 dx\,dy\,dz\left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} \end{aligned} \quad (5.81)$$

5.3.2.1 Second diagram

The second possible topology is shown in figure

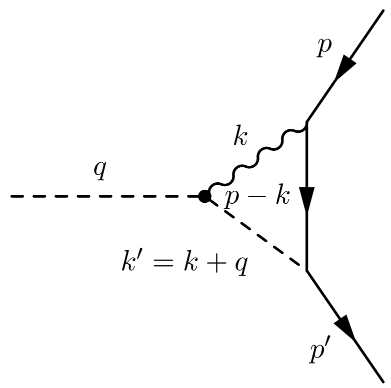


FIGURE 5.7: Second type of one loop correction to the B-Yukawa vertex.

This time the amplitude will look rather complicated, because of the presence of various propagators and vertices. The expression is

$$\begin{aligned} (-i\hbar)^2(-iy)(-ig)(-g)\int\frac{d^dk}{(2\pi)^d}\Big\{\Big[\frac{\eta_{\mu\nu}}{k^2}-(1-\xi_A)\frac{k_\mu k_\nu}{k^4}\Big]\times\Big[\frac{\eta_{\lambda[\rho}\eta_{\tau]\delta}}{k'^2}-2(1-\xi_B)\frac{k'_{[\lambda}\eta_{\delta][\rho}k'_{\tau]}}{k'^4}\Big]\times \\ \times\bar{u}(p')\gamma^\mu\Big[i\frac{(\not{p}-\not{k}+m)}{(p-k)^2-m^2}\Big]\sigma^{\rho\tau}u(p)\times\Big[(k+2q)^\nu\eta^{\alpha[\lambda}\eta^{\delta]\beta}+2q^{[\lambda}\eta^{\delta][\alpha}\eta^{\beta]\nu}+2k'^{[\alpha}\eta^{\beta][\lambda}\eta^{\delta]\nu}\Big]\Big\} = \end{aligned} \quad (5.82)$$

$$
\begin{aligned}
=&\Big\{\frac{1}{k^2}\Big[(k+2q)_\mu\eta^{\alpha[\lambda}\eta^{\delta]\beta}+2q^{[\lambda}\eta^{\delta][\alpha}\eta^{\beta]}_\mu+2k'^{[\alpha}\eta^{\beta][\lambda}\eta^{\delta]}_\mu\Big]\\
&-\frac{1-\xi_A}{k^4}\Big[k\cdot(k+2q)\eta^{\alpha[\lambda}\eta^{\delta]\beta}k_\mu+2q^{[\lambda}\eta^{\delta][\alpha}k_\mu+\\
&+2k'^{[\alpha}\eta^{\beta][\lambda}k_\mu\Big]\Big\}\times\bar u(p')\gamma^\mu\Big[i\frac{(\not p-\not k+m)}{(p-k)^2-m^2}\Big]\sigma^{\rho\tau}u(p)\times\Big[\frac{\eta_{\lambda[\rho}\eta_{\tau]\delta}}{k'^2}-2(1-\xi_B)\frac{k'_{[\lambda}\eta_{\delta][\rho}k'_{\tau]}}{k'^4}\Big]=\\
=&\Big\{\frac{1}{k^2[(p-k)^2-m^2]}\bar u(p')\Big[(\not k+2\not q)(\not p-\not k+m)\eta^{\alpha[\lambda}\eta^{\delta]\beta}+2q^{[\lambda}\eta^{\delta][\alpha}\gamma^{\beta]}(\not p-\not k+m)+\\
&+2k'^{[\alpha}\eta^{\beta][\lambda}\gamma^{\delta]}(\not p-\not k+m)\Big]\sigma^{\rho\tau}u(p)\\
&-\frac{1-\xi_A}{k^4[(p-k)^2-m^2]}\bar u(p')\Big[k\cdot(k+2q)\not k(\not p-\not k+m)\eta^{\alpha[\lambda}\eta^{\delta]\beta}+\\
&+2q^{[\lambda}\eta^{\delta][\alpha}k^{\beta]}\not k(\not p-\not k+m)+2k'^{[\alpha}\eta^{\beta][\lambda}k^{\delta]}\not k(\not p-\not k+m)\Big]\sigma^{\rho\tau}u(p)\times\\
&\times\Big[\frac{\eta_{\lambda[\rho}\eta_{\tau]\delta}}{k'^2}-2(1-\xi_B)\frac{k'_{[\lambda}\eta_{\delta][\rho}k'_{\tau]}}{k'^4}\Big]\Big\}.
\end{aligned}
$$

At this point we have four terms, one of which is gauge independent while the other three will depend on the gauge choice. We will treat the four terms separately, in order for the calculation to be more clear. For the first, gauge-independent term, we get

$$
\begin{aligned}
&\frac{1}{k^2}\frac{1}{(p-k)^2}\frac{1}{k'^2}\bar u(p')\Big[(\not k\not p-k^2+2\not q\not p-2\not q\not k+m\not k+2m\not q)\eta^{\alpha[\lambda}\eta^{\delta]\beta}\eta_{\lambda[\rho}\eta_{\tau]\delta}+\\
&\quad+2q^{[\lambda}\eta^{\delta][\alpha}\gamma^{\beta]}\eta_{\lambda[\rho}\eta_{\tau]\delta}(\not p-\not k+m)+2k'^{[\alpha}\eta^{\beta][\lambda}\gamma^{\delta]}\eta_{\lambda[\rho}\eta_{\tau]\delta}(\not p-\not k+m)\Big]\sigma^{\rho\tau}u(p)=\\
=&\frac{1}{k^2}\frac{1}{(p-k)^2}\frac{1}{k'^2}\bar u(p')\Big[(\not k\not p-k^2+2\not q\not p-2\not q\not k+m\not k+2m\not q)\sigma^{\alpha\beta}+2\gamma^{[\beta}(\not p-\not k+m)\not q\gamma^{\alpha]}\\
&\quad-2\gamma^{[\beta}(\not p-\not k+m)\gamma^{\alpha]}\not q+2k'^{[\alpha}\gamma_\tau(\not p-\not k+m)\gamma^{\beta]}\gamma^\tau-2k'^{[\alpha}\gamma_\tau(\not p-\not k+m)\gamma^\tau\gamma^{\beta]}\Big]u(p).
\end{aligned}
$$

We can already argue that the only terms which will give a logarithmic divergence are quadratic k terms, namely, the k^2 factor in the first term and the last two in brackets. So we can limit to consider only those terms.
Having a closer look we get

$$
\begin{aligned}
&2k'^{[\alpha}\gamma_\tau(\not p-\not k+m)\gamma^{\beta]}\gamma^\tau-2k'^{[\alpha}\gamma_\tau(\not p-\not k+m)\gamma^\tau\gamma^{\beta]}=\\
&=4k'^{[\alpha}\gamma_\tau(\not p-\not k+m)\gamma^{\beta]}\gamma^\tau-4k'^{[\alpha}\gamma_\tau(\not p-\not k+m)\eta^{\beta]\tau}=\\
&4k'^{[\alpha}\gamma_\tau(\not p-\not k+m)\gamma^{\beta]}\gamma^\tau-4k'^{[\alpha}\gamma^{\beta]}(\not p-\not k+m);
\end{aligned}
$$

Where we have used the anticommutation relation of γ matrices. Now, since we are always interested in quadratic k terms, we can replace k' with k and drop all other terms.

$$
=-4k^{[\alpha}\gamma_\tau\not k\gamma^{\beta]}\gamma^\tau+4k^{[\alpha}\gamma^{\beta]}\not k=-4l^{[\alpha}l_\mu(4\eta^{\beta]\mu}-\epsilon\gamma^\mu\gamma^{\beta]})+4l^{[\alpha}\gamma^{\beta]}\not l=-\frac{2}{d}l^2\sigma^{\alpha\beta}
$$

where in the last equation we subsituted k with l and then plugged in $l^\mu l^\nu\to\frac{l^2}{d}\eta^{\mu\nu}$.

Summing up with the previous result and rearranging everything in terms of $l = k - yp$ and $\Delta = -(1-x-y)q^2 + y^2m^2$ we get that the divergent part is

$$\begin{aligned}
-2i\frac{d+2}{d}\int_0^1 dx\,dy\,dz\delta(x+y+z-1)\int\frac{d^dl}{(2\pi)^d}\frac{l^2}{(l^2-\Delta)^3}\sigma^{\alpha\beta} = \\
=\frac{d+2}{2}\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}\int_0^1 dx\,dy\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}\sigma^{\alpha\beta}.
\end{aligned} \tag{5.83}$$

Then we move on to the second term. This time the divergent part will be of order k^4. Having this in mind we have

$$\begin{aligned}
-\frac{2(1-\xi_B)}{k^2[(p-k)^2-m^2]k'^4}\bar{u}(p')\Big[(\not{k}+2\not{q})(\not{p}-\not{k}+m)(k'^{[\alpha}\gamma^{\beta]}\not{k}'-\not{k}'k'^{[\alpha}\gamma^{\beta]})+ \\
+2q^{[\lambda}\eta^{\delta][\alpha}\gamma^{\beta]}(\not{p}-\not{k}+m)k'_{[\lambda}\eta_{\delta][\rho}k'_{\tau]}\sigma^{\rho\tau}-k'^{[\alpha}\eta^{\beta]}_{[\rho}k'_{\tau]}\not{k}'(\not{p}-\not{k}+m)\sigma^{\rho\tau}\Big]u(p).
\end{aligned}$$

Again, we can let $k' = k$ and then put directly $k = l$. We can already see that the second term only has a maximum k order of 3, thus we can drop it. The divergent k^4 terms will be just

$$-k^2[k^{[\alpha}\gamma^{\beta]}k_\mu\gamma^\mu - k_\mu\gamma^\mu k^{[\alpha}\gamma^{\beta]}] + k^2[k^{[\alpha}\gamma^{\beta]}k_\mu\gamma^\mu - k_\mu\gamma^\mu k^{[\alpha}\gamma^{\beta]}] = 0.$$

So the divergent part of this term accidentally cancels out. Then we have the third term, which is

$$\begin{aligned}
-\frac{1-\xi_A}{k^4[(p-k)^2-m^2]k'^2}\bar{u}(p')\Big[k\cdot(k+2q)\not{k}(\not{p}-\not{k}+m)\sigma^{\alpha\beta}+2q_{[\rho}\eta^{[\alpha}_{\tau]}k^{\beta]}\not{k}(\not{p}-\not{k}+m)\sigma^{\rho\tau}+ \\
-2k^2(k^{[\alpha}\gamma^{\beta]}\not{k}-\not{k}k^{[\alpha}\gamma^{\beta]})\Big].
\end{aligned}$$

Also here we can drop the second term as long as it has odd powers of k and does not give a divergent contribute. The divergent part is the sum of the two l^4 terms , where $l = k - xp$

$$-l^4\sigma^{\alpha\beta} + 2\frac{l^4}{d}\sigma^{\alpha\beta} = \frac{2-d}{d}l^4\sigma^{\alpha\beta}.$$

So the total divergent contribution of this term will be

$$\begin{aligned}
6i(-\frac{1}{2})(1-\xi_A)\int_0^1 dx\,dy\,dz\delta(x+y+z+t-1)\int\frac{d^dl}{(2\pi)^d}\frac{l^4}{(l^2-\Delta)^4}\sigma^{\alpha\beta} = \\
=3(1-\xi_A)\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}\int_0^1 dx\,dy\,dz\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}\sigma^{\alpha\beta},
\end{aligned} \tag{5.84}$$

where $\Delta = -x(1-2x)m^2 - yq^2$.

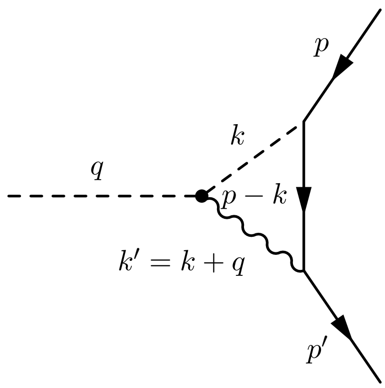
Finally, we have the fourth term

$$\frac{2(1-\xi_B)(1-\xi_A)}{k^4[(p-k)^2-m^2]k'^4}\bar{u}(p')\Big[k^2(-k^2)[k^{[\alpha}\gamma^{\beta]}\not{k} - k^{[\alpha}\not{k}\gamma^{\beta]}] - 2k^2k'_{[\lambda}\eta_{\delta][\rho}k'_{\tau]}]k'^{[\alpha}\eta^{\beta][\lambda}k^{\delta]}\sigma^{\rho\tau}\Big],$$

where for simplicity we have considered only maximum order k term and omitted the second term because we have already seen it gives no divergent contribution. However, expanding the last term and substituting $k' = k$ we get

$$2k^2k'_{[\lambda}\eta_{\delta][\rho}k'_{\tau]}]k'^{[\alpha}\eta^{\beta][\lambda}k^{\delta]}\sigma^{\rho\tau} = k^4k^{[\alpha}\eta^{\beta]}_{[\rho}k_{\tau]}\sigma^{\rho\tau} = k^4[k^{[\alpha}\gamma^{\beta]}\not{k} - \not{k}k^{[\alpha}\gamma^{\beta]}].$$

This means that even this last term has an accidental cancellation of the divergent part. Then we have to consider also another diagram, but with interchanged photon and antisymmetric propagator, as is shown in figure: The amplitude will look somewhat



similar to the previous graph :

$$(-i\hbar)^2(-iy)(-ig)(-g)\int\frac{d^dk}{(2\pi)^d}\Big\{\Big[\frac{\eta_{\mu\nu}}{k'^2} - (1-\xi_A)\frac{k'_\mu k'_\nu}{k^4}\Big]\times\Big[\frac{\eta_{\lambda[\rho}\eta_{\tau]\delta}}{k^2} - 2(1-\xi_B)\frac{k_{[\lambda}\eta_{\delta][\rho}k_{\tau]}}{k^4}\Big]\times$$
$$\times\bar{u}(p')\sigma^{\lambda\delta}\Big[i\frac{(\not{p}-\not{k})}{(p-k)^2}\Big]\gamma^\nu u(p)\times\Big[(k-q)^\mu\eta^{\alpha[\rho}\eta^{\tau]\beta} - 2q^{[\rho}\eta^{\tau][\alpha}\eta^{\beta]\mu} + 2k^{[\alpha}\eta^{\beta][\rho}\eta^{\tau]\mu}\Big]\Big\} = \tag{5.85}$$

$$= -iy\hbar^2 g^2 \int \frac{d^d k}{(2\pi)^d} \Big\{ \frac{1}{k'^2} \Big[(k-q)_\nu \eta^{\alpha[\rho}\eta^{\tau]\beta} - 2q^{[\rho}\eta^{\tau][\alpha}\eta_\nu^{\beta]} + 2k^{[\alpha}\eta^{\beta][\rho}\eta_\nu^{\tau]} \Big] +$$
$$- \frac{1-\xi_A}{k'^4} \Big[k' \cdot (k-q)\eta^{\alpha[\rho}\eta^{\tau]\beta} k'_\nu - 2q^{[\rho}\eta^{\tau][\alpha}k'^{\beta]}k'_\nu + 2k^{[\alpha}\eta^{\beta][\rho}k'^{\beta]}k'_\nu \Big] \Big\} \times$$
$$\times \bar{u}(p')\sigma^{\lambda\delta} \Big[i \frac{(\not{p} - \not{k})}{(p-k)^2} \Big] \gamma^\nu u(p) \times \Big[\frac{\eta_{\lambda[\rho}\eta_{\tau]\delta}}{k^2} - 2(1-\xi_B) \frac{k_{[\lambda}\eta_{\delta][\rho}k_{\tau]}}{k^4} \Big] =$$
$$= -iy\hbar^2 g^2 \int \frac{d^d k}{(2\pi)^d} \Big\{ \frac{1}{k'^2(p-k)^2} \bar{u}(p')\sigma^{\lambda\delta} \Big[(\not{p} - \not{k})(\not{k} - \not{q})\eta^{\alpha[\rho}\eta^{\tau]\beta} - 2(\not{p} - \not{k}) q^{[\rho}\eta^{\tau][\alpha}\gamma^{\beta]} +$$
$$+ 2(\not{p} - \not{k}) k^{[\alpha}\eta^{\beta][\rho}\gamma^{\tau]} \Big] u(p) +$$
$$- \frac{1-\xi_A}{k'^4(p-k)^2} \bar{u}(p')\sigma^{\lambda\delta} \Big[(\not{p} - \not{k}) k' \cdot (k-q)\eta^{\alpha[\rho}\eta^{\tau]\beta} \not{k}' - 2(\not{p} - \not{k}) q^{[\rho}\eta^{\tau][\alpha}k'^{\beta]}\not{k}' +$$
$$+ 2(\not{p} - \not{k}) k^{[\alpha}\eta^{\beta][\rho}k'^{\tau]}\not{k}' \Big] u(p) \Big\} \times \Big[\frac{\eta_{\lambda[\rho}\eta_{\tau]\delta}}{k^2} - 2(1-\xi_B) \frac{k_{[\lambda}\eta_{\delta][\rho}k_{\tau]}}{k^4} \Big].$$

At this point again we split different terms for simplicity. Also, we can omit from the start terms with odd powers of k since we have already seen that these do not contribute to the determination of divergences. We will compute only such divergences, so we can safely use $k' = k$ and look only at highest powers of k. Bearing this in mind, the gauge independent part yields

$$\frac{1}{k'^2(p-k)^2 k^2} \bar{u}(p')[-k^2\sigma^{\alpha\beta} - 2k^{[\alpha}\gamma^{\beta]}\gamma^\delta \not{k}\gamma_\delta + 2k^{[\alpha}\gamma^\lambda\gamma^{\beta]}\not{k}\gamma_\lambda] u(p).$$

The divergent part is then just the same as in the previous graph.

As for the second part we have

$$- \frac{2(1-\xi_B)}{k'^2(p-k)^2k^4} \bar{u}(p')[-k^2\not{k}\gamma^{[\alpha}k^{\beta]} + k^2\gamma^{[\alpha}\not{k}k^{\beta]} - k^2k^{[\alpha}\not{k}\gamma^{\beta]} + k^2k^{[\alpha}\gamma^{\beta]}\not{k}] = 0,$$

so also this time it cancels accidentally.

The third part reads

$$\frac{1-\xi_A}{k'^4(p-k)^2k^2} \bar{u}(p')[-k^4\sigma^{\alpha\beta} - 2k^2\gamma^{[\beta}k^{\alpha]}\not{k} + 2\not{k}\gamma^{[\beta}\not{k}k^{\alpha]}\not{k}] u(p) =$$
$$\frac{1-\xi_A}{k'^4(p-k)^2k^2} \bar{u}(p')[-k^4\sigma^{\alpha\beta} - 4k^2\gamma^{[\beta}k^{\alpha]}\not{k}] u(p),$$

and again the contribution is equal to the one obtained before. Finally, for the last term

$$\frac{2(1-\xi_B)(1-\xi_A)}{k'^4(p-k)^2k^4} \bar{u}(p')[-k^4\not{k}\gamma^{[\alpha}k^{\beta]} + k^4\gamma^{[\alpha}\not{k}k^{\beta]} - k^4k^{[\alpha}\not{k}\gamma^{\beta]} + k^4k^{[\alpha}\gamma^{\beta]}\not{k}] = 0.$$

So we can conclude that the divergent part of this diagram is exactly equal to the first diagram we considered. Also, the contribution from diagrams with W_μ is proportional to the one just computed.

5.3.3 Corrections to the external fermion leg

As regards the eternal fermion leg, we have the usual corrections due to QED diagrams, plus a new self-energy kind of diagram coming from the interactions with $B_{\mu\nu}$. Such diagram is represented in figure

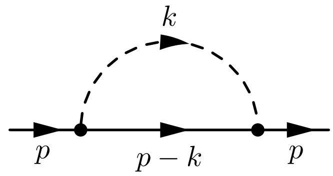


FIGURE 5.8: Additional 1-loop correction to the fermion propagator

and its amplitude can be easily written down with the help of Feynman rules:

$$(-iy)^2\int\frac{d^dk}{(2\pi)^d}\sigma^{\alpha\beta}\Big[\frac{i(\not p-\not k+m)}{(p-k)^2-m^2}\Big]\sigma^{\mu\nu}\times\Big[\frac{\eta_{\alpha[\mu}\eta_{\nu]\beta}}{k^2}-2(1-\xi_B)\frac{k_{[\alpha}\eta_{\beta][\mu}k_{\nu]}}{k^4}\Big]= \tag{5.86}$$

$$\begin{aligned}=&-iy^2\int\frac{d^dk}{(2\pi)^d}\Big\{\frac{1}{k^2[(p-k)^2-m^2]}\Big[\sigma_{\mu\nu}(\not p-\not k+m)\sigma^{\mu\nu}\Big]+\\&-\frac{2(1-\xi_B)}{k^4[(p-k)^2-m^2]}\Big[\sigma^{\alpha\beta}(\not p-\not k+m)\sigma^{\mu\nu}k_{[\alpha}\eta_{\beta][\mu}k_{\nu]}\Big]\Big\}.\end{aligned}$$

Focusing on the first term we have

$$\sigma_{\mu\nu}(\not p-\not k+m)\sigma^{\mu\nu}=2\gamma_\mu\gamma_\nu(\not p-\not k+m)\gamma^\mu\gamma^\nu-2\gamma_\mu\gamma_\nu(\not p-\not k+m)\gamma^\nu\gamma^\mu=-2d(2+d)m,$$

where the momentum part vanishes by symmetry. So, in the massless fermion case this term just cancels out. For the second, gauge dependent, part we have

$$\begin{aligned}&\sigma^{\alpha\beta}(\not p-\not k+m)\sigma^{\mu\nu}k_{[\alpha}\eta_{\beta][\mu}k_{\nu]}=\sigma^{\alpha\beta}(\not p-\not k+m)(k_{[\alpha}\gamma_{\beta]}\not k-k_{[\alpha}\not k\gamma_{\beta]})=\\=&\gamma^\alpha\gamma^\beta(\not p-\not k+m)k_{[\alpha}\gamma_{\beta]}\not k-\gamma^\alpha\gamma^\beta(\not p-\not k+m)k_{[\alpha}\not k\gamma_{\beta]}-\gamma^\beta\gamma^\alpha(\not p-\not k+m)k_{[\alpha}\gamma_{\beta]}\not k+\\+&\gamma^\alpha\gamma^\beta(\not p-\not k+m)k_{[\alpha}\not k\gamma_{\beta]}.\end{aligned}$$

Now we will look at the terms with $\not{p}$ and borrow our result for the ones with $\not{k}$.

$$=k_{[\alpha}p_\mu k_\nu\gamma^\alpha\gamma^\beta\gamma^\mu\gamma_{\beta]}\gamma^\nu - k_{[\alpha}p_\mu k_\nu\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu\gamma_{\beta]} + k_{[\alpha}p_\mu k_\nu\gamma^\beta\gamma^\alpha\gamma^\mu\gamma_{\beta]}\gamma^\nu - k_{[\alpha}p_\mu k_\nu\gamma^\beta\gamma^\alpha\gamma^\mu\gamma^\nu\gamma_{\beta]} =$$
$$= -\left[k_\alpha p_\mu k_\nu\gamma^\alpha\gamma^\mu\gamma^\nu + 2k_\beta p_\mu k_\nu\eta^{\mu\beta}\gamma^\nu + 2k_\alpha p_\mu k_\nu\eta^{\mu\nu}\gamma^\alpha + k_\beta p_\mu k_\nu\gamma^\nu\gamma^\mu\gamma^\beta + 2k_\alpha p_\mu k_\nu\eta^{\mu\alpha}\gamma^\nu +\right.$$
$$\left.+k_\beta p_\mu k_\nu\gamma^\beta\gamma^\mu\gamma^\nu + k_\alpha p_\mu k_\nu\gamma^\nu\gamma^\mu\gamma^\alpha + 2k_\beta p_\mu k_\nu\eta^{\mu\nu}\gamma^\beta\right] = -4(\not{k}\not{p}\not{k} + 2(p\cdot k)\not{k}).$$

This result implies that the part with $\not{k}$ instead of $\not{p}$ gives $12k^2\not{k}$. Whereas for the part with m:

$$m[k_\alpha k_\mu\gamma^\alpha\gamma^\beta\gamma_\beta\gamma^\mu - k_\beta k_\mu\gamma^\alpha\gamma^\beta\gamma_\alpha\gamma^\mu - k_\alpha k_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\gamma_\beta + k_\beta k_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\gamma_\alpha - k_\alpha k_\mu\gamma^\beta\gamma^\alpha\gamma_\beta\gamma^\mu +$$
$$+k_\beta k_\mu\gamma^\beta\gamma^\alpha\gamma_\alpha\gamma^\mu + +k_\alpha k_\mu\gamma^\beta\gamma^\alpha\gamma^\mu\gamma_\beta - k_\beta k_\mu\gamma^\beta\gamma^\alpha\gamma^\mu\gamma_\alpha] = 2(8+d)mk^2.$$

So in total we are left with

$$2[6k^2\not{k} - 2\not{k}\not{p}\not{k} - 4(p\cdot k)\not{k} + (8+d)mk^2)].$$

Now let us introduce Feynman parameters so that

$$\frac{1}{k^4[(p-k^2)-m^2]} = \int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\frac{2}{D^3} \tag{5.87}$$

and in this case $D = l^2 - \Delta$, where $l = k - xp$ and $\Delta = -x(1-2x)m^2$. Next thing to do is to rewrite all in terms of l and drop odd powers:

$$12(l^2 + x^2p^2 + 2x(p\cdot l))(\not{l}x\not{p}) - 4\not{l}\not{p}\not{l} - x^2p^2 - 2(p\cdot l)\not{l} + 6m^2(l^2 + x^2p^2 + 2x(p\cdot l)) =$$
$$=12xl^2\not{p} + 24x\frac{l^2}{d}\not{p} - 4\not{l}\not{p}\not{l} - 2\frac{l^2}{d}\not{p} + 6m^2l^2 = 3l^2\not{p}(2x+1+2m^2).$$

So the divergent part for this diagram will be

$$-12i(1-\xi_B)\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\not{p}(2x+1+2m^2)\int\frac{d^dl}{(2\pi)^d}\frac{l^2}{(l^2-\Delta)^3} =$$
$$= 12(1-\xi_B)\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}\not{p}\int_0^1 dx\,dy(2x+1+2m^2)\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}} \tag{5.88}$$

5.3.4 Counterterms

Now we have all the ingredients to evaluate the counterterms, introduced in the original Lagrangian to get rid of all divergences. To this end we just have to implement the renormalization conditions, i.e. equate the sum of divergent parts of diagrams plus the counterterms to zero. So, for δ_B, the counterterm that cancels divergences of the external $B_{\mu\nu}$ leg, we have that the sum of all divergences is

$$\Big\{\eta^{\alpha[\mu}\eta^{\nu]\beta}p^2\Big[\frac{16}{3}y^2-\frac{13}{6}g^2-\frac{13}{8}g'^2+\frac{7}{4}(1-\xi_B)g^2+\frac{21}{16}(1-\xi_B)g'^2+\frac{19}{24}(1-\xi_A)g^2+\frac{19}{32}(1-\xi_W)g'^2\Big]+$$
$$+p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}\Big[-\frac{64}{3}y^2-5g^2-\frac{15}{4}g'^2+2(1-\xi_B)g^2+\frac{3}{2}(1-\xi_B)g'^2-\frac{31}{3}(1-\xi_A)g^2+$$
$$-\frac{31}{4}(1-\xi_W)g'^2\Big]\Big\}\frac{i\hbar^2}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{\frac{d}{2}-2}}=$$
$$=\Big\{\eta^{\alpha[\mu}\eta^{\nu]\beta}p^2\Big[\frac{128y^2+(g^2+\frac{3}{4}g'^2)(9-42\xi_B)-19\xi_Ag^2-\frac{57}{4}\xi_Wg'^2}{24}\Big]+$$
$$+p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}\Big[\frac{-64y^2-(g^2+\frac{3}{4}g'^2)(40+6\xi_B)+31\xi_Ag^2+\frac{93}{4}\xi_Wg'^2}{3}\Big]\Big\}\frac{i\hbar^2}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{\frac{d}{2}-2}}.$$

We can see that, since there is not a gauge invariance and, hence, the Ward-Takahashi identity does not hold, the two tensorial structures have different divergent factors. However we first introduced in our Lagrangian for the antisymmetric tensor a term which does not affect the only dynamical degree of freedom, B_L. So the part proportional to $p^{[\mu}\eta^{\nu][\beta}p^{\alpha]}$ coming from that choice, even if divergent, has not dynamical influence. This means that the counterterm we need to introduce has to cancel divergences of the the first term of the previous equation. This is done by imposing

$$\delta_B=\frac{128y^2+(g^2+\frac{3}{4}g'^2)(9-42\xi_B)-19\xi_Ag^2-\frac{57}{4}\xi_Wg'^2}{24}\frac{\hbar^2}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}}. \tag{5.89}$$

As regards the corrections to the vertex, divergences sum up to

$$\Big[4(1-\xi_A)yg^2+\frac{3}{2}(1-\xi_W)yg'^2+3y(g^2+\frac{3}{8}g'^2)\Big]\sigma^{\alpha\beta}\frac{1}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}}=$$
$$=\frac{y}{(4\pi)^2}[4(g^2+\frac{3}{8}g'^2)-4g^2\xi_A-\frac{3}{2}g'^2\xi_W]\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}},$$

which implies

$$\delta_y=\frac{y^2}{(4\pi)^2}[4(g^2+\frac{3}{8}g'^2)-4g^2\xi_A-\frac{3}{2}g'^2\xi_W]\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}}. \tag{5.90}$$

Finally, the divergent corrections to the fermion propagator are (computated by imposing the renormalization conditions, i.e. taking the derivative with respect to $\not p$ in (5.88))

$$-\frac{g^2+\frac{3}{8}g'^2}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}}+\frac{10(1-\xi_B)y^2}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}}=-\frac{g^2+\frac{3}{8}g'^2-10(1-\xi_B)y^2}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}}=\delta_\Psi.$$

We can combine these results to obtain our final goal: the beta function of the theory.

$$\begin{aligned}\beta(y,g,g') =& M\frac{\partial}{\partial M}(-\delta_y + \frac{1}{2}y(\delta_B + 2\delta_\Psi)) = \\ & \frac{y}{(4\pi)^2}\Big\{-y\Big[4(g^2+\frac{3}{8}g'^2) - 4g^2\xi_A - \frac{3}{2}g'^2\xi_W\Big] + (\frac{22}{3} - 5\xi_B)y^2 - \frac{g^2+\frac{3}{8}g'^2}{2}\Big\}.\end{aligned} \tag{5.91}$$

Obviously this result has no physical meaning because of the dependence on gauge parameters ξ_B, ξ_A and ξ_W. The beta function, which is supposed to describe the behavior of the coupling constant at different energy scales, must be an object independent on gauge choice.

Also, dependence on ξ_A and ξ_W implies that somehow $SU(2)\otimes U(1)$ symmetry is broken. This fact is probably a result of the gauge fixing term used to covariant quantize the antisymmetric tensor field, which formally breaks $SU(2)\otimes U(1)$. Further work is needed to see if implementing a different gauge fixing term with a covariant derivative can restore the symmetry and provide new interactions which, in turn, can cancel the dependence on ξ_A and ξ_W. Also, the possibility to add ghost fields for the antisymmetric tensor field should be investigated as a possible cure to this gauge dependence problem.

Chapter 6

Conclusions

In this work we studied a model alternative to the SM in which a fundamental scalar particle is eliminated from the theory and the breaking of electro-weak symmetry is achieved through the introduction of an antisymmetric tensor field coupled to fermions with a "B-Yukawa" interaction term. The basic idea of dynamical symmetry breaking, taken in analogy with what happens in Technicolor models, is the key concept of this model. If the coupling constant of the B-Yukawa term was proven to have a negative β function, then the strong dynamics of this coupling would generate some fermion-antifermion condensate $\langle \bar{\Psi}\Psi \rangle$ which can play the role of the Higgs particle, generating masses to gauge bosons W and Z and also to fermions.
This theory does not present typical problems associated with a fundamental scalar, such as hierarchy, unnaturalness and vacuum stability, and for this reason could be a more appealing way to explain the mechanism of the electro-weak symmetry breaking.
Even after the discovery of a scalar particle at LHC in 2012, many properties of such particle are to be unraveled, such as its possible compositeness or its parity properties, and must still be checked experimentally to prove that it is the fundamental Higgs particle of the SM. So, these alternative models in which there is no fundamental scalar are still worth investigating.
Starting from a first model proposed by Wetterich [20], we removed classical instabilities by modifying the Lagrangian for the antisymmetric tensor field. This new Lagrangian presents a gauge freedom that, in analogy with what happens with gauge fields, forbids a straightforward computation of the propagator due to the presence of a null eigenvector for the quadratic terms. By adding a suitable gauge-fixing term, we constructed a covariant propagator with which we performed the computation of the relevant Feynman diagrams in momentum space for the evaluation of the β function of the B-Yukawa coupling, which is expected to be negative to allow the formation of the condensate. What we found is a gauge dependent value of β, which is obviously unacceptable. The

dependence on the gauge parameters of the W_μ and A_μ fields, in particular, implies that somehow the introduction of the B-Yukawa interaction breaks $SU(2) \otimes U(1)$ symmetry. This phenomenon could be a result of the form of gauge fixing term introduced previously, and a substitution of the usual derivative with a covariant one may be the solution of this gauge dependence. Also, the introduction of ghost fields should be investigated as a possible cure to this problem. Further work is needed to complete these analysis and to see if the gauge independent β function is indeed negative, a necessary condition to provide the condensate and to give mass to fermions and gauge bosons. A further step would then be the study of possible experimental signatures of this model at current experiments at LHC.

Appendix A

Graviton Propagator In Momentum Space

A.1 Introduction

We know that the propagator for the graviton propagator in covariant gauges is given by:

$$i[{}_{\rho\sigma}\Delta^{ab}_{\alpha\beta}](x,x') = -2\hbar\kappa^2\left(\eta_{\rho(\alpha}\eta_{\beta)\sigma} - \frac{1}{D-2}\eta_{\rho\sigma}\eta_{\alpha\beta}\right)i\Delta^{ab}_0(x,x') + 4(1-\xi)\hbar\kappa^2\partial_{(\rho}\eta_{\sigma)(\alpha}\partial_{\beta)}\int d^Dz\, i\Delta^{ac}_0(x,z)(\sigma^3)^{cd}\Delta^{db}_0(z,x'). \quad (A.1)$$

Where

$$i\Delta^{ab}_0(x,x') = \frac{\Gamma(\frac{D-2}{2})}{4\pi^{D/2}}\frac{1}{(\Delta x^2_{ab})^{(D-2)/2}} \quad (A.2)$$

is the Keldysh propagator for a massless scalar field in Minkowski space. So, in order to calculate the graviton propagator in momentum space, we need to first compute the scalar propagator in momentum space.

A.2 The massless scalar propagator in momentum space

We want to calculate the Feynman time-ordered propagator in momentum space, i.e. $\tilde{\Delta}^{++}_0(k_\mu)$. In order to do so we have to consider $\Delta x^2_{++} = -(|t-t'|-i\varepsilon)^2 + \|\vec{x}-\vec{x}'\|$ in

(1). We also know that it satisfies the equation

$$\partial^2 i\Delta_0^{++}(x,x') = i\delta^D(x-x'). \tag{A.3}$$

Now, inserting a Fourier ansatz for the propagator we get:

$$\partial^2 \int d^D(x-x')e^{-ik\cdot(x-x')} i\tilde{\Delta}_0^{++}(k_\mu) = \int d^D(x-x')ie^{-ik\cdot(x-x')} \tag{A.4}$$

$$\Longrightarrow -k_\mu k^\mu i\Delta_0^{++}(k_\mu) = i \Longrightarrow i\Delta_0^{++}(k_\mu) = -\frac{i}{k_\mu k^\mu} \tag{A.5}$$

At this point we still don't know what the exact Feynman prescription is for such propagator. So all we can do is just guess the prescription and verify if it gives us the correct Feynman propagator in real space after performing an anti Fourier transformation. So, we can rewrite A.5 as:

$$i\tilde{\Delta}_0^{++}(k_\mu) = \frac{i}{2k}\left[\frac{1}{k_0 - k + i\varepsilon} - \frac{1}{k_0 + k - i\varepsilon}\right] \tag{A.6}$$

where $k = \|\vec{k}\|$. Then

$$i\Delta_0^{++}(x-x') = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \int \frac{dk_0}{2\pi} i\tilde{\Delta}_0^{++}(k_\mu)e^{-ik_0(t-t')} \tag{A.7}$$

The result of the last integral in the complex k_0 plane is (given that the poles are shifted as in (A.6))

$$\frac{i}{2k}\frac{1}{2\pi}\left[\vartheta(t-t')(-2\pi i)e^{-ik(t-t')} - \vartheta(t'-t)(2\pi i)e^{ik(t-t')}\right] \tag{A.8}$$

Thus

$$i\Delta_0^{++}(x-x') = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \frac{1}{2k}\left[\vartheta(t-t')e^{-ik(t-t')} + \vartheta(t'-t)e^{ik(t-t')}\right] \tag{A.9}$$

$$= \frac{1}{2(2\pi)^{D-1}} \int_0^\infty \frac{dk k^{D-2}}{2k} \int_0^{2\pi} d\vartheta_1 \int_0^\pi d\vartheta_2 \sin\vartheta_2 \cdots \int_0^\pi d\vartheta_{D-2} \sin^{D-3}\vartheta_{D-2} e^{ik\|x-x'\|\cos\vartheta_{D-2}} \times$$
$$\times \left[\vartheta(t-t')e^{-ik(t-t')} + \vartheta(t'-t)e^{ik(t-t')}\right] \quad \text{(A.10)}$$

The part with the ϑ functions can be rewritten in terms of the absolute value of the time difference. Besides, in order to maintain the integral finite we insert a factor $e^{-\varepsilon k}$, which just gives an imaginary shift in the time difference.

$$= \frac{1}{2(2\pi)^{D-1}} \int_0^\infty \frac{dk k^{D-2}}{2k} \int_0^{2\pi} d\vartheta_1 \int_0^\pi d\vartheta_2 \sin\vartheta_2 \cdots \int_0^\pi d\vartheta_{D-2} \sin^{D-3}\vartheta_{D-2} e^{ik\|x-x'\|\cos\vartheta_{D-2}} \times$$
$$\times\, e^{-ik(|t-t'|-i\varepsilon)} \quad \text{(A.11)}$$

The integral over ϑ_{D-2} is

$$\int_0^\pi d\vartheta \sin^{D-3}\vartheta e^{ik\|x-x'\|\cos\vartheta} = \int_{-1}^1 dq(1-q^2)^{\frac{D-4}{2}} e^{ik\|x-x'\|q} \quad \text{(A.12)}$$

$$= \sqrt{\pi} \quad \Gamma\left(\frac{D-2}{2}\right)\left(\frac{k\|x-x'\|}{2}\right)^{\frac{3-D}{2}} J_{\frac{D-3}{2}}(k\|x-x'\|) \quad \text{(A.13)}$$

At this point we can use the formula for the integral over k of the form

$$\int_0^\infty e^{-\alpha x} J_\nu(\beta x)x^\nu = \frac{(2\beta)^\nu \Gamma(\nu+\frac{1}{2})}{\sqrt{\pi}(\alpha^2+\beta^2)^{\nu+\frac{1}{2}}}$$

Substituting $\alpha = i(|t-t'|-i\varepsilon)$, $\nu = \frac{D-3}{2}$ and $\beta = \|x-x'\|$ we get:

$$\frac{(2\|x-x'\|)^{\frac{D-3}{2}}\Gamma(\frac{D-2}{2})}{\sqrt{\pi}(-|t-t'|^2 - i\varepsilon + \|x-x'\|^2)^{\frac{D-2}{2}}} \quad \text{(A.14)}$$

that, including the constants comining from (A.13) and from the angular integrations of (10) finally gives:

$$\frac{2^{-3}\Gamma(\frac{D-2}{2})}{(\Delta x^2_{++})^{\frac{D-2}{2}}} \frac{\Gamma(\frac{D-2}{2})}{\pi^{D-1}} \Omega_{D-2} = \frac{\Gamma(\frac{D-2}{2})}{4\pi^{D/2}} \frac{1}{(\Delta x^2_{++})^{\frac{D-2}{2}}} \quad \text{(A.15)}$$

This means that the prescription that we guessed in A.6 is indeed correct since it gives the original propagator in real space.

A.3 The graviton propagator in momentum space

At this point we are ready to compute the whole graviton propagator in momentum space. To do so we just have to substitute

$$i\Delta_0^{++}(x-x') = \int \frac{d^D}{(2\pi)^D} k i\tilde{\Delta}_0^{++}(k) e^{ik\cdot(x-x')}$$

in both terms of (1). The first one is trivial and gives:

$$\begin{aligned}(-2\hbar\kappa^2) \int \frac{d^D k}{(2\pi)^D} \left(\eta_{\rho(\alpha}\eta_{\beta)\sigma} - \frac{1}{D-2}\eta_{\rho\sigma}\eta_{\alpha\beta}\right) i\tilde{\Delta}_0^{++}(k) e^{ik\cdot(x-x')} = \\ = (-2\hbar\kappa^2) \int d^D k \left(\eta_{\rho(\alpha}\eta_{\beta)\sigma} - \frac{1}{D-2}\eta_{\rho\sigma}\eta_{\alpha\beta}\right) \frac{-i}{k_\mu k^\mu} e^{ik\cdot(x-x')}\end{aligned} \quad (A.16)$$

The second involves a convolution of two scalar Keldysh propagators and gives:

$$\begin{aligned}&4(1-\xi)\hbar\kappa^2 \int d^D k \int d^D k' \int d^D z \quad \partial_{(\rho}\eta_{\sigma)(\alpha}\partial_{\beta)} i[\tilde{\Delta}_0^{++}(k)\tilde{\Delta}_0^{++}(k') - \tilde{\Delta}_0^{+-}(k)\tilde{\Delta}_0^{-+}(k')] e^{-ik(x-z)} e^{-ik'(z-x')} \\ &= 4(1-\xi)\hbar\kappa^2 \int d^D k \quad \partial_{(\rho}\eta_{\sigma)(\alpha}\partial_{\beta)} i[\tilde{\Delta}_0^{++}(k)\tilde{\Delta}_0^{++}(k) - \tilde{\Delta}_0^{+-}(k)\tilde{\Delta}_0^{-+}(k)] e^{-ik(x-x')} = \\ &= 4(1-\xi)\hbar\kappa^2 \int d^D k \quad i[-k_{(\rho}\eta_{\sigma)(\alpha}k_{\beta)}][\tilde{\Delta}_0^{++}(k)\tilde{\Delta}_0^{++}(k) - \tilde{\Delta}_0^{+-}(k)\tilde{\Delta}_0^{-+}(k)] e^{-ik(x-x')}.\end{aligned} \quad (A.17)$$

To find the final expression for the propagator in momentum space is just matter to substitute the expressions for the Keldysh scalar propagators with the correspondent $i\varepsilon$ prescriptions. For the $\tilde{\Delta}_0^{+-}(k)$ and $\tilde{\Delta}_0^{-+}(k)$ of course the prescription is different from the one for the time ordered propagator, and it corresponds to consider alternatively only one of the poles in (A.6).

So, namely,

$$i\tilde{\Delta}_0^{+-}(\vec{k}, \Delta t) = \frac{1}{2k} e^{-ik\Delta t}$$

$$i\tilde{\Delta}_0^{-+}(\vec{k}, \Delta t) = \frac{1}{2k} e^{ik\Delta t}$$

Which can be expressed, respectively, as:

$$i\tilde{\Delta}_0^{+-}(k) = \int d(\Delta t) \quad e^{ik_0\Delta t} \frac{1}{2k} e^{-ik\Delta t} = \frac{\pi}{k}\delta(k_0 + k) = 2\pi\delta(k_\mu k^\mu)\theta(-k_0)$$

$$i\tilde{\Delta}_0^{-+}(k) = \int d(\Delta t) \quad e^{ik_0\Delta t} \frac{1}{2k} e^{ik\Delta t} = \frac{\pi}{k}\delta(k_0 - k) = 2\pi\delta(k_\mu k^\mu)\theta(k_0)$$

When we take the product of these two terms, we have:

$$\tilde{\Delta}_0^{+-}\tilde{\Delta}_0^{-+} = 4\pi^2\delta(k_\mu k^\mu)\delta(k_\mu k^\mu)\theta(-k_0)\theta(k_0) = 0$$

So the final result for the time ordered graviton propagator in momentum space is:

$$i[_{\rho\sigma}\Delta^{++}_{\alpha\beta}](k) = (-2\hbar\kappa^2)\left(\eta_{\rho(\alpha}\eta_{\beta)\sigma} - \frac{1}{D-2}\eta_{\rho\sigma}\eta_{\alpha\beta}\right)\frac{-i}{k_\mu k^\mu - i\epsilon} - 4(1-\xi)i\hbar\kappa\frac{k_{(\rho}\eta_{\sigma)(\alpha}k_{\beta)}}{(k_\mu k^\mu - i\epsilon)^2} \tag{A.18}$$

provided that we use the correct $i\epsilon$ prescription of (A.6). To obtain the anti-Feynman graviton propagator and the Wightman functions we only need the anti-Feynman scalar propagator $\tilde{\Delta}_0^{--}(k_\mu)$. This can be obtained in a completely analogous fashion as it was done for the time ordered scalar propagator in section 2. The prescription for such propagator will simply be $k_\mu k^\mu \to k_\mu k^\mu + i\epsilon$ i.e. $i\tilde{\Delta}_0^{--}(k_\mu) = \frac{i}{k_\mu k^\mu + i\epsilon}$. Thus, substituting this expression into A.1, we have:

$$i[_{\rho\sigma}\Delta^{--}_{\alpha\beta}](k) = (-2\hbar\kappa^2)\left(\eta_{\rho(\alpha}\eta_{\beta)\sigma} - \frac{1}{D-2}\eta_{\rho\sigma}\eta_{\alpha\beta}\right)\frac{i}{k_\mu k^\mu + i\epsilon} + 4(1-\xi)i\hbar\kappa\frac{k_{(\rho}\eta_{\sigma)(\alpha}k_{\beta)}}{(k_\mu k^\mu + i\epsilon)^2} \tag{A.19}$$

For the Wightman function we have to calculate the quantity

$$[\tilde{\Delta}_0^{++}(k)\tilde{\Delta}_0^{+-}(k) - \tilde{\Delta}_0^{+-}(k)\tilde{\Delta}_0^{--}(k)] = -2\pi\delta(k_\mu k^\mu)\theta(-k_0)\left[\frac{1}{k_\mu k^\mu - i\epsilon} + \frac{1}{k_\mu k^\mu + i\epsilon}\right]$$

Then we can write the resulting expression:

$$\begin{aligned} i\tilde{\Delta}^{+-}(k_\mu) = 2\pi\delta(k_\mu k^\mu)\theta(-k_0)\Big\{&(-2\hbar\kappa^2)\Big(\eta_{\rho(\alpha}\eta_{\beta)\sigma} - \frac{1}{D-2}\eta_{\rho\sigma}\eta_{\alpha\beta}\Big) + \\ &+ 4(1-\xi)\hbar\kappa^2 i[k_{(\rho}\eta_{\sigma)(\alpha}k_{\beta)}]\Big[\frac{1}{k_\mu k^\mu - i\epsilon} + \frac{1}{k_\mu k^\mu + i\epsilon}\Big]\Big\} \end{aligned} \tag{A.20}$$

And a similar expression holds for the other one:

$$\begin{aligned} i\tilde{\Delta}^{-+}(k_\mu) = 2\pi\delta(k_\mu k^\mu)\theta(k_0)\Big\{&(-2\hbar\kappa^2)\Big(\eta_{\rho(\alpha}\eta_{\beta)\sigma} - \frac{1}{D-2}\eta_{\rho\sigma}\eta_{\alpha\beta}\Big) + \\ &+ 4(1-\xi)\hbar\kappa^2 i[k_{(\rho}\eta_{\sigma)(\alpha}k_{\beta)}]\Big[\frac{1}{k_\mu k^\mu - i\epsilon} + \frac{1}{k_\mu k^\mu + i\epsilon}\Big]\Big\} \end{aligned} \tag{A.21}$$

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