UTRECHT UNIVERSITY

The influence of grain size on densification of firn

by

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Abstract

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In this thesis we have made a model for the densification of firn under overburden pressure. In this model the grain size growth speed is only influenced by temperature. Next a new model for grain size growth is researched, also taking into account the temperature gradient and the density. However, the results from this model correspond less well with the data of four Antarctic ice cores then the original model.

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Chapter 1

Introduction

1.1 Antarctic mass balance

In order to make predictions about the future sea level change it is extremely important to understand the distribution of ice across the Antarctic continent. Nowhere in the world, more solid water than on Antarctica can be found therefore a small change in the mass of the Antarctic ice can lead the a substantial rise of the sea level. However, measuring the mass of ice spread over an entire continent is all but trivial. Satellite altimeters can be used to measure the height of the ice sheet. The lower part of the ice sheet is composed of ice of known density. The upper tens of meters are made of firn, snow still compressing to ice. Since we only know the volume of the ice sheet, in order to calculate the mass, we need to know the density throughout the sheet. Therefore errors in the density and the thickness of the firn layer lead to errors in the mass of the layer, and thereby in thereby in the mass of the entire ice sheet. The relative error of a firn layer of 100 m on the mass of an ice sheet of a few kilometers is not that big, however we are mostly interested in the changes in the mass, which means this effect becomes significant.

1.1.1 Antarctic water cycle

Since on almost the entire continent the temperature hardly ever rises above 273 K, melt is virtually non-existent. The water cycle goes therefore mostly as follows. Above the seas surrounding Antarctica water evaporates. This precipices on Antarctica, mostly in the form of snow. Not that this is very much, therefore Antarctica is often called the largest desert in the world. Since this snow little evaporates and melts it stays there until new snow fall on top it. Then due to the weight of the overlying snow layers, the firn compresses. This process continues until the snow becomes ice. This ice flows from the center to the edges where it breaks off into icebergs and melts in the sea.

1.1.2 Densification of Firn

Because the temperatures and accumulation rates in Antarctica are low, densification is a slow process. It can take up to centuries or even millennia, until fresh snow, with a density between 200 and 500 kgm^{-3}, is compressed to glacier ice, with a density of more than 900 kgm^{-3}. To complicate matters, the accumulation rate and temperature vary largely across the continent. This means that the upper layer, consisting of firn has a size that can vary between in the order of ten meter along the relatively warm coastline and more than 100 meter in the much colder interior [S.R.M. Ligtenberg and van den Broeke, 2011]. In a thick firn layer, more air is contained than in a shallow firn layer. This mean that changes in firn densification can change the snow surface without changing the snow mass.

1.2 Current densification models

Considering the above mentioned importance of understanding the densification, it should be no surprise that a lot of models have been proposed to understand this behavior. The firn densification process is however, not yet fully understood. Most models are semi-empirical. Some are purely empirical, others are based on sintering theories, but have their coefficients bases on laboratory experiments or ice core data. All the models agree reasonably well with ice core data, but the differ enormously in their sensitivity on physical conditions such as temperature and accumulation rate. So while we can reasonably model densification, the underlying physics is not yet resolved. An (incomplete) list of current densification models can be found in [Robert J. Arthern and Thomas, 2010].

1.3 Model

The model used in this thesis is an extension on the model proposed in [Robert J. Arthern and Thomas, 2010]. In this semi-empirical model they combine an expression for creep in a medium with cylindrical pores with the equation for Nabarro-Herring creep. In this model compactation is dependant of density, temperature, overburden pressure and grain size. In the model of [Robert J. Arthern and Thomas, 2010] a rather simple equation for grain size growth is used.

In their paper, [Flanner and Zender, 2006] propose a more complicated model for grain size growth. This model takes into account a variety of parameters, like temperature gradient and density. This research strives to combine the above mentioned densification model with two different models for grain size growth. Both models are then used to fit the modelled density profile to a detailed density profile from ice core measurements. This is the compared with to see whether this gives better results.

Chapter 2

Theory and model

2.1 Theory

2.1.1 Nabarro-Herring Creep

In this model we assume the densification is caused by Nabarro-Herring creep[Coble, 1970] which means it obeys the following equation:

$$\frac{d\rho}{dt} = k_c(\rho_i - \rho)exp(-E_c/RT)\sigma/r^2 \tag{2.1}$$

with r the grain radius, ρ the density of the firn, ρ_i the density of ice, , R is the gas constant, T is the temperature, σ the overburden pressure due to overlying snow layers, E_c the activation energy for self-diffusion of water molecules through the ice and k_c is a constant which has two values, one for low density firn, and one for high density snow. Nabarro-Herring creep is a form of diffusion creep. The atoms diffuse trough the firn, thereby filling up the holes between the grains. It describes granular sliding across dimensions comparable to the grain size. This model assumes the grains are jammed around the cylindrical pore, so first diffusion must occur before sliding is possible.

2.1.2 Snow loading

The equation for the overburden pressure is rather trivial. Since it is an one-dimensional model all the snow that was above a certain layer remains on top of it, so any change in the pressure must come from the addition of new snow. Therefore the equation becomes:

$$\frac{d\sigma}{dt} = \dot{b}g \tag{2.2}$$

with $\dot{b}$ the accumulation rate, and g the gravitational acceleration.

2.1.3 Temperature

The temperature satisfies the heat-conduction equation:

$$\rho c \frac{dT}{dt} = \frac{\partial}{\partial z}[\kappa \frac{\partial}{\partial z} T] \tag{2.3}$$

with c the heat capacity of ice, and κ the thermal conductivity of snow. Air is a good isolator compared to ice. The thermal conductivity is thus dependant on the amount of air in the firn, which means it is a function of density. [N. Calonne and Geindreau, 2011] gives for this

$$\kappa = 2.5 \times 10^{-6}\rho^2 - 1.23 \times 10^{-4}\rho + 0.024 \tag{2.4}$$

with ρ in kgm^{-3} and κ in $Wm^{-1}K^{-1}$.

2.1.4 Simple grain growth

[Robert J. Arthern and Thomas, 2010] use in their paper for grain growth the following equation:

$$\frac{dr^2}{dt} = k_g exp(-E_g/RT) \tag{2.5}$$

with k_g a constant and E_g the activation energy for grain growth. [Robert J. Arthern and Thomas, 2010] simplify this further, assuming that effects of temperature fluctuations near the surface can be neglected once intergraded over sufficient time. They replace the temperature T with the mean annual temperature T_{av}. In this research this simplification is not made. Since the purpose of this research is to compare two models for grain size, simplifications in one of the two models can alter the results.

2.1.5 Complex grain growth

In this thesis we test a new model (the SNICAR model) proposed by [P. Kuipers Munnke and van de Berg, 2011] based on research by [Flanner and Zender, 2006]. They suggest the following equation:

$$\frac{dr}{dt} = (\frac{dr}{dt})_0 \frac{\eta}{(r - r_0) + \eta}^{\frac{1}{\kappa}} \tag{2.6}$$

with r_0 the initial size of fresh grains,$(\frac{dr}{dt})_0$, η and κ are coefficients retrieved from a lookup table with the dimensions temperature, temperature and density, which essentially means that the entire formula is a lookup table. This model has the following general behaviour A larger temperature gradient causes a larger concentration gradient, because the difference in evaporation will be stronger. This will lead to more diffusion and this will lead to a larger growth rate. Also a lower density leads to a larger growth rate. This table is made using laboratory results, and as mentioned by [P. Kuipers Munnke and van de Berg, 2011] the model fits the results very accurate, with a mean square error of approximately 3 percent.

2.2 Data

2.2.1 Roosevelt Island

The data used in this thesis comes from two different origins. The first dataset comes from Roosevelt Island. It was collected by [Conway] between November and December of 1997. Firn cores were taken from Roosevelt Island, which is an ice dome within the Ross Ice Shelf. The core was divided into sections of between 20 cm and 50 cm. Of each section the density was measured. Three cores were collected to a depth of approximately $17m$. All three cores were used in this research. For the mean annual temperature, the annual temperature fluctuation amplitude and the accumulation rate, the data of RACMO2 were used, which were modelled by [J.T. M. Lenaerts and Munneke, 2012]. They were 246.0 K, 15.9 K and $240.8kgm^{-2}a^{-1}$ respectively

2.2.2 Siple Dome

The second source of data is measured by Dr. Gregg Lamorey and Mr. David White in december 1996 in Siple Dome. This is an ice dome near the coast of western Antarctica. A similar procedure as in the previous section was used, only here the sampling interval was approximately 1 m. Here also weather data from RACMO2 was used. This gave values of 234.48 K, 16.9 K and $15.08kgm^{-2}a^{-1}$. Here only one of the profiles was used.

2.3 Model

2.3.1 Algorithm

As mentioned before, an one-dimensional model is used. First it computes the initial conditions: a pack of 100 layers of snow, each layer with a mass of $10kg$. A slight temperature gradient is introduced to exclude problems with zero temperature gradients. Then equation 2.1 up to and including 2.6 are computed, as well as the accumulation in that period. If the total accumulation exceeds a certain value, a new layer is added, shifting the other layers one layer down. The new layer starts with a grain size and density corresponding to the initial grain size and density chosen in the model. The temperature of the layer is the air temperature, modelled by the annual temperature with a sinusoidal variation. This is repeated over 200000 or 3000000 timesteps, each of $\frac{1}{2000}th$ of a year.

2.3.2 Optimization

Next the initial grain size is optimized using a golden ratio search. This search algorithm works under the assumption that the optimizing function, in this case the mean square difference between the model and the measurements, is unimodal. First for a given density two boundaries, r_1 and r_2 are chosen. Next two other points are chosen, r_3 and r_4 which are related by the fact that $\frac{r_4-r_3}{r_3-r_1} = \frac{r_3-r_1}{r_2-r_1} = \varphi$ hence the name golden ratio search. Then if $f(r_3)$ is smaller than $f(r_4)$ then, because the function is unimodal, the minimum must lie between r_1 and r_4 and the procedure is repeated with the new boundaries and of course using the known value of $f(r_3)$ as one of the intermediate points. Of course if $f(r_3)$ is larger than $f(r_4)$ it works the other way around.

Using this procedure we have determined the optimal grain size for a given density. Then this search algorithm is repeated, this time optimizing the density. Still for each density the grain size is first optimized using a golden ratio search.

2.3.3 Constants and initial conditions

Since for each site the temperature, annual temperature fluctuations and the accumulation rates are known, these values are used, assuming sinusoidal temperature variation and constant snowfall. This last assumption is not very realistic, but this does not influence the results very much. However since the heat conductivity equation is a differential equation also a boundary condition at the bottom of the pack is necessary. Fortunately from heat theory is known that the amplitude decreases exponentially for increasing depth, so we can safely assume constant temperature at the bottom of the snow pack. The initial snow pack has a density equal to the density of fresh snow, which follows from the optimizing search. The initial value for the grain size comes also from the search. The value of k_c in equation 2.1 is not constant. According to the research done by [Robert J. Arthern and Thomas, 2010] it is approximately $9.2 \times 10^{-9} kg^{-1} m^3 s$ for densities below $550 kgm^{-3}$ and approximately $3.7 \times 10^{-9} kg^{-1} m^3 s$ for densities above. Following a line of reasoning in [Flanner and Zender, 2006], this sudden change might be because this density is approximately the density of a stack of ice spheres of equal size. There is a laboratory-bases estimate of k_c of $1.6 \times 10^{-10} kg^{-1} m^3 s$, but here we give preference to measurements in the field, so in this research the values from [Robert J. Arthern and Thomas, 2010] are used. Further used are the values of respectively $42.4 kJmol^{-1}$ and $60.0 kJmol^{-1}$ for E_g and E_c. For the gas constant R the value of $8.3145 JK^{-1} mol^{-1}$ is used. Other constants used are $k_g = 1.3 \times 10^{-7} m^2 s^{-1}$, $g = 9.81 ms^{-2}$, $\rho_i = 917 kgm^{-3}$ and $c = 2.0 \times 10^3 Jkg^{-1} K^{-1}$.

Chapter 3

Results

3.1 Model differences

A typical temperature profile is shown in figure 3.1. This is a rather standard profile for these circumstances (an one-dimensional solid with a flux of heat at the top layer which varies sinusoidal in time). As can be seen this results in a temperature profile which varies sinusoidal in depth, but with a exponentially decreasing amplitude. To illustrate this, the amplitude at the top is equal to the annual temperature amplitude of 16 K. Over a depth of only half a meter, this decreases to only 6 K.

The comparison between the models can be seen in figure 3.2. These graphs give the value of r^2 (and thereby the grain size area) plotted against the depth. For each model a fit to the Roosevelt Island 210 core was performed. The results of this model were used for these graphs. This also means they have a slightly different initial value. The flat section at the highest depths are model anomalies. The differences are clear. While the Standard Grain Model (SGM) shows a slightly increasing slope, the Complex Grain model starts with a rapid slope, which subsequently slowly decreases. This can easily be explained. As can be seen, equation 2.5 is for constant temperature(which is practically the case for any depth greater than 4 m) a linear equation. The fact that the slope is still slightly increasing comes because this is a linear equation in time. From mass conservation Sorge's Law [Bader, 1960] can be deduced:

$$v(z) = \frac{\dot{b}}{\rho(z)} \tag{3.1}$$

The equation is only valid under the assumption of an unchanging 'steady state', but is still approximately correct. And since density increases with increasing depth, speed decreases and therefore grain size growth per meter grows as well.

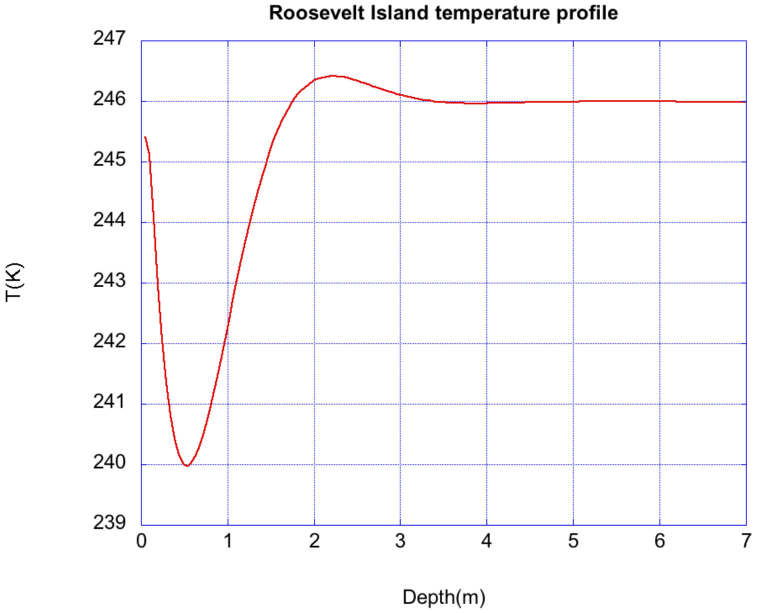

FIGURE 3.1: Typical temperature profile

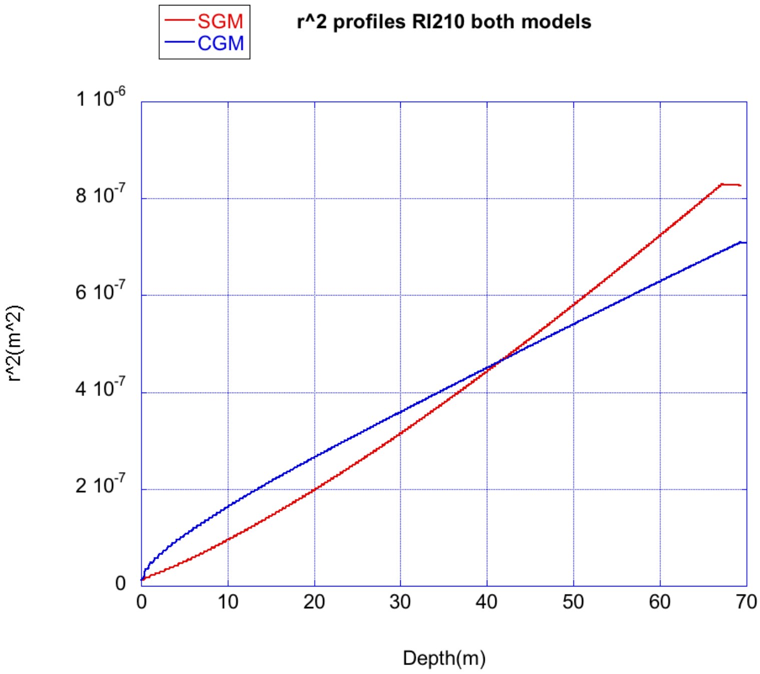

FIGURE 3.2: Overview of grain area for different models

As mentioned in 2.1.5, the complex grain size model predicts a higher grain size growth with increasing temperature gradient en decreasing density. Since both the temperature gradient(see again figure 3.1) decreases and the density increases with increasing depth, the growth rate is for low depth very high, because of the enormous temperature gradient, and then decreases for higher depths. The effects of Sorge's Law, as mentioned above, temper this effect partially, but not entirely.

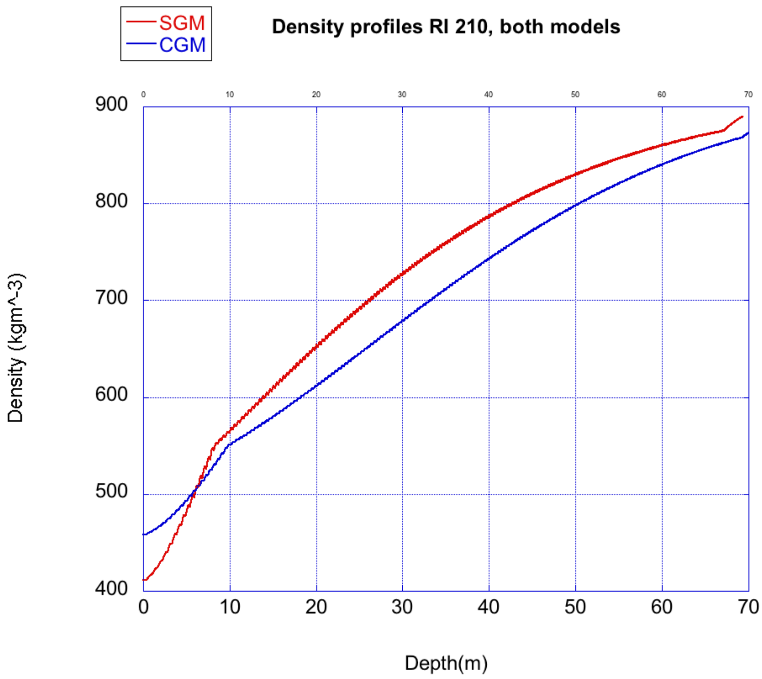

FIGURE 3.3: Density profiles for both models

The typical density profile can be seen in figure 3.3. Both models render the same general behaviour. At first, the densification is very slow, but rapidly increasing. This is because the pressure is increasing relatively rapidly. Next the density reaches $550 kgm^{-3}$. This means the constant k_c in equation 2.1 changes it value, leading to an instant decrease in densification rate. Following that the point is reached where the effect of the growth of the grain size exceeds the increase in pressure leading to a decrease in densification rate. Furthermore an other effect can be seen; the annual temperature fluctuation which show up, like the annual rings in a tree, in the modelled core. This is caused by two effects. First of all the densification itself is temperature dependant, which results in layers that spend a relative long time in warmer regions having a higher density. However, this effect is one-off, and gets soon averaged out and dominated by the other effect. This is fact that layers that spend a relative long time in warmer regions have also a larger grain size. This is a persistent effect and is therefore dominating at depths higher than a few meters. However, this effect gets slightly thwarted, and this prevents it from exploding, which it, as can be seen in the figure, clearly does not do. Since equation 2.1 is density dependant, layers with a high density experience a lower densification rate, and thus the density difference are smoothed out. This is also why the fluctuations are much smaller for higher densities.

Knowing the characteristics of the profiles it is easy to explain the differences. Since the grain size is much higher in the CGM model for low depths, densification goes slower. However, since the grain size increases less in the CGM, the second derivative of the density is much lower. At a depth of approximately 32 m, their densification rates are equal; the by this time only slightly higher grain size of the CGM is compensated by its lower pressure(because the overlying layers have a lesser density). After that the CGM has a higher densification rate because of its lower grain size.

3.2 Comparison with measurements

In figures 3.4 up to and including 3.11 the results of the comparison of both models with measurements. For each core the best fit is shown, as well as the original data. The difference between figure 3.4 and figure 3.5 is striking. While the SGM model line in 3.5 looks like represents the depth averaged density, the CGM model line in 3.4 looks more like a more or less randomly drawn line, which only approximately has the same slope. The same can be seen in figures 3.6 and 3.7, and to a lesser extent also in figures 3.8 and 3.8.

But also other features catch the eye. While the model predicts the trend very well, the annual oscillations do not match. However upon looking more clearly it can be seen that the period of these fluctuations have a period of a least two year. The measurements are not sensitive to distinguish the annual oscillations. The fluctuations seen in the measurements are probably due to variations in the yearly snowfall and temperature.

The values for initial density and grain size are listed in tables 3.1 and 3.2.

Ice Core	$r_{0,SGM} 10^{-4} m$	$r_{0,CGM} 10^{-4} m$
Roosevelt Island	1.06	1.09
Roosevelt Island 210	1.10	1.09
Roosevelt Island 350	1.06	1.08
Siple Dome H	1.81	1.86

Table 3.1: Initial values for grain size for each best fit

In table 3.2 can be seen that initial values of the grain size seems to be approximately equal. The initial values for the density however are not. This is exactly what we expect since the Complex growth model gives a lower densification rate for low depths, so in

Ice Core	$\rho_{0,SGM}(kgm^{-3})$	$\rho_{0,CGM}(kgm^{-3})$
Roosevelt Island	392	458
Roosevelt Island 210	411	458
Roosevelt Island 350	382	443
Siple Dome H	357	445

TABLE 3.2: Initial values for density for each best fit

order to have approximately the same density in the middle of the ice core, at 10 m, the initial density of the Complex growth model must be greater.

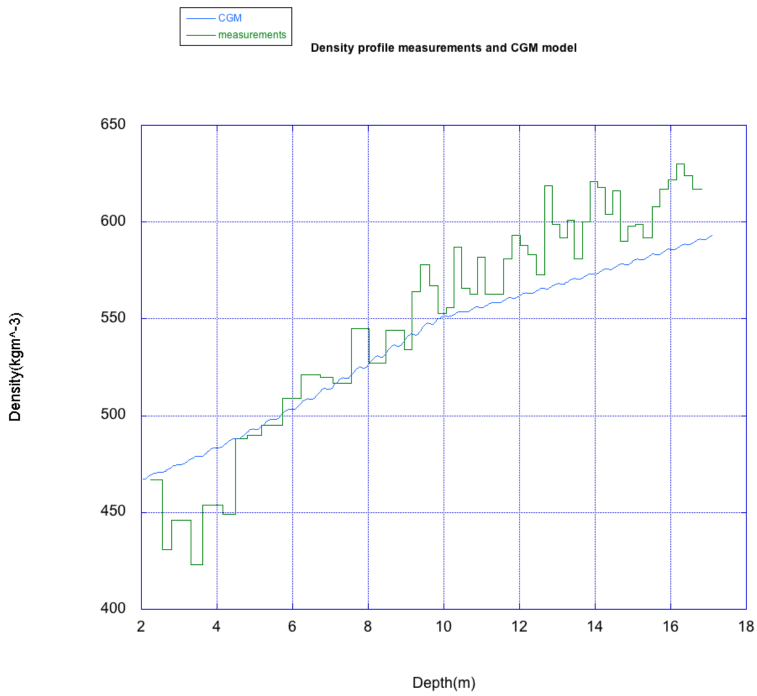

FIGURE 3.4: Comparison of CGM model with measurements of Roosevelt Island 210

Further research has shown that the assumption to use constant temperature made by [Robert J. Arthern and Thomas, 2010] referred to in section 2.1.4 is not correct. First of all, as mentioned before in this section, the density fluctuations due to seasonal temperature fluctuations remain present in the density proflie down to a depth of a least 100 m Further modelling has learned that apart from missing this behaviour, models not taking into account the seasonal fluctuation in temperature predict values for the density which are higher by approximately $10kgm^{-3}$ for depths of $50 - 100m$.

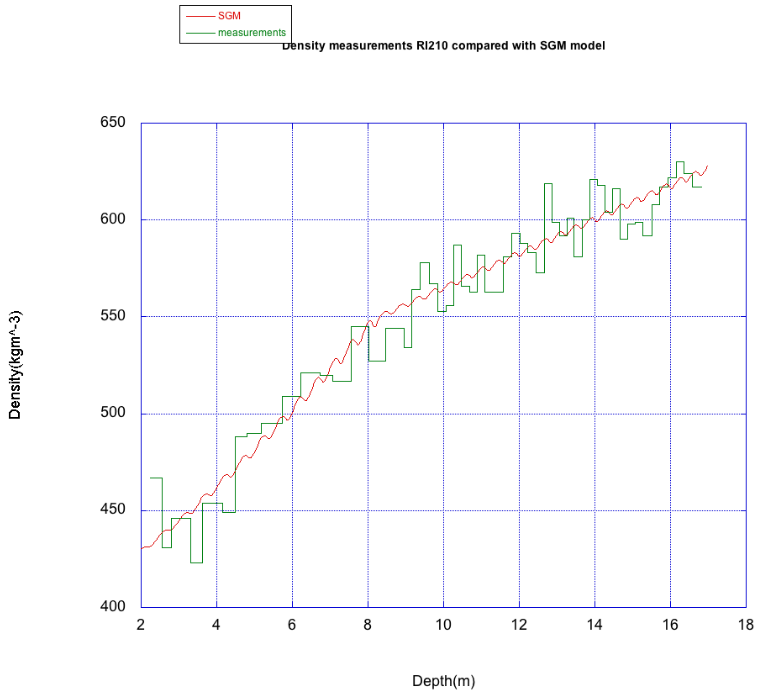

FIGURE 3.5: Comparison of SGM model with measurements of Roosevelt Island 210

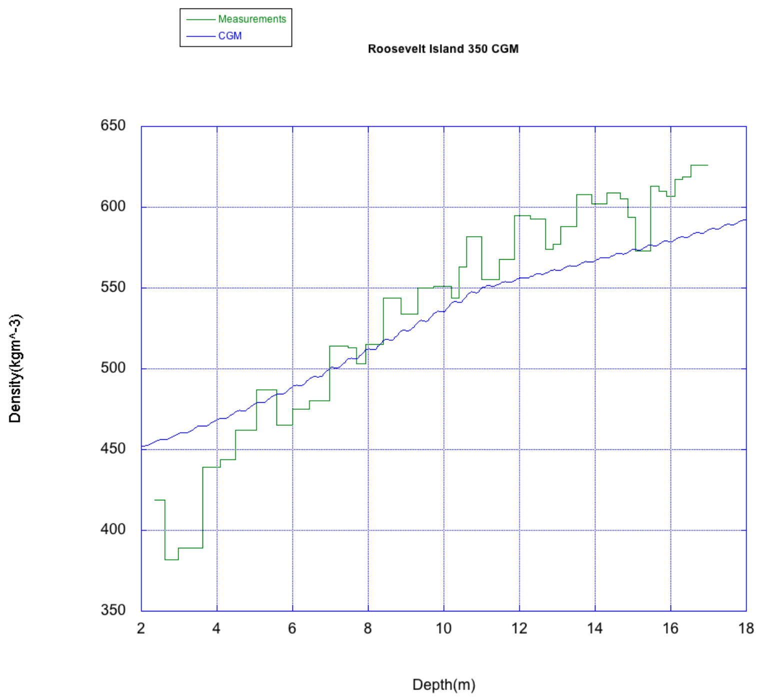

FIGURE 3.6: Comparison of CGM model with measurements of Roosevelt Island 350

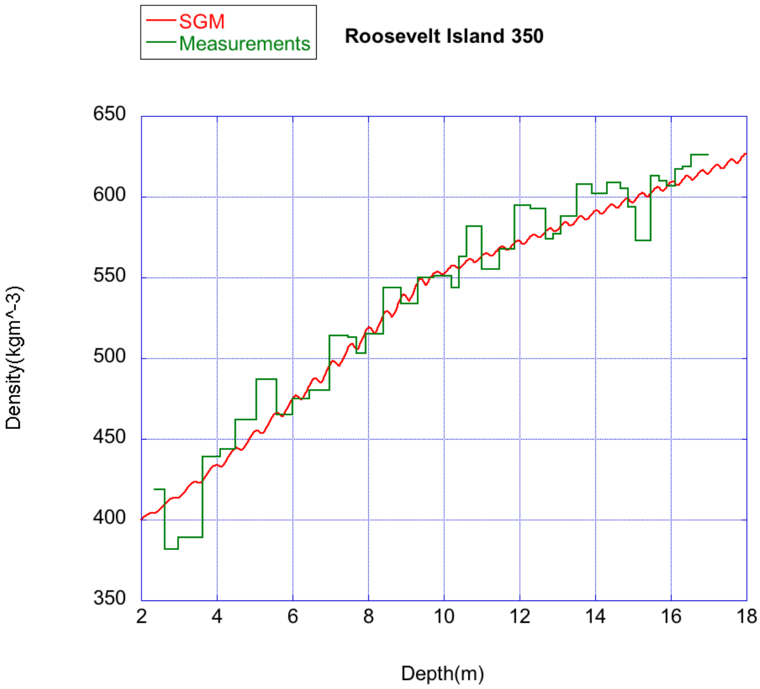

FIGURE 3.7: Comparison of SGM model with measurements of Roosevelt Island 350

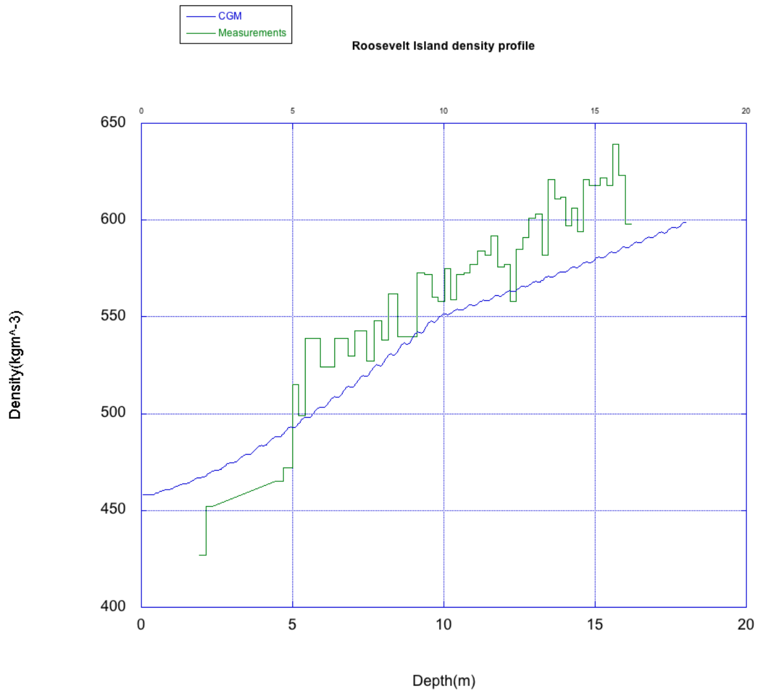

FIGURE 3.8: Comparison of CGM model with measurements of Roosevelt Island

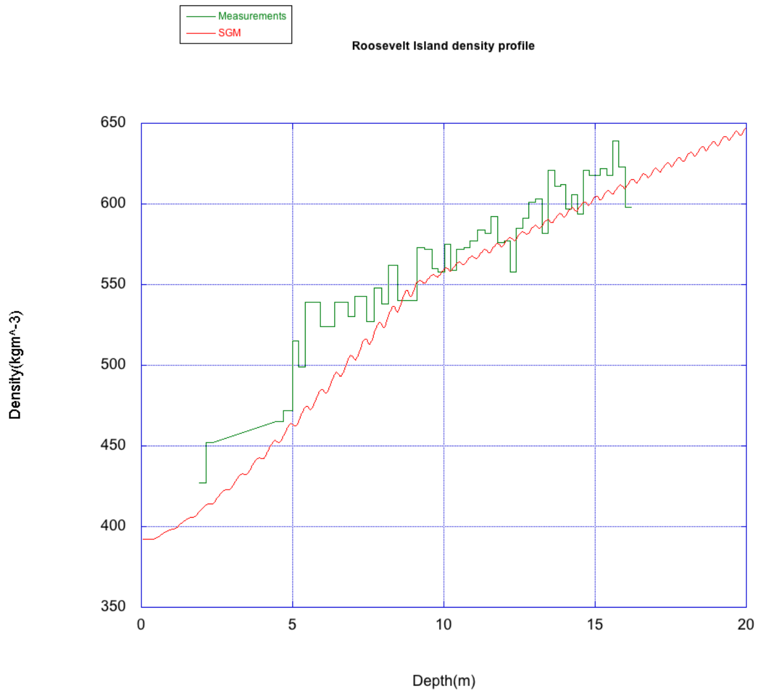

Figure 3.9: Comparison of SGM model with measurements of Roosevelt Island

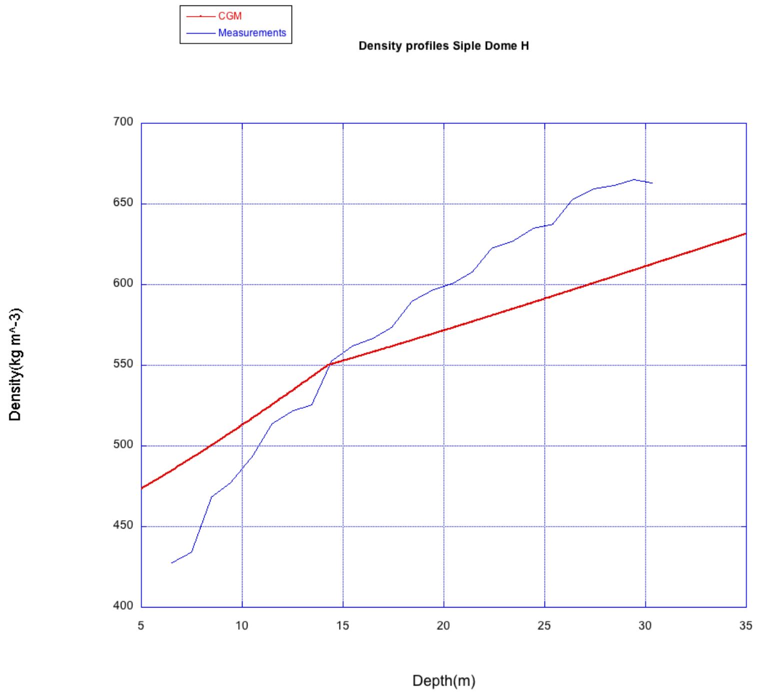

Figure 3.10: Comparison of SGM model with measurements of Siple Dome H

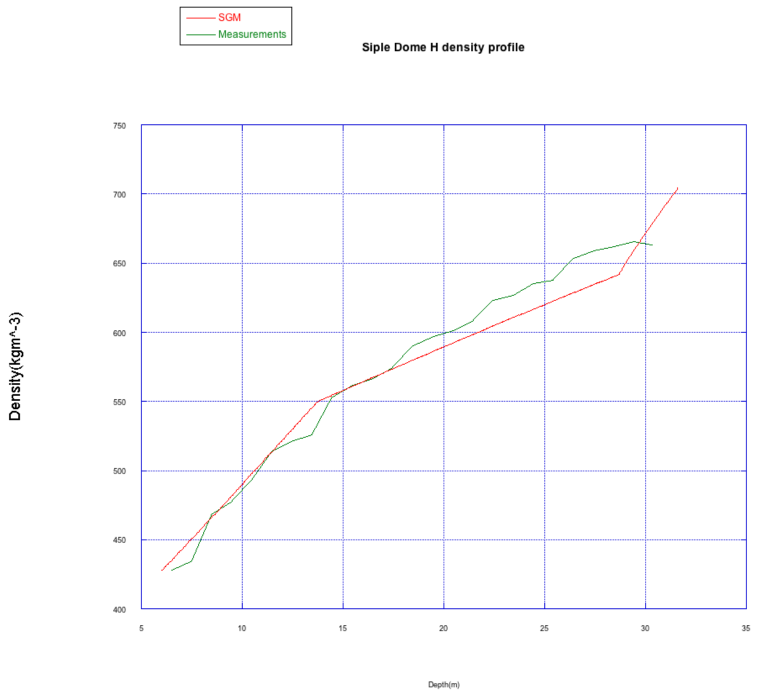

FIGURE 3.11: Comparison of SGM model with measurements of Siple Dome H

Chapter 4

Discussion

4.1 Result Processing

In order to quantify the difference between the model and the measurement, for each ice core the square distance with the model is calculated for each measurement of the density. This is them averaged over all data points. The results can be found in table 4.1.

Ice Core	Simple Grain Model	Complex Grain Model
Roosevelt Island	12	14
Roosevelt Island 210	7	14
Roosevelt Island 350	8	18
Siple Dome H	10	29

Table 4.1: Mean square differences between models and ice core measurements

First of all, it can be seen that both models work rather well. Only the CGM modelling of the Siple Dome H core is different is such a way we can dismiss the model. However as was already visible in the preceding section, the SGM model gives far better results then the CGM model. In three of the four cores the mean square difference is lower by a factor two. With the fourth core, the SGM model is still better, and the fact that the CGM model is not much worse, is more because the SGM model gives below-average results then because the CGM is excelling.

4.2 Limitations

Like any model, this model has its limitations. First of all, it does not result in a continues depth profile, the only thing you know is points. In the z-direction, this poses not much of a problem. Simulations with more datapoints gave the same results. The same goes for the timesteps. Shorter timesteps do not alter the results very much. Longer timesteps do however. Longer timesteps results in a no longer converging solution. This influences largely the time needed for a simulation.

A more severe problem is of course that the assumption that the weather is constant is far from valid. Any deviations of the measured values form the model can come from both a flaw in the model and a sudden change in temperature, accumulation rate or even possibly the initial grain size and the initial density. However the differences between the two models, under the current densification model and using this data, is sufficiently large in order not to influence the conclusions.

Also a big problem is the fact the model computes with non-physical equations. Equations of which we do not even know if they are valid under these conditions. If equation 2.1 is only slightly incorrect, then this effect is completely dominating any effects from flaws in the grain size models.

Furthermore, as can be seen in figure 3.4 the density profiles are not detailed enough to incorporate the yearly fluctuations. Since this inhabits a large part of information about the temperature dependence, this an huge loss.

The conclusions are also negatively influenced by the presence of only four ice cores, from only two independent sources. It would be far more testing if more ice cores, from independent sources were used.

Chapter 5

Conclusions

5.1 Conclusion

Based on the data, we are save to conclude that the simple grain growth model is preferable over the complex grain growth model.

5.2 Future research

5.2.1 This thesis

As mentioned in section 4.2, a more definitive answer could be given using more data sets of independent source. This would mean more different temperatures and accumulation rate thereby being more capable to test the influence of these variables. This could also include the use of ice cores of greater depth, thereby checking the validity of the model for greater depths.

There is however also an other factor. The values of k_c given in section 2.3.3 are empirical. This value is calculated assuming the Simple Grain Growth Model. It is a rather bold assumption to assume that this constant is the same for the Complex grain growth model as well, especially since they predict so different grain sizes for low densities. Further research is needed to sort this out.

5.2.2 Other research

As mentioned in the introduction, this subject is far from finished. A lot could be contributed by more research into the equations involved in densification. A second part of research, which is closely related to the first, is the measurement of new ice cores. In any case more data is useful in solving the puzzle, high resolution density profiles are absolutely necessary in order to investigate grain size growth. Which brings me to the third part of research. The two in this thesis mentioned models for grain size growth are not the only one and both probably not correct. Therefore, with the help of high-resolution density profiles, research in this area must continue as well.

Chapter 6

Acknowledgements

I would like to thank my family, for all the support given to me these past months. Of course, this would not have been possible without the help of all the people at IMAU, for giving me a pleasant environment to work in. Also my gratitude goes to my fellow students at the IMAU-student room, for those little bits of help when needed and an always pleasant atmosphere. I also can not thank enough thank the people who were willing to endure sub-zero temperatures to gather my data, so I did not need to. But I also want to name out a few people in particular. Anna von der Heydt and Joke van Dijk, the bachelor project supervisors, for their general help during the project. Jan-Willem Meijerink, for his visible and invisible coaching and never giving up the faith. And Willem Jan van de Berg, I dare to say I could not have done this with a different supervisor.

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