Bachelor Thesis
A one-dimensional flowline model applied to Kongsvegen

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21st June 2013

Abstract

A basic version of a one-dimensional numerical model describing a glacier, provided by Prof. Dr. Oerlemans, was slightly adapted and expanded to describe a well documented glacier. Kongsvegen (Spitsbergen, Svalbard) is known to be a surging glacier. Though the internal processes causing the surge were not part of this model, surging behaviour could be simulated by temporarily increasing the sliding parameter. After approximating the local geometry and establishing the duration of a necessary initial period in which the glacier builds up a sufficient volume of ice, the current conditions could be approximated and an artificially induced surge took place. The sliding parameter was increased with a Gaussian curve, leading to a value at its peak of roughly 20 times that of the quiescent phase.

Contents

1 Introduction

Svalbard is an archipelago in the Arctic Ocean under Norwegian rule. A large portion of these islands' surface is made up of glaciers, many of which are of the surging variety. The largest island is Spitsbergen and it is on the west coast of this island that one might find the glacier known as Kongsvegen.

Glaciers can be used as indicators of climate change. Summer temperature, winter precipitation and front position of glaciers provide us with a statistical relation, enabling us to observe climate change. For one specific glacier, however, the local geometry can make it respond quite uniquely.[9] The model described in this thesis concerns only one glacier and its specific data is taken from observations of Kongsvegen. A problem with this particular glacier is that it is known to surge.

1.1 Surging glaciers

Glaciers that show non-seasonal, cyclical behaviour of advancement in response to internal processes are known as surging glaciers. Long periods of time called quiescent phases, which may last over a century, are interrupted by short periods of a few years up to a decade, known as surges. During a surge, the glacier front advances swiftly and the glacier length may increase with several kilometres in a remarkably short period of time. This behaviour is not to be confused with glaciers simply showing a fast flow which is sustained over long periods. The surge is but a phase that eventually passes and is followed by a retreating glacier length as the glacier enters a new quiescent phase. After a number of decades, another surge occurs and subsequently subsides. Nor is this behaviour to be confused with straightforward responses to climate change, such as the sudden onset of a glacial period in which glacier lengths are expected to increase due to declining temperatures. Surging behaviour occurs due to internal forces within the glacier itself, responding to climate change only indirectly. The recurrence interval is indeed influenced by climate change, and surging behaviour might be turned off completely.[10]

The workings of this phenomenon are still not fully understood. Several factors may contribute to surging behaviour and this might relate to the specific type of surge that takes place. In the quiescent phase, glaciers move through both shearing (creep) and sliding of the ice. The latter depends on the lubrication of the glacier bed. There exists a clear divide between two views on surging glaciers. The hard-bed view hypothesises the surge being caused by the deterioration of an efficient system of channels in the glacier bed, trapping water in linked cavities. The subsequent change in water pressure would then lead to an increase in glacial sliding, causing the rapid advancement. The soft-bed hypothesis states that deformation of the underlying sediment is the key factor, affecting water pressure through sediment displacement and stimulating surge behaviour in that way. The latter view was favoured for explaining Svalbard's surging glaciers by Jiskoot et al.[4] In between surges, the glacier flow is insufficient to achieve a steady state and the glacier loses ice below and builds up in its reservoir(s). For a more detailed description of surging mechanics, see, for example, the 1986 article by McMeeking and Johnson [5] or the 1989 article by Fowler[2].

Almost a hundred of Svalbard's glaciers are commonly identified as surging glaciers, the precise requirements of a surge and identifying its initial phases forming points of discussion. Estimates range between 13% and a staggering 90% of the archipelago's glaciers being of the surging variety. Furthermore, extremely long quiescent phases and changes in climate conditions might make it difficult to correctly identify surges. Svalbard's glaciers are notorious for having relatively slow surges and extremely long quiescent phases of up to 500 years. Due to these long quiescent phases, many surge events have only been observed once or might predate scientific observation entirely. Often, these surges have only been studied after the onset of the event, as surges are difficult to recognise in an early stage. Extensive crevassing is thought to be a good indication of an upcoming surge. It is usually only after a surge has been recognised that scientists study sattelite images to investigate its development.[10]

In their 2009 article, Sund et al. suggest a three stage development of surges. Stage 1 concerns surface lowering in the glacier's upper parts, while thinning down below defines stage 2. Only in the third stage does the glacier show a prominent acceleration, which is when most would recognise it as a surge. As suggested by Sund et al., some glaciers may display what they call 'partial surges'. These are defined as surges which do not pass the second stage. This can easily be misinterpreted as a response to climate change instead of a response to internal changes. This misinterpretation may arise due to thinning of the glacier due to mass displacement up top and ablation at lower elevation, creating the appearance of a climate response. Partial surges are not to be confused with mini-surges, which are characterised by being very brief. These small mass displacements might cause the quiescent phase to lengthen, as the glacier fails to build up its reservoir. And yet, it might also enlarge the coming surge by allowing a more stable ice build-up through small releases during the process.[10]

Other problems with identifying surges include, for example, problems with the glacier's given name. It is not uncommon that a glacier may be fed by several tributaries which might have different names. Furthermore, these tributaries might surge independently, complicating the recognition of a surge. It is also more difficult to recognise a surge on a rather small glacier, adding to the wide range in Svalbard's surge glacier estimates. Surge intervals can vary between distinct glaciers and within the same glacier as mass balance is affected by changing climate conditions. These changes have more effect on long, thin glaciers at low elevations as they are more exposed to ablation.[10]

1.2 Kongsvegen

Kongsvegen (King's Road/Highway) is a polythermal glacier on Spitsbergen's northwestern coast (78°51'N, 12°30'E); the ablation area is frozen to the mountains, but the accumulation area is known to be temperate. Flowing into Kongsfjorden, its calving front is joined with that of Kronebreen (Crown Glacier). The two streams meet about 5 km from this calving front, split along a large moraine. The glacier is fed by several basins, of which the largest are known as Kongsvegen and Sidevegen. The Kongsvegen basin is estimated at 102 km^2, judging from maps from 1966, and at a length of 25.8 km. Its surface slope was measured in 1990, ranging from 0.5° to 2.5°. Few crevasses have been reported, while large supraglacial channels have been observed in the ablation area.[6] As already mentioned, surface crevassing could be a sign of surges. Several of these crevasses seem to have been filled up with subglacial sediment during Kongsvegen's surges.[11] The calving front is quite active, in high contrast with the nearly stationary part of the front where it ends on land. Kongsvegen's bed is mostly fine-grained sandstone from the Middle and Upper Carboniferous and Permian age.[6]

The front positions of both Kongsvegen and Kronebreen have been documented in several studies since 1965, showing that a steady retreat of these glaciers has been interrupted by rapid advancement in what appear to be glacial surges. Kongsvegen's last surge ended in 1948 and since then the glacier front has retreated over 4 km to some 25 km. Crevasses were still visible on air records from 1956. The front position is known to have advanced up to 2 km from its 1938 position during this surge. Kronebreen or possibly a small tributary probably surged in 1869. The mass balance for most large glaciers in Svalbard has only been measured sporadically, but Kongsvegen has been measured every year since 1987 by the Norsk Polarinstitutt.[3][6]

Surface velocities have been measured by several expeditions in the front area. Surface velocities upstream of the junction with Kronebreen have shown to be roughly one hundred times as high for Kronebreen (1-3 m/d) compared to Kongsvegen. This is explained by Kronebreen's much larger accumulation area and high basal motion. Not surprisingly, Kongsvegen is far less crevassed in its lower regions than Kronebreen.[6]

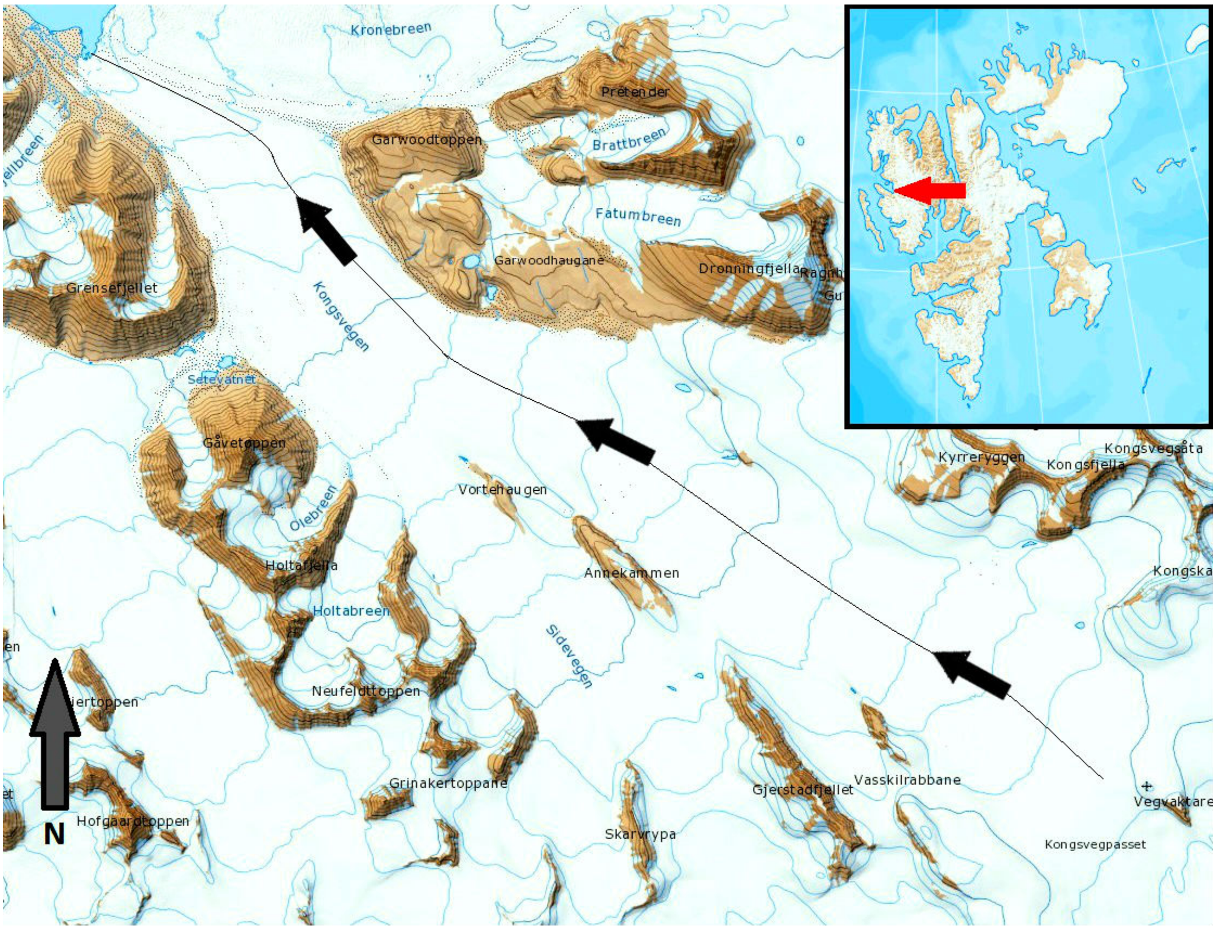

Figure 1: Map of Kongsvegen and the surrounding area, copyright © Norwegian Polar Institute. An approximation of the flowline is indicated.

2 Model Description

In this chapter, some of the model's relevant theory is explained. The model is based on similar programs used by J. Oerlemans in his earlier works.[7][8] The reader is referred to these articles, among others, for a more detailed description.

2.1 The relevant relations

As mentioned in the 1997 article[8], the relevant equations include:

$$Vertically\ integrated\ continuity: \qquad \frac{\partial H}{\partial t} = -\frac{\partial UH}{\partial x} - \frac{\partial UH}{\partial y} + \dot{b} - \dot{b}_b \qquad (1)$$

Here the flowline is represented by the x-axis and y is the vertical coordinate. H is ice thickness, U is the vertical mean horizontal ice velocity, $\dot{b}$ is the specific net balance and $\dot{b}_b$ is the smelt at the base. The latter does not play any role in the model. The specific net balance incorporates all accumulation and ablation, but not calving at the front where Kongsvegen meets the fjord. The altitude above sea level for which accumulation equals ablation, and thus for which $\dot{b} = 0$, is known as the equilibrium line altitude and this plays a central role in the model as it is defined directly.

$$Glacier\ width\ at\ the\ surface: \qquad w_s = w_0 + \lambda_w H \qquad (2)$$

The glacier's cross-section is represented by a trapezoid. The actual width at the surface of the glacier is w_s while the width at the bed is w_0. The factor λ_w represents the steepness of the trapezoid. For lack of a better description, this factor is taken to be 0 and therefore the trapezoidal shape is essentially an oblong shape.

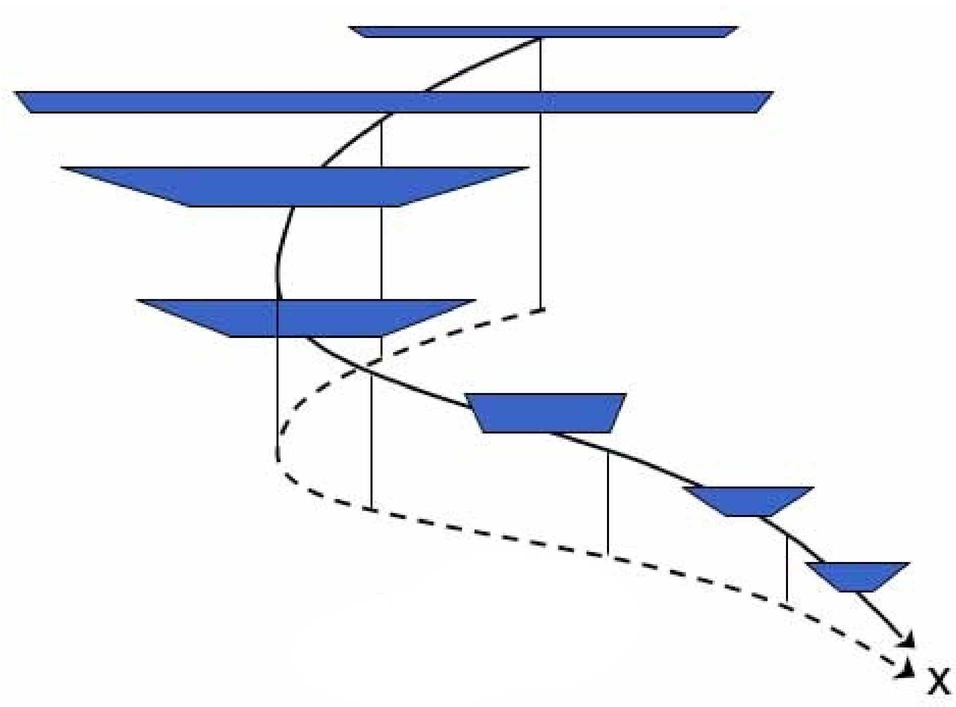

Figure 2: This figure illustrates the flowline and the trapezoidal cross sections centered on this line. The figure was taken from lecture notes provided by Dr. Oerlemans, which are not available online.

$$Evolution\ of\ glacier\ cross\ section: \qquad \frac{\partial S}{\partial t} = \frac{\partial \bar{U} H}{\partial x} + \dot{b} \tag{3}$$

Integrating over y results in equation (3). S is the area of the cross section and $\bar{U}$ is the ice velocity in the flow direction averaged over this cross section. Naturally, this means S is defined as in (4).

$$Surface\ of\ glacier\ cross\ section: \qquad S = H(w_0 + \frac{1}{2}\lambda H) \tag{4}$$

By means of substitution of (4) into (3), this leads to a relation for $\frac{\partial H}{\partial t}$.

$$Change\ in\ ice\ thickness: \qquad \frac{\partial H}{\partial t} = \frac{-1}{w_0 + \lambda_w H} \frac{\partial}{\partial x}[(w_0 + \frac{1}{2}\lambda_w H) H \bar{U}] + \dot{b} \tag{5}$$

Using a Weertman sliding law, an expression can be found for deformation and sliding speeds using flow parameters, ice thickness and the driving stress. This is based on the assumption that the right-hand relation holds for the average velocity as well.

$$Horizontal\ ice\ velocity: \qquad \bar{U} = U_d + U_s = f_d H \tau_d^3 + f_s \frac{\tau_d^3}{H} \tag{6}$$

The flow parameters f_d and f_s absorb the relevant parameters for deformation and sliding, respectively. The exact values of the flow parameters are not known. If enough historical data of the glacier's velocities were available and a steady state could be achieved, a more advanced model might be able to deduce both f_d and f_s. The current model does not produce a steady state when all parameters are free to evolve in time, preventing this model from obtaining these values. Since there is, however, no obvious reason to doubt these parameters as being substantially different from other glaciers', the values were taken from those found at Nigardsbreen ($f_d = 1.9 \cdot 10^{-24} Pa^{-3} s^{-1}$ and $f_s = 5.7 \cdot 10^{-20} Pa^{-3} m^2 s^{-1}$).[8] The model requires slightly altered dimensions, resulting in $f_d = 6.0 \cdot 10^{-17} Pa^{-3} yr^{-1}$ and $f_s = 1.8 \cdot 10^{-12} Pa^{-3} m^2 yr^{-1}$. Expression (7) was used for the driving stress, with density ρ, gravitational acceleration g and surface elevation h.

$$Driving\ stress: \qquad \tau_d = \rho g H \left| \frac{\partial h}{\partial x} \right| \tag{7}$$

Using this expression, the change in ice thickness can be expressed as in (8).

$$Change\ in\ ice\ thickness: \qquad \frac{\partial H}{\partial t} = \frac{-1}{w_0 + \lambda_w H} \frac{\partial}{\partial x}[D \frac{\partial h}{\partial x}] + \dot{b} \tag{8}$$

The parameter D in this expression is the diffusivity, which understandably depends on width, ice thickness and surface slope. Effects like viscosity, drag and lateral variations in H have been absorbed in f_d. The diffusivity, as calculated with $\lambda = 0$, is then defined as in (9).

$$Diffusivity: \qquad D = w_0 \tau^3 (f_d H^2 + f_s H) \tag{9}$$

2.2 The procedure

This is a vertically integrated flowline model programmed in Fortran-90, calculating ice flow along a one-dimensional flowline (essentially the x-axis). Grid-points are placed 100 metres apart, to provide a sufficiently sound resolution. A time loop is performed, calculating for each grid point the driving stress, diffusivity, ice thickness, etc. Derivatives are approximated using central finite differences. Weighted averages are taken for the driving stress and diffusivity to smoothen their gradient and thereby stabilise the calculations. When the domain limit, 40 km or grid point $i = 400$, is reached, the model performs a time step of one thousandth of a year. This continues for a specified number of years and parameters such as ice volume and glacier length are calculated for each year.

The resulting output is split into two files. One describes the calculated parameters for all grid points after the final iteration of the time loop. The other file concerns evolution through time in steps of whole years for those parameters describing the entire glacier. The duration of computation depends on several factors, but typical runs lasted up to five minutes.

2.3 Additions to the model

In order to allow this existing model to approximate the Kongsvegen glacier, several additions and adaptations had to be implemented. Initially, the glacier bed was described by an unsuitable function. The glacier's width was a constant value instead of an approximation of Kongsvegen actual varying width. The specific net balance against altitude was a simple function that needed to be replaced with a better fitting one. Equilibrium line altitude (ELA) was also a constant instead of varying in time. Furthermore, the existing flow parameters allowed for an emulation of surge behaviour. Most of these approximations were done using results from the 1998 article by Melvold and Hagen.[6]

These improvements were based on approximations of the Kongsvegen geometry. Other alterations or implementations would be of little interest to the reader, but they included additional quantities to be calculated, such as ice volume. This model neither simulates calving nor incorporates the processes causing surging behaviour. These severe limitations are obvious points of interest for future work. For an example of a study concerning a self-surging model, see the 1975 article by Budd.[1]

2.4 Implementing the glacier bed

It is obvious that a good approximation of Kongsvegen's bed will make the model more realistic when modelling this particular glacier. Luckily, the 1998 article by Melvold and Hagen provides us with a more than sufficiently accurate elevation profile of this bed.[6] Because the model becomes unstable for strong gradients, the elevation is smoothed by taking several data points from the provided figure and finding a suitable fit for them.

This resulted in function 10, using the least squares method to find the fit as plotted in figure 3.

$$\begin{aligned} bed(i) \sim 2.189 \cdot 10^{-10} \cdot (i-250)^4 - 0.131 \cdot 10^{-5} \cdot (i-250)^3 + \\ 0.003 \cdot (i-250)^2 - 0.143 \cdot (i-250) - 45.275 \end{aligned} \tag{10}$$

The bed was taken to be at -45.0 m after $i = 250$, that is after 25 km, to simulate the glacier flowing into the fjord.

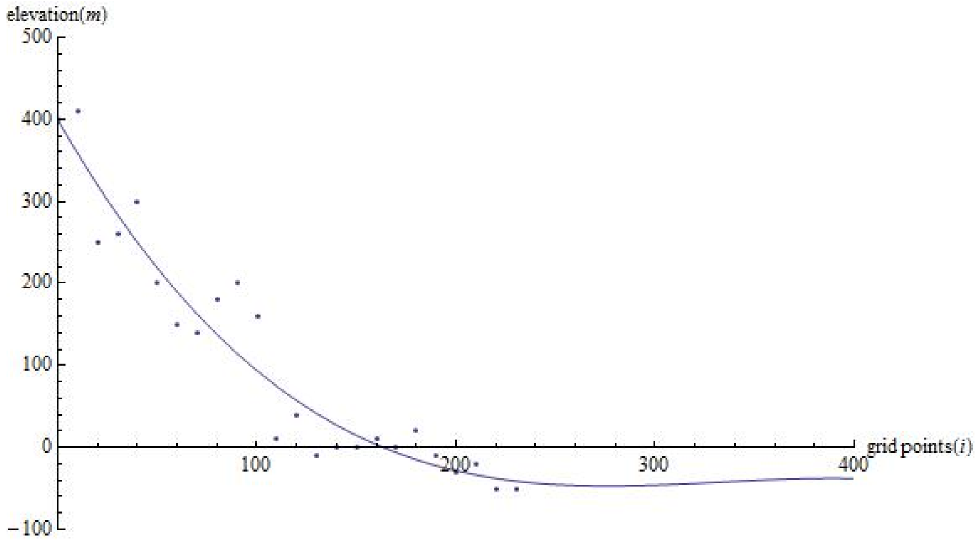

Figure 3: The bed elevation in metres above sea level for Kongsvegen glacier against the horizontal distance in kilometres. This shows data points taken from the 1998 article by Melvold and Hagen and a subsequent least square fit to those data points.

2.5 Variable width

At first, the glacier's width was taken to be a constant and thus irrelevant for changes in ice thickness. It should be quite obvious that a glacier's width affects its ice thickness, though. Again, the aforementioned article provided an overly accurate profile. Kongsvegen flows through a relatively broad valley, they report.[6] Width was approximated using:

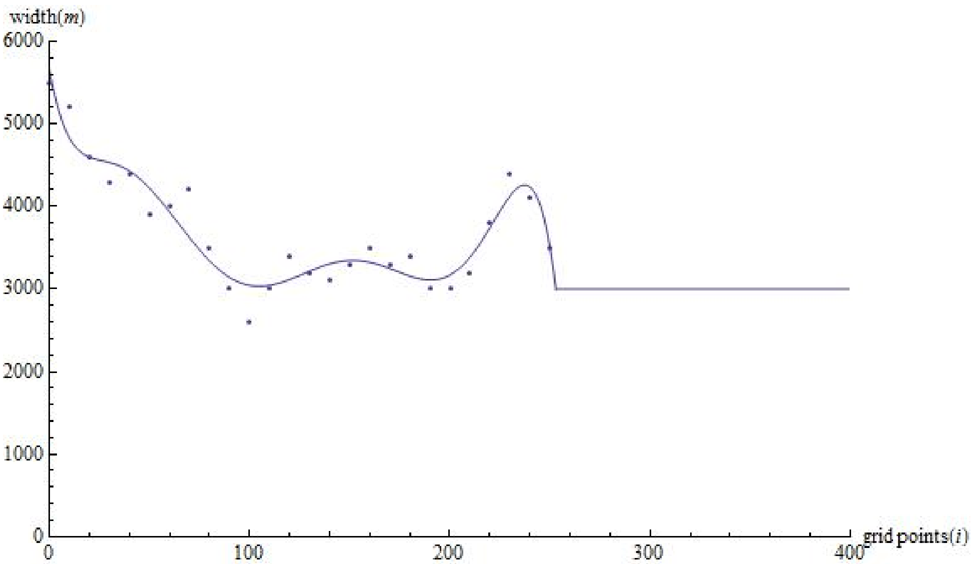

Figure 4: The glacier width in metres for Kongsvegen glacier against the horizontal distance in kilometres. This shows data points taken from the 1998 article by Melvold and Hagen and a subsequent least square fit to those data points. The width is kept constant at 3000 m after $i = 250$ *or 25 kilometres.*

Resulting in function 11, again using the least squares method.

$$\begin{aligned} width(i) \sim -3.760 \cdot 10^{-11} \cdot (i-250)^7 - 3.355 \cdot 10^{-8} \cdot (i-250)^6 - \\ 0.118 \cdot 10^{-4} \cdot (i-250)^5 0.002 \cdot (i-250)^4 - 0.193 \cdot (i-250)^3 - \\ 8.719 \cdot (i-250)^2 - 143.34 \cdot (i-250) + 3499.2 \end{aligned} \tag{11}$$

This is kept constant at 3000 m after 14 km. Steepness of the valley's edges is not taken into account, meaning the trapezoid shape is essentially an oblong.

2.6 Specific net balance

The specific net balance was first taken to be a simple relation, as expressed in 12, where $\beta = 0.007$.

$$\dot{b}(i) = \beta \cdot (ht(i) - ELA) \tag{12}$$

Obviously, this is not ideal when studying a specific glacier. A more accurate picture can be achieved by fitting a function for $\dot{b}$ based on, again, the article by Melvold and Hagen.[6] They reported the specific net balance for the years '86 -'87 and '92 -'93, together with a mean for the entire 9 year period of observations ('87-'94). A least squares fit based on the latter would give expression 13, with a and b parameters determined by the fit.

$$\dot{b}(i) = a \cdot \arctan(\frac{ht(i)}{b}) \tag{13}$$

There is, however, a problem with this function. All dependence on the equilibrium line altitude, and thus on climate conditions, has been removed from the relation for $\dot{b}$. To alter this, function 14 was implemented:

$$\dot{b}(i) = 2.32641 \cdot \arctan(\frac{ht(i) - ELA}{508.987}) \tag{14}$$

The function now equals the fit above for an ELA of 490 m, which is the average ELA that was measured and thus the ELA for which the fit must have been established.[6] Thanks to this expression, $\dot{b}$ responds to changes in ELA by shifting the graph up and down.

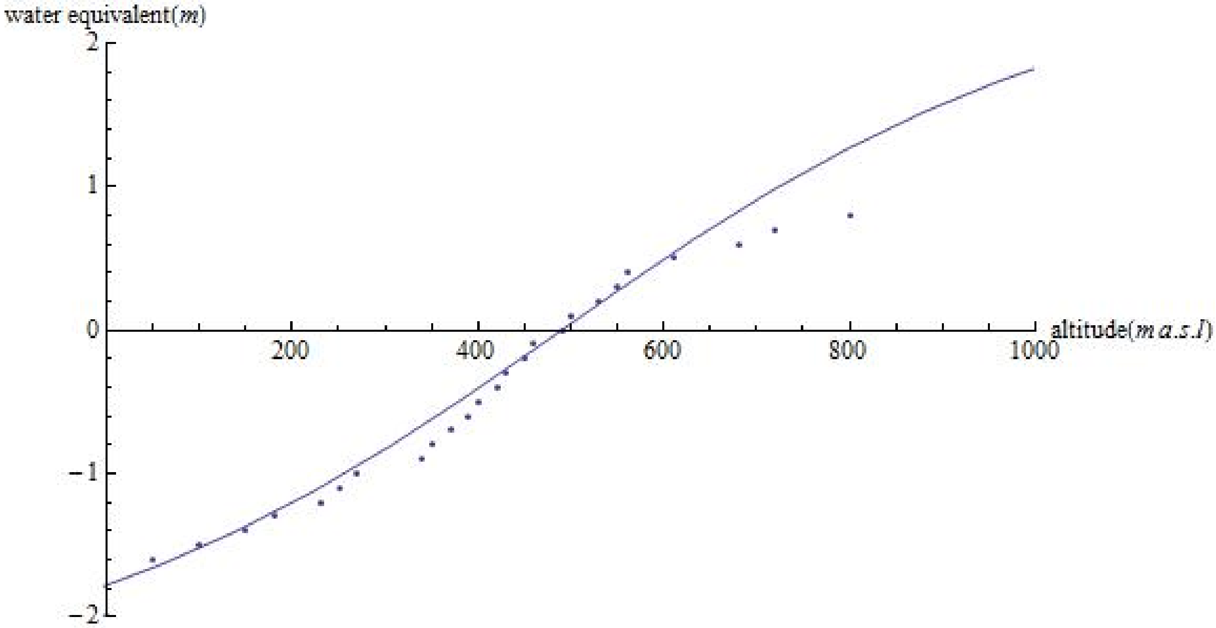

Figure 5: The specific net balance $\dot{b}$ as water equivalent in metres against altitude above sea level in meters for Kongsvegen glacier. This shows data points taken from the 1998 article by Melvold and Hagen and a subsequent least square fit to those data points.

3 Results

The model starts with no ice at all. There is simply an ice-free valley and a given value for ELA. This means that the climate might prevent a glacier from appearing in the model, unless conditions are favourable for ice formation.

3.1 Equilibrium line altitude

The equilibrium line altitude serves as an expression of climate conditions in this model. A higher value of ELA indicates a higher temperature, or otherwise less favourable climate, since the glacier's ablation area is increased. Although the exact conversion between ELA and for example temperature is unknown, fluctuations in ELA simulate fluctuations in climate. To illustrate the effect of this altitude on the glacier, see figure 6 for the glacier length and volume when there is no variation in ELA. Melvold and Hagen report a mean ELA of 490 m above sea level between 1987 and 1994. The highest measured value was 580 m while the lowest was only 380 m.[6] Notice how these values do not produce the desired result. Glacier length ought to be around 25 km and has retreated only a few kilometers in half a century. For these constant values of ELA, however, the glacier seems either not to exist or to continue growing until it exceeds the model's domain. Figure 6 does not actually show the latter, but increasing the number of time loops has indicated this to be true. A steady state is nowhere to be seen.

The reader is reminded that the equilibrium line altitude is the altitude above sea level for which the glacier experiences a specific net balance of zero, meaning the glacier's ice at that altitude will have equal ablation and accumulation. Though the elevation of the glacier's bed may exceed this altitude, the glacier itself may be too thin to have its surface reach so high.

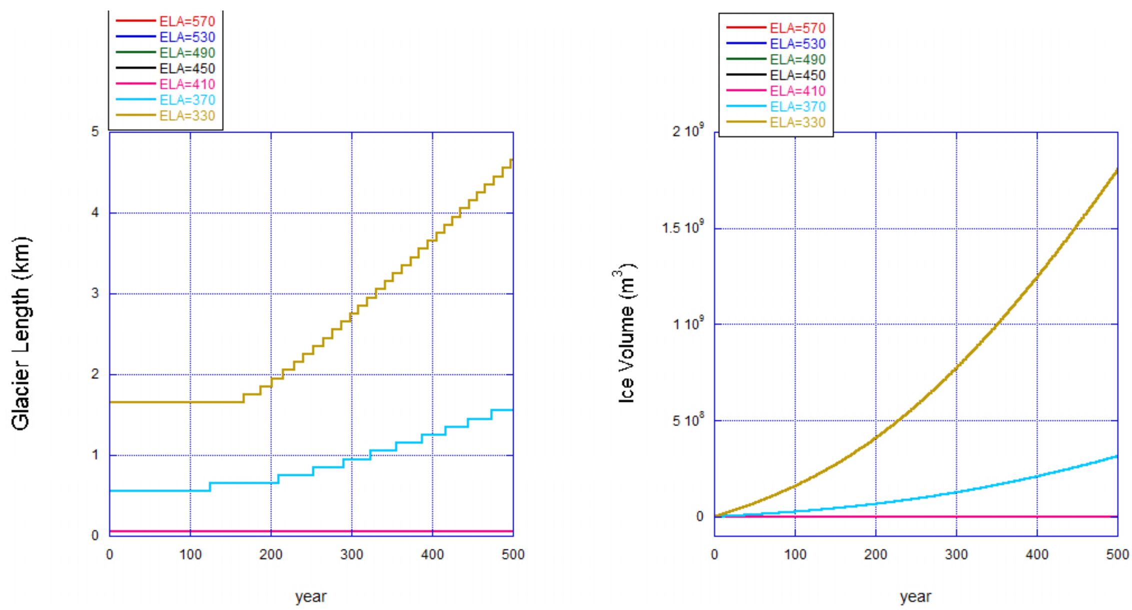

Figure 6: These figures show the glacier length in kilometres and the ice volume in cubic metres over time for various constant values of ELA. Note how some lines appear to be missing, these are hidden behind the continuous zero-value line for ELA=410 at the bottom of both figures. The steps, which are more pronounced in the length graph but visible in both figures, are due to the model's precision. A smaller spacing of the grid points would result in smaller steps while increasing the computation time.

Since climate varies over time, it makes sense to define a function for ELA that allows such a variation. The effect of climate on the equilibrium line altitude is a complex one as there is a role for precipitation, temperature, etc.

$$ELA = a * \sin(\frac{2 * \pi * year}{b}) + c \tag{15}$$

As is visible in figures 7 and 8, equation (15) with certain parameters a, b, and c, creates a glacier more similar to the actual conditions than the constant ELA.

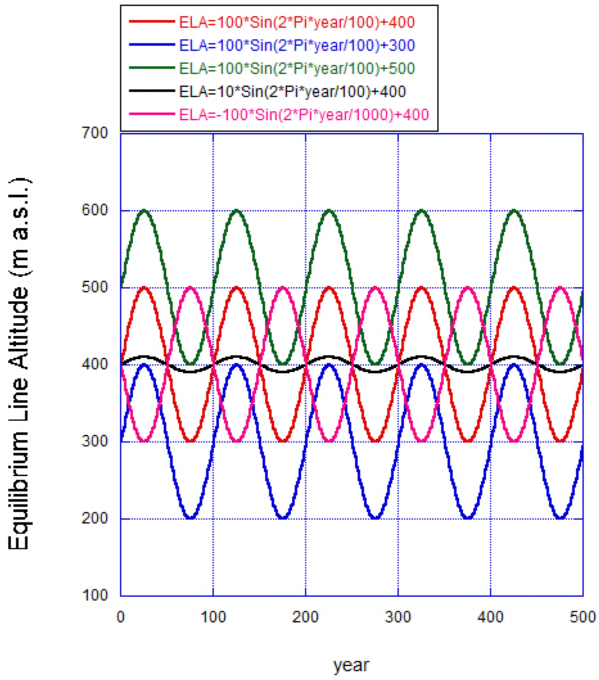

Figure 7: Variation of the Equilibrium Line Altitude in time for various sine functions.

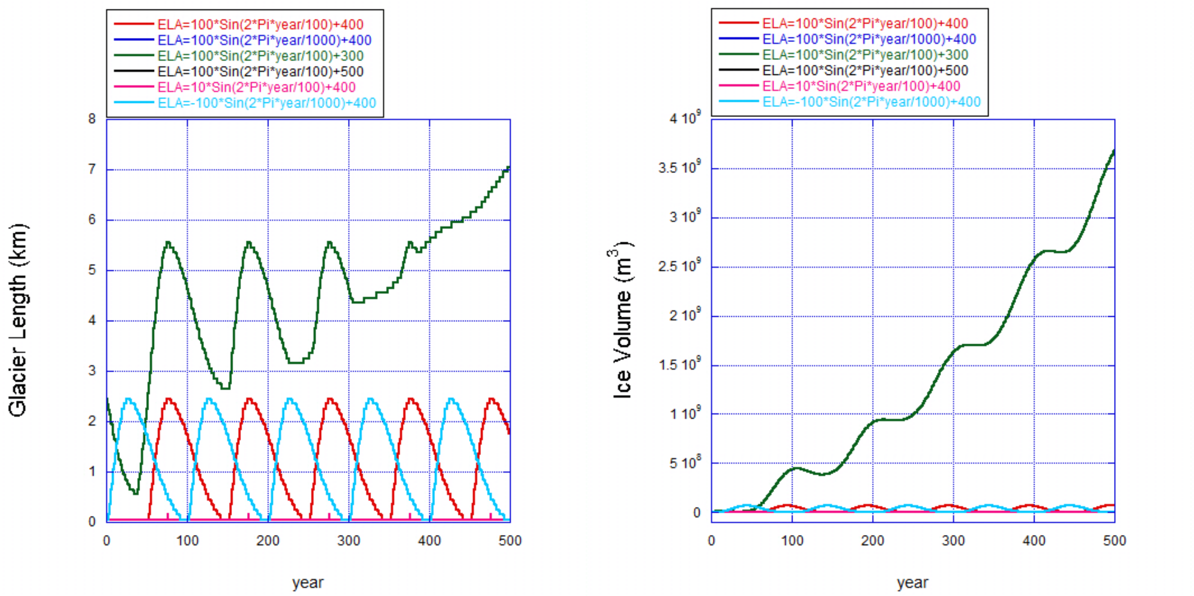

*Figure 8: These figures show the glacier length in kilometres and the ice volume in cubic metres over time for several varying values of ELA. Note how some lines appear to be missing, these are hidden behind the continuous zero-value line for ELA=10*Sin(2*Pi*year/100)+400 at the bottom of both figures. These are the values of ELA for which the glacier refuses to grow (in both length and volume).*

It would appear from both figures 6 and 8 that the glacier fails to grow below a certain value for ELA and does, in fact, manage to grow above this value. The only variable ELA for which the glacier doesn't completely disappear during the less favourable periods, is $ELA = 100 * Sin(2 * Pi * year/100) + 300$. Due to the constant 300 term, ELA is constantly above a certain threshold value. This tipping point seems to be somewhere around $ELA = 390$ m, but its exact value was not investigated further. The model seems to respond rather strongly to deviation from this value and there might not be any steady states due to a lack of calving. Evidently, the model requires a initial period in which the glacier builds up ice before current conditions can be applied. At high values of ELA, the entire glacier experiences a negative net balance and therefore a net melt. Kongsvegen has been reported as retreating while ELA varies between 380 m and 580 m.[6] So it is not surprising that entering these values into the model without initially building up sufficient ice volume does not result in any useful results. The model apparently demands this initial build up. This way, steady states might be achieved, but the exact function of ELA in the real world remains a mystery. Future research could therefore be aimed at applying proxy data for past temperature to discover the most appropriate function for ELA.

For the purposes of this simulation, an ice build-up period of half a millennium is applied for $ELA = 100$ m, followed by raising the temperature to the more realistic and actual mean value of $ELA = 490$ m. This creates enough ice for the current conditions to be simulated. The resulting graphs are visible in figure 9.

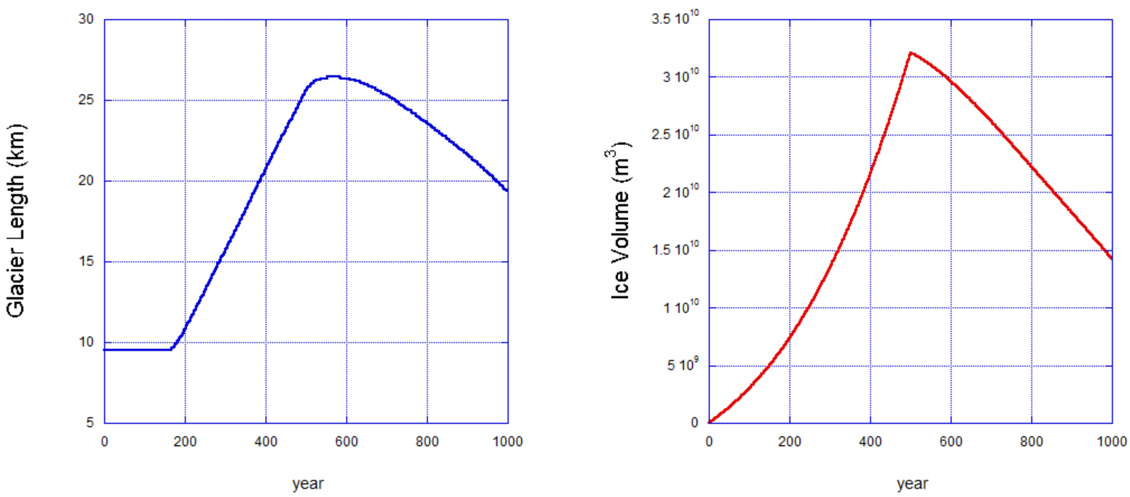

Figure 9: These two figures show the glacier length in kilometres and the ice volume of the glacier in cubic metres for an initial period of 500 years at ELA=100 m and a subsequent period of 500 years of ELA=490 m.

From these figures, it is apparent that the ice volume reacts more quickly to the change in ELA than the glacier length, which seems to take its time slowing down in comparison. This leads one to suspect a relatively rapid reaction in ice thickness. Mass conservation would demand a reaction in ice volume, but the response in glacier length could be a rather slow one. Therefore, figure 10 was plotted. Figure 10 does indeed show that the ice thickness reacts relatively sharply to the change in ELA, which should explain the sudden response in ice volume.

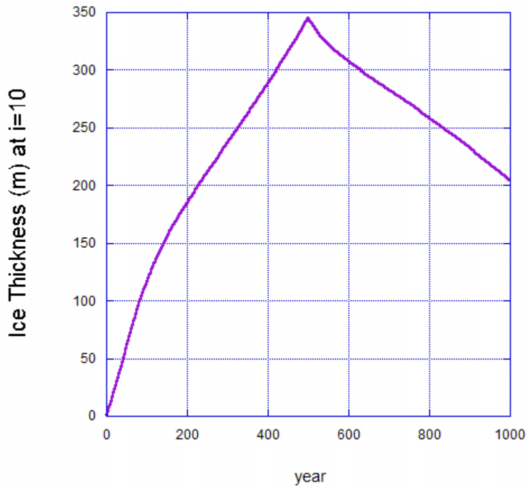

Figure 10: Variation of the ice thickness in time for an initial period of 500 years at ELA=100 m and a subsequent period of 500 years of ELA=490 m. Ice thickness was taken at i=10 or 1 km.

3.2 Surges

Since the model does not involve sediment displacement, water drainage or any other factor directly involved in causing surging behaviour as it is understood, surges can only be enforced through deliberate intervention. Essentially, the effect of a surge is a dramatic increase in the sliding parameter f_s. In order to show that the model does allow surges, this parameter is made variable in time. The definition of the sliding parameter was replaced with a Gaussian curve as described in (16), with certain parameters a,b and c. Naturally, a represents the factor by which f_s increases, b represents the moment in time at which the peak occurs and c affects the duration.

$$f_s = f_s(1 + a \cdot \exp{(-\frac{(year - b)^2}{2c^2})}) \tag{16}$$

In this way, the Gaussian curve causes an increase in sliding around a predetermined year for a predetermined duration. The aim is to find an increase in glacier length of approximately $1 - 3$ km. Figure 11 shows the increase of the sliding parameter f_s in time and figure 12 the subsequent increase in glacier length and ice volume. Notice how an increase in surge duration as well as the factor by which the sliding parameter is increased both affect the intensity of the surge's effect on length and volume. Without more accurate data of Kongsvegen, the exact circumstances of a surge like the one in 1948 cannot be implemented.

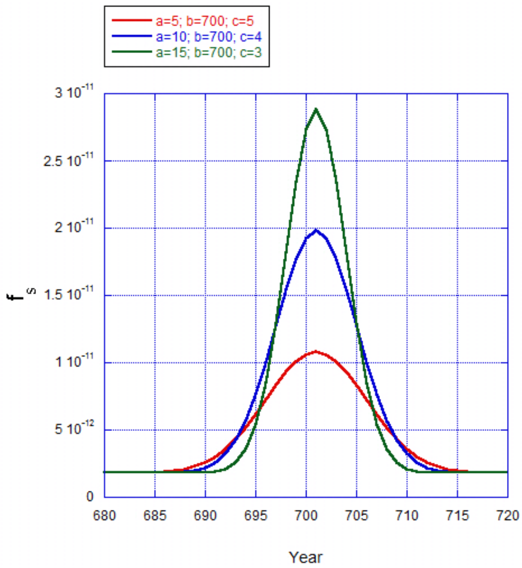

Figure 11: This shows the evolution of f_s with time in $Pa^{-3}m^2yr^{-1}$ for an initial period of 500 years at ELA=100 m and a subsequent period of 500 years of ELA=490 m.

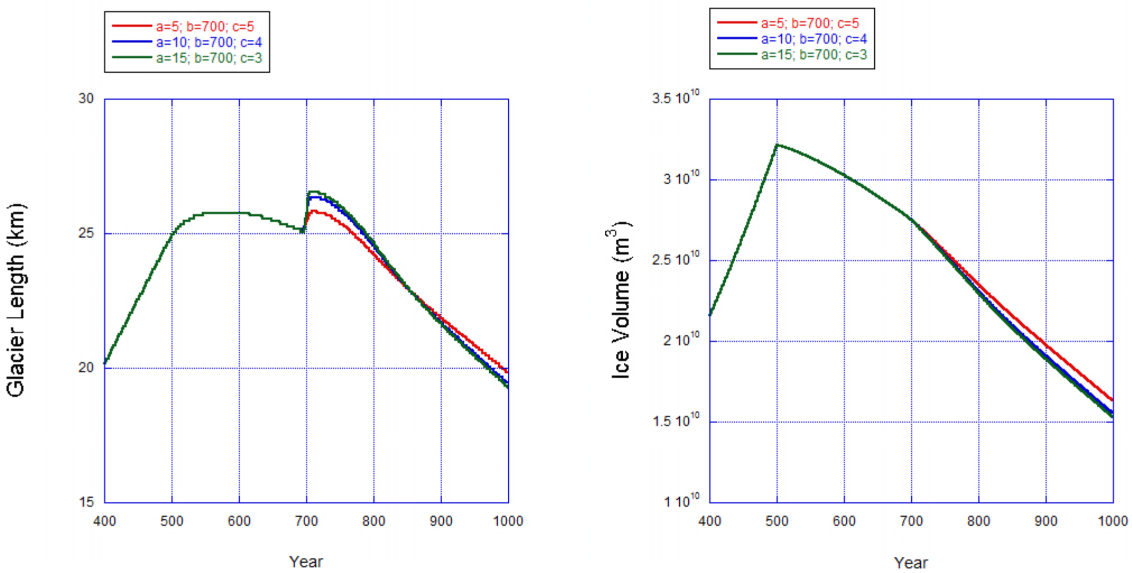

Figure 12: These figures show the glacier length in kilometres and the ice volume of the glacier in cubic metres for an initial period of 500 years at ELA=100 m and a subsequent period of 500 years of ELA=490 m.

To illustrate the surge in more detail, figures 13 and 14 illustrate the surge defined by $a = 15, b = 700$ and $c = 3$. The figures were plotted for the years 685 - 715 in steps of five. Year 685 is just before the increase in f_s, while year 715 marks the return to its previous value. This illustrates the development of the surge, with the peak in the sliding parameter at year 700. Additionally, year 783 is plotted, as it is the year when the glacier length has finally retreated to its value before the surge (25.15 km).

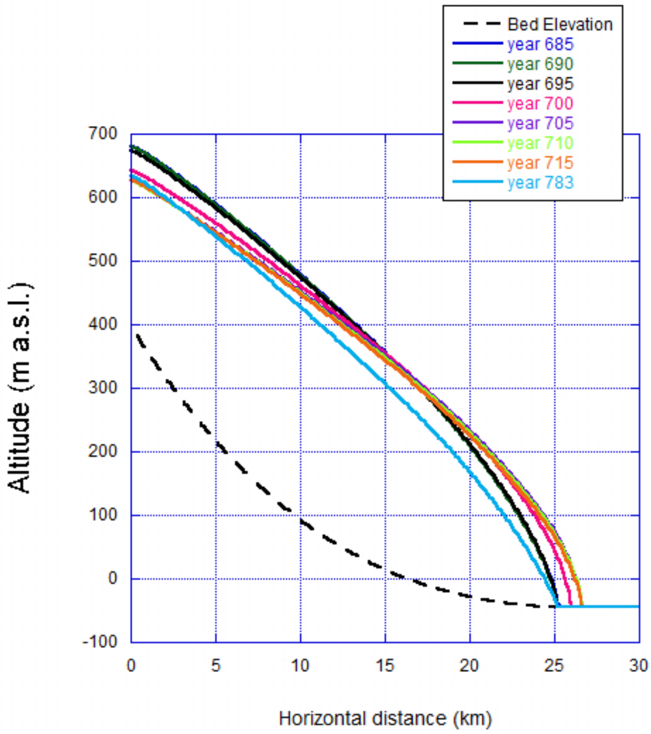

Figure 13: The surface elevation for several years during a surge and the bed elevation.

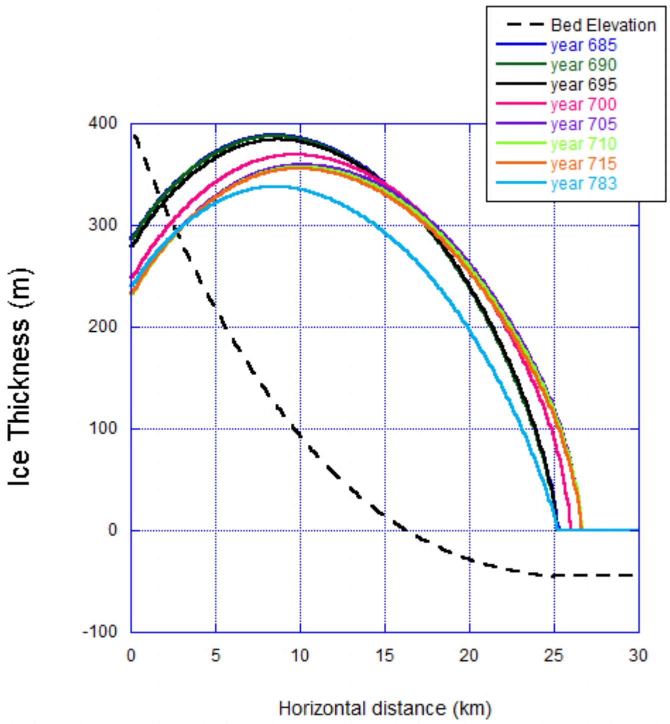

Figure 14: The ice thickness plotted for the same years as the previous figure. Surface elevation is simply the sum of bed elevation and ice thickness.

Figures 13 and 14 show how the surge develops. With time, the glacier lengthens and thins out. By the time the sliding parameter f_s has returned to its initial value, the glacier is significantly thinner at higher elevation but also significantly longer (26.55 km to be exact). By the time the glacier length has returned to its previous value, after 783 years or 95 years after the onset of the surge, the entire glacier is much thinner overall than it was before. After this point, the quiescent phase continues as ice builds up, in preparation for another surge at some point in the distant future.

Another way to look at the surge is through the ice velocities. Figures 15 and 16 show the sliding and deformation velocities (u_s and u_d). Note how u_s is small and relatively level for all years but two. The peak in f_s coincides with a dramatic increase in u_s, as it should according to (6). This also shows how the surge is relatively short compared to the width of the Gaussian curve in f_s. The sliding velocity u_s responds quite slowly to the increase in the sliding parameter, only dramatically increasing near the very peak of the curve in f_s.

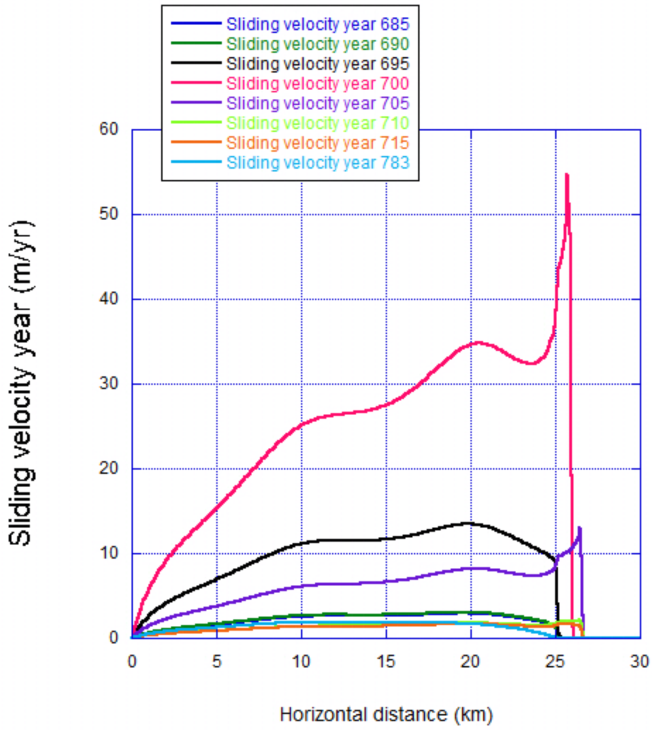

Figure 15: Sliding velocities against the horizontal distance in kilometres for the same eight snap-shots in time as figures 13 and 14.

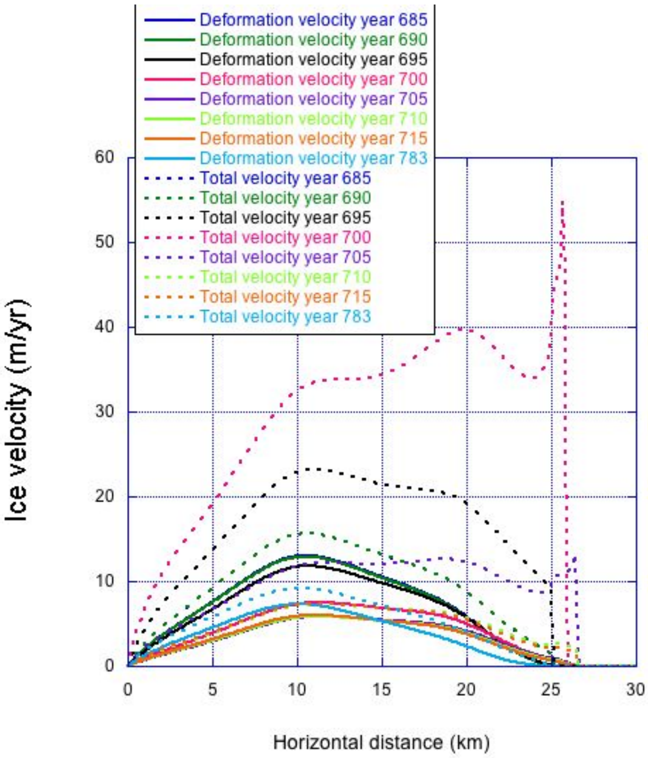

Figure 16: Deformation velocities and the total velocities, the summation u_s and u_d, against the horizontal distance in kilometres.

These figures together with figures 13 and 14 successfully describe Kongsvegen's surging behaviour within the limits of this model and the available data.

4 Discussion

The model suffers from several limitations (such as calving not being taken into account) and unknowns (such as the flow parameters). Future research might improve upon the model's present condition.

Since several functions for ELA may satisfy the need for building up ice before actual conditions can be applied, little can be said about Kongsvegen's future from this model. Improving further upon the present model, however, might narrow the gap. Many of the current functions are based on crude and strongly smoothed fits, which may be imperfect even within the limitations of the model. For example, the varying width might be improved by incorporating the slope of the valley sides. Also, if the model could be adapted to accept more detailed data instead of these smooth fits, reality ought to be approximated more closely.

The most obvious need for improvement lies with climate data. If one could find a relation between the equilibrium line altitude and the actual local temperature, precipitation, etc., combined with proxy data describing that climate, the glacier could be closely examined and predictions could be made.

A recurring problem with this program is the uncertainty which knobs to turn, as it were. The specific net balance, bed elevation and glacier width were chosen to be taken from measurement data. Subsequently, the equilibrium line altitude, an initial ice build-up period and the flow parameters (or rather the sliding parameter) were adjusted to study the effects. Though this might seem a rather straightforward combination, other combinations are not unthinkable and might lead to equally informative results. If only there had been more time to work on these things.

5 Conclusions

Expanding upon the initial model as if it were a template, several approximating expressions were implement in order to simulate the Kongsvegen glacier on Spitsbergen, Svalbard. Taking measurement data from the 1998 article by Melvold and Hagen, three least-square fits were fed to the model; bed elevation, glacier width and the specific net balance. This did not result in a steady state for any value of the Equilibrium Line Altitude. Though there might still be a value of ELA for which there is almost a steady state, the geometry seems to make this model extremely sensitive to deviations from this value. Calving might be the main issue here, preventing a steady state from being achieved.

An initial ice build-up period was required before the current value of $ELA = 490$ metres could be applied. This was set at 500 years of $ELA = 100$ metres. This then allowed the model to approximate Kongsvegen's actual length of roughly 25 kilometres.

Surging behaviour was successfully simulated in the model by adjusting the sliding parameter f_s through use of a Gaussian curve for that parameter as a function of time. This resulted in an increase in the sliding velocity and glacier length in accordance with typical surges. The glacier does not retreat to its previous length until 95 years after the onset surge, taken to be 15 years before the peak of the Gaussian curve.

The sliding velocity and thus the glacier length does not respond substantially to the increase of the sliding parameter unless it is increased by at least a factor of 5. In order for the surge to satisfy typical advancement of the glacier front, the sliding parameter should peak at 20 times its original value, or thereabouts. This result implies quite a dramatic increase of f_s during a surge.

6 Acknowledgements

Thanks go to my fellow Bachelor students also doing their thesis at the Institute for Marine and Atmospheric research Utrecht for their help, tips and support. The same goes for the Master students with whom we shared our working space. Thanks to A.S. von der Heydt for her contributions as coordinator of the Bachelor Research group at IMAU. And, of course, special thanks to J. Oerlemans for a great many explanations, comments, suggestions, articles and the opportunity to complete my Bachelor thesis under his supervision.

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