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On the foundations and interpretations of Bohm's approach to quantum mechanics

THESIS

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"Tanto en el conocimiento como en la vida real, cada obra y cada ser humano representa una humanidad individualizada" L.O.

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Part I

Introduction

1 General introduction

Quantum mechanics apparently stands against preceding classical deterministic world views, where if the precise location and momentum of every particle in a system is known, their past and future values for any given time is determined by the governing Newtonian laws. It is true that classical statistics uses non-deterministic techniques to describe the behavior of a great number of particles; nonetheless, this indeterminacy is not inherent in nature, but only used for practical considerations.

With the advent of quantum mechanics, physicists and philosophers have developed different interpretations about the role of probabilities, where ontological or epistemic positions have been claimed. In one particular interpretation, indeterminism has been assumed in the sense that uncontrollable and unpredictable real events are actually occurring in nature. In the same spirit, predictions and observations of irregular processes have envisaged a real statistical behavior of these particular events. Is this point of view reliable? Although quantum mechanics is in accordance with all observable phenomena in the domains of the atomic level, any mechanical description at this level seems to be limited. Rather, it seems that quantum mechanics is a mathematical model consisting of theoretical entities only capable to predict probabilities of outcomes before any measurement has been performed. The actual problem is, how should this be interpreted?

The first steps toward this interpretation were taken by the early founders, defining the mathematical objects of quantum mechanics e.g. the wave function, as merely epistemic *useful tools*, used to statistically predict outcomes of measurements among systems of several particles. Later on, the *Uncertainty Principle* was discovered by Werner Heisenberg, introducing indeterminacy in the values of physical quantities, and this was followed by a standard realistic belief that nature was in fact governed by chance. However, although Bohr, Heisenberg, Von Neumann and other physicists, developed together *the Copenhagen interpretation* or *Orthodox Quantum Mechanics* (OQM), there are sharp differences among them, e.g. the debate about realism and instrumentalism. This point is of great importance in the history and philosophy of quantum mechanics but for the time being this controversial debate will not be considered.

In this thesis, one eventually realizes that OQM is not the only theoretical account to describe the nature of the micro-world. There exists another quantum theory as well, such as Bohm's theory (BQM). This particular approach is in agreement with the phenomena predicted by the usual quantum theory, but yields a very different interpretation. In fact, probability is merely epistemic, because an ensemble of objective trajectories of real particles is formulated with a gamut of initial conditions. Every trajectory obeys the Schrödinger equation in a similar manner that a particle obeys Newtonian laws in classical physics. But in contrast to the latter, the quantum effects observed in the experiments follow from the manifestation of a new kind of entity, which has not been mentioned so far: the quantum potential. In fact, this theoretical entity behaves different from the classical potential, and possesses new intriguing properties, besides other physical and ontological aspects.

In the following pages, a review of both interpretations is given. In the first part, general aspects of both interpretations are discussed, avoiding technical aspects that are not so important for the purpose of this thesis. Later on, the problems concerning the foundational aspects among all Bohmian interpretations proposed so far are written. Finally the purpose of my thesis is explained and then solved. Let us begin with.

2 Quantum mechanics interpretation

At first, some concepts that are used in this thesis are defined. A philosophical position with regard to these definitions is not required, but only their meaning will be given.

(a) Physical entity. Entities that are ontic, existing and real, and may generally be subjected to experiments. However, there is an essential question here: Whether or not systems, states or any other theoretical entities are actual? Or are they theoretical constructions? To avoid this problem, *(Physical)* is written when real entities are considered, while *physical* is written in the case theoretical constructions are considered.

(b) Fundamentality. Fundamental means (Physical).

(c) Language. The mathematical language, endowed with logical deductive structure and meaning.

(d) Meaning. Sentence's meaning in a particular language. In particular, the meaning of mathematical propositions. Meaning will be construed as the designation of a (Physical) counterpart to the mathematical objects but also as the designation with epistemic or artificial objects, not necessarily real, e.g. physical entities, epistemic probabilities, and so on.

(e) Interpretation. It is the act of assigning meaning to linguistic terms and explanations. In particular, assigning meaning to mathematical theories and to explain them, e.g. philosophical commitments, space and time conceptions, and so on.

Once the preceding concepts have been defined, it is important to explain the first interpretation known as the Copenhagen interpretation of quantum mechanics or orthodox interpretation.

3 Orthodox interpretation

In early XX century the first attempt to predict the phenomena of the micro-world emerged by one particular theoretical model that was different from the classical description. The first insights commonly known as early quantum mechanics were carried out by Bohr's atomic model and the earlier discoveries of Planck and Einstein. The first steps were brought about by assumptions that were, in principle, non systematic. It was not until 1925 that a broader and complete theoretical system emerged along the lines of Schrödinger and Heisenberg, and later, axiomatized by Von Neumann. Several postulates form the basis for the theory, where a connection between the physical properties of a system and the mathematical language employed therein had been suggested.

It is remarkable that these physicists were engaged in a project aiming to establish a new theory that contradicts classical physics and under which the only reference at hand was the empirical knowledge provided by recent experiments. In this context, the only way to build up a complete physical theory was to develop a coherent interpretation. Therefore, the first interpretation was proposed during the second half of 1920, namely the *The Copenhagen interpretation* or *Orthodox interpretation.*

Along the lines of non-pragmatic positions, this particular interpretation is not an additional theory to the current predictions of the time, but was specially proposed to explain these predictions. Moreover, Von Neumann's postulates are not the basic and primary axioms of all interpretations. Rather, they basically form the corresponding axioms of the orthodox interpretation, because they were developed once this interpretation had been proposed.

Now, the Copenhagen interpretation is formulated considering the following statement:

Von Neumann's postulates are the primary and basic axioms of OQM.

3.1 The postulates, their interpretation and their problems

The whole content of the orthodox interpretation is elucidated in the following quotation[1]:

> There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.[23, p.12]

What Bohr tried to say here corresponds to philosophical reflections developed between 1920 and 1930. Whether Bohr's earlier ideas were coherent with his latest works is still controversial; nonetheless, the epistemic character of OQM tends to cease the debate in the majority of textbooks.

For the time being, the epistemic view will be described and then a discussion of other interpretations will be given.

According to this view, there is no quantum world, quantum theory is rather part of a mathematical description used to predict outcomes of measurements. In fact, Bohr claimed that mathematical objects such as the wave function, hermitian operators, and so on, are neither real entities, nor abstract objects that belong to certain (Physical) descriptions. According

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[1] Aage Petersen, "The philosophy of Niels Bohr", Bulletin of the Atomic Scientists Vol. 19, No. 7 (1963).

to Bohr, nature cannot be described in terms of abstractions, much less one can say that the descriptions are actually the reality. The mathematical model may only predict the results obtained in experiments, as long as this model comprises physical states, systems and the evolution of those systems[2].

In this sense, Bohr rejects that mathematical objects and properties must have a counterpart in the (Physical) world. The discussion about nature as something objectively perceptible turns out to be an epistemological issue: quantum physics is about statistical predictions of possible outcomes in particular situations i.e. measurements performed in ensembles, rather than the description of individual atomic events before, during and after the measurement, which are unpredictable from this point of view.

Now, to understand the preceding claim it is important to go in depth into the technical content of quantum mechanics. Von Neumann's postulates will be explained and later the most important points of the Copenhagen interpretation will be enumerated. In addition, Bohr's complementary principle and the uncertainty principle will be discussed.

The physical state of a system provides information of the system at one particular time. In classical physics, in the case of a one-body system, the state is determined by the position and momentum of the particle. Whereas in the case of a field, the state is determined by a complex function of position and time. Conversely, in quantum mechanics the physical state is determined by the wave-function and a wave-functional, respectively. However, three important questions arise here: How position and momentum are related to the wave function? What does the state means? and How does it evolve in time?

To answer these questions, Von Neumann's postulates are enumerated:

1. *State postulate.* Every physical system corresponds to a Hilbert space $\mathcal{H}$, and the state of the system is completely described by an unitary vector in $\mathcal{H}$. A composite physical system corresponds to the direct product of the Hilbert spaces of the subsystems.

2. *Observables postulate.* Every physical quantity $\mathcal{A}$ of the system corresponds to a self-adjoint operator A in $\mathcal{H}$.

3. *Spectrum postulate.* The possible outcomes which can be found upon measurement of a physical quantity $\mathcal{A}$, corresponding to an operator A, are values from the spectrum of A. It is worth to remember here that the spectral theorem says that every self-adjoint operator A has an orthonormal basis of eigenvectors $|a\rangle_i$ in $\mathcal{H}$ with their corresponding eigenvalues a_i in $\mathbb{R}$ satisfying:
$$A\,|a_i\rangle = a_i\,|a_i\rangle$$

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[2] In this respect, one should be careful: as stated, (Physical) entities in this thesis refers to real entities existing in the world whereas physical entities are theoretical constructions belonging to a model of nature, but not part of it.

From this, any arbitrary state may be expressed in their corresponding basis of eigenstates:

$$|\psi\rangle = \sum_{i=1}^{N} c_i |a_i\rangle \tag{3.1}$$

where N is the dimension of $\mathcal{H}$ and c_i is a complex number.

4. *Born postulate* If the system is in a state $|\psi\rangle \in \mathcal{H}$, and a physical quantity $\mathcal{A}$, corresponding to an operator A with a discrete spectrum $\operatorname{Spec} A$, is being measured, then the probability to find the outcome $a_i \in \operatorname{Spec} A$, is equal to

   $$\mathrm{Prob}^{|\psi\rangle}(a_i) = |c_i|^2$$

   And, if we let $\langle\psi|$ operates on both side of the equation (3.1):

   $$\mathrm{Prob}^{|\psi\rangle}(a_i) = |c_i|^2 = |\langle a_i|\,\psi\rangle|^2 = \langle\psi|\,P_{a_i}\,|\psi\rangle\,,$$

   where P_{a_i} is the projector (projection of $|\psi\rangle$ on $|a_i\rangle$) from the spectral decomposition of A[3].

5. *Schrödinger postulate.* As long as no measurements are taking place on the system, the temporal evolution of the system is described by an unitary transformation,

   $$|\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle\,.$$

   This is an internal and mathematical evolution and the entire information of the system contained in $|\psi\rangle$ is preserved by this Schrödinger equation. In this manner the description of the specific mathematical object $|\psi\rangle$ is deterministic, but there is no relation with the (Physical) world in the sense that, without the process of measurement, $|\psi\rangle$ does not have any (Physical) significance.

6. *Projection postulate.* If the system is in a state $|\psi\rangle \in \mathcal{H}$ and a measurement is made on a physical quantity $\mathcal{A}$ corresponding to an operator A with discrete spectrum, and the outcome of the measurement is the eigenvalue $a_i \in \operatorname{Spec} A$, then the system is, immediately after the measurement, in the eigenstate

   $$|\psi\rangle \rightsquigarrow \frac{P_{a_i}\,|\psi\rangle}{\|P_{a_i}\,|\psi\rangle\|}.$$

   As we may notice in the present postulate, the state of the system $|\psi\rangle$ has collapsed into the $|a_i\rangle$ state from all over the eigenstates as a result of the measurement. In doing so,

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[3] It is important to introduce the projector operators $P_{a_i} = |a_i\rangle\langle a_i|$ in order to express all the testable predictions of quantum mechanics in terms of expectation values in the state $|\psi\rangle$.

$|c_i|^2$ must be construed not as a the probability of the outcome *being* a_i before, during and after the measurement, but to *find* a_i by the measurement.

The first four postulates connect the undefined theoretical concepts ‘physical system’, ‘state’, ‘quantity’ and ‘measurement’ to mathematical concepts. In the literature the postulates 3 and 4 are sometimes combined into the so-called *measurement postulate*. The last two postulates determine the evolution of the states in time. As observed, these set of rules used for computing the most probable outcomes of measurements do not explain the measurement process. There is a division between deterministic processes governed by the Schrödinger equation from the statistical process of measurement.

According to Bohr, quantum mechanics is not in the world, rather, it is a mathematical tool that helps to predict probable outcomes of a particular ensemble, rather than objective individual events. (Physical) entities such as electrons and other particles are not directly observable, and one cannot talk about the existence of these entities that are beyond the measurement process. For this reason, the possible outcomes predicted by the quantum theory do not pertain to the values of independent properties of (Physical) objects, they rather come out from the total classical description of the measurement process (experimental device together with (Physical) entities).

Moreover, Bohr believed that only classical systems may be described in terms of common language. In this manner, Bohr concludes that quantum mechanics provides statistical descriptions of interactions between the measurement device and (Physical) entities, in terms of classical concepts.

> [...] It is decisive to recognise that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. The argument is simply that by the word ”experiment” we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics.[24, p.209]

In the following lines we will enumerate the most important points of the Copenhagen interpretation in a systematic way:

3.2 The complementarity principle

Niels Bohr first introduced the complementarity principle in a paper called *The Quantum Postulate and the Recent Development of Atomic Theory.* In fact, he claimed that (Physical) objects have multiple properties that appear to be contradictory during measurement processes. There are situations where different objects behave in a different manner depending on the observation method, e.g. wave and particle. However, it is in principle not possible to observe these different properties at the same time. On the same footing, it is also not possible to separate quantum systems from the measuring setup, because quantum objects have multiple properties depending on the experiment. Therefore, systems are defined by the totality of observations among different experimental arrangements.

> This crucial point, which was to become a main theme of the discussions reported in the following, implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear. In fact, the individuality of the typical quantum effects finds its proper expression in the circumstance that any attempt of subdividing the phenomena will demand a change in the experimental arrangement introducing new possibilities of interaction between objects and measuring instruments which in principle cannot be controlled. Consequently, evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects.[24, pp.209-210]

3.3 The uncertainty principle

In 1927 Werner Heisenberg published the commonly known *Uncertainty Relations*. These relations arise from fundamental principles in the domains of quantum mechanics, where a lack of knowledge is manifested by performing instantaneous measurements upon pairs of physical properties. In fact, we cannot know simultaneously both physical properties, the more precisely one is measured, the less precisely the other may be controlled, predicted and determined. For instance, the instantaneous measurement of both momentum and position proves to be impossible. The knowledge of the outcome of the former produces indeterminacy in the outcome of the later. In this respect, different interpretations of the uncertainty relations have been given, but definite claims have not been grounded and have remained tentative though.

According to Heisenberg, the uncertainty principle reflects a fundamental and inherent feature of (Physical) reality, and does not follow from our ignorance, the observational success or practical considerations. Conversely, along the lines of Bohr, these relations do not possess any (Physical) meaning but only represent lack of predictive knowledge, because they are merely mathematical abstractions. This debate is still controversial.

3.4 The assumption of completeness

According to the Copenhagen interpretation:

> The wave-function is associated with an individual physical system. It provides the most complete description of the system that is, in principle, possible. The nature of the description is statistical, and concerns the probability of the outcomes of all conceivable measurements that may be performed on the system.[2, p.9]

First, it is important to remark that only systems within the domain of quantum mechanics are considered. Thereby, the assumption of completeness fix the limits of knowledge about quantum systems. If statistical predictions correspond to the complete conceivable knowledge that may be described, and this description is provided by the wave function, then one cannot know more than what is actually contained in the wave function. Under these circumstances, two different views arise:

(I) First, in the case of having a complete description of individual (Physical) systems through ontological probabilities, then indeterminism remains intrinsically embedded in the nature of the system and the meaning of these probabilities is independent of our ignorance.

(II) Second, probabilities are not real entities but are tools that describe all possible information to be obtained by means of quantum mechanical measurements.

Following along the lines of (II), Bohr is convinced that the wave function does not possess any (Physical) significance and it is just a mathematical tool, representing an ensemble of possible outcomes. From this epistemological point of view, the ontological assumption that nature is, in principle, grounded by chance cannot be accepted. Nonetheless, despite these mutual exclusive interpretations, this thesis will not focus on the ontological approach, and will assume that the Copenhagen interpretation is epistemic. In other words, the fact that the wave function is anything more than a theoretical concept will be accepted.**??**espite this conflict, the idea of completeness of quantum mechanics turns out to be problematic, because one cannot give a complete description of real individual systems in the context of measurements (particles, fields, and so on.), but only from an objective point of view, where quantum systems are individually described independent of the measurement.

3.5 Summary

Considering the basic postulates, i.e., the complementarity principle, the assumption of completeness, and the uncertainty principle, it is important to emphasize here that reality in Bohr's epistemic conception is restricted to the context of measurements. There is a reality that one can observe and describe within the context of measurements, but not beyond observation. In this respect, Bohr does not deny the existence of real entities; rather, there are electrons and atoms, but their properties cannot be determined beyond the context of measurement; their properties are described by mathematical tools in terms of classical mechanics.

Moreover, although this thesis assumes one particular point of view, there is no real consensus about the epistemic or objective character of OQM. The debate about the meaning of the uncertainty relations and the interpretation of probabilities is still controversial. However, besides these intriguing questions, one should point out that OQM is naturally statistical and predictions in the domains of quantum mechanics have shown to be correct.

4 Preliminaries to Bohm's theory

Along the lines of De Broglie's pilot-wave theory, Bohm published a pair of papers in 1952 ([5] and [6]), from which a new quantum theory emerged. In his early writings Bohm appeals to his theory (BQM), as an evidence of the existence of a hidden variable theory within quantum mechanics, rejecting with it Von Neumann's proof. But, it was not until Bell exposed his theorem in his seminal 1964 paper, that BQM entered into the debate and was recognized to be an unique reliable example of a hidden variable theory with a non-local essential structure.

Although later interpretations of BQM tried to ascribe to this theory a realistic, objective, and deterministic picture, that was never his intention. Bohm himself never regarded his theory BQM as an ultimate and fundamental explanation of quantum processes. He was rather convinced on a different approach arising from philosophical ideas which aimed coherence and harmony among all fields of knowledge. In fact, he mainly proposed BQM to show, with the aid of a concrete and consistent example, that alternative interpretations of the quantum theory were in fact possible. This would serve him as the starting point for further developments.

In the second part of this thesis an explanation of these intriguing questions is attempted, but for the time being it is important to go in depth into BQM to understand the formalism and the original content of his 1952 papers. We will call BQM the theory that Bohm himself regarded as merely 'concrete example'. This means that this theory may not be considered, as the foundational and philosophical basis of Bohm's original thought, but just a self-consistent theory that is grounded in a objective, realistic and deterministic basis besides the problems and inconsistencies that Bohm himself subsequently confessed. Let us begin with.

4.1 Introduction to Bohm's theory

According to BQM, a physical system comprises a wave field and a collection of particles that form one inseparable and ontological entity. However, particles can be approximately described alone objectively, determinately, and independent of the context of measurement, because there are only two real parameters in the formalism: the particle positions and their apparent conjugate momentum given by the wave function, which behaves differently compared to the classical fields. In fact, the wave function turns out to be a guiding field acting upon the particles without reciprocal action and living in a multidimensional configuration space. Thus, although one can describe the motion of particles by definite trajectories, particles and waves are not mutually exclusive and autonomous entities. Both might belong to an ontological entity of higher order instead, from which none of them can be seen as independent.

Despite the possibility to describe individual (Physical) processes (with objective particles), BQM and classical physics are ultimately very different. Following along the lines of mechanistic and deterministic descriptions of point particles and fields, classical mechanics holds that the state of a system is described by the position and momentum of every particle in the presence of external fields. Conversely, in the case of BQM, the state of the system is fully described by the positions of the particles and a wave field with non-classical properties, including the presence of a quantum potential that is not external to the system. Moreover, this wave field is a singled-valued function of all positions at one instant of time determining the particle dynamics and the evolution of the system, whereas in classical mechanics the evolution of the field is rather determined by the particle dynamics. However, before making explicit the differences between BQM and classical physics, the Hamilton-Jacobi formulation is introduced in appendix 1(14), for those who wish to follow a detail description of 1952 papers in their original form.

The necessity to use Hamilton-Jacobi formulation in Bohm's approach is still controversial. There are plenty of arguments, some of which regard H.J. formulation as intrinsic to the theory, but also as a tool (or convenience) for a good intuition and visual description. For this reason, in the next part of this thesis, the starting point to assess the interpretation of the formalism and the meaning of theoretical entities is given.

Peter. R. Holland belongs to the branch of Bohmians whom think that Hamilton-Jacobi formulation is not intrinsic to the formalism and is merely a basis for comparison. He developed an extended interpretation of Bohm's early works called 'Quantum Theory of Motion' (QTM). It is important to point out that some elements of QTM, compared to BQM, have proved to be different. Later on these differences are shown and the reliability of some Holland's claims are challenged.

Moreover, according to QTM interpretation, since the Hamilton Jacobi formulation does not possess a fundamental significance in this theory, Bohm's approach does not add new mathematical concepts in addition to the formalism employed in OQM. This formulation is part of the same formalism, but only the form of the equations changes. In fact, QTM goes beyond the mathematical structure, because this interpretation does not assume any important statement that cannot be derived from the mathematical structure of OQM. Therefore, according to QTM, the Hamilton Jacobi formulation has been introduced to construct an intuitive quantum formulation and to reduce the classical approach as a particular and approximative case, considering material processes in space and time.

4.2 The Hamilton's principal function S in Hamilton-Jacobi theory

The derivation of the equations of motion has been introduced in the appendix 1 (14), given the complete specification of what is called 'the Hamilton's principal function' (S).

One important fact is that once the initial conditions of the equations have been fixed, there is just one trajectory corresponding to all solutions S. This is due to the fact there are different solutions to S that obey the same Hamilton-Jacobi equation, i.e, the integral constants that may be added, and also because this equation uniquely determines the motion of a particle, once the initial conditions are given. However, in the appendix 2 (15), this argument is corroborated in the classical framework but not in BQM. For *Classical mechanics* the following statements hold:

(a) Fixing $S(q,t)$, a family of trajectories may be traced, all of them with initial positions q_0.

(b) One common trajectory is obtained from different solutions S_1 and S_2 with fixed common initial conditions $\nabla S_{0_1} = \nabla S_{0_2}$.

For quantum mechanics the first holds while the second does not hold (See in detail in 5.3). In the usual classical framework, the evolution of the field S is given by the particle dynamics via the Lagrangian. However, in quantum mechanics the field dynamics S makes possible to obtain the particle dynamics via the Hamilton-Jacobi equation and the law of motion.
In this case, S may be thought as a field that manifests itself through the motion of the particles. In fact, it cannot be influenced by the particle's motion but *guides* all particles at the same time. In the context of BQM, the complete information of fields and particles should be provided, including S. S is therefore not external to the system, rather, it is internal and must be considered part of one undivided entity composed by the wave-field and the particles together. This is clarified in 5.3.

4.3 The importance of probability in Hamilton-Jacobi theory

In the case of classical mechanics, the initial conditions of every system are well defined. Once we have the equation of motion and their initial conditions, the evolution of the system is already determined. Unfortunately, to know their exact values is not always possible due to practical limitations. For this reason, to give a probabilistic description has proved to be necessary to advance in the knowledge of the system. This has been achieved by inferring several possible trajectories from varying the initial conditions of the system. Although the theory remains deterministic, the evolution of the statistical ensemble corresponding to the current knowledge of the system is not deterministic, and to obtain exact predictions of individual particles turns out to impossible.

Hamilton-Jacobi theory is connected with a probabilistic description, in the sense that considers an ensemble of particle trajectories with different initial conditions given the same solution S. However, we know that ultimately only one track is realized, and regardless of our degree of knowledge, the particle's motion is already determined by the Hamilton-Jacobi equation.

In the classical statistical framework, a probability density $\delta(q,t)$ is defined. In fact, it is the density of particles per unit volume at any given position and time. In this case, it is related with the ensemble of several copies of the same particle which differ by their initial positions. A general result is mentioned here: the number of particles in the ensemble is preserved given by the following conservation law:

$$\frac{\partial \delta}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{\delta \nabla S}{m} \right) = 0 \tag{4.1}$$

This equation together with the Hamilton-Jacobi equation are field equations if S and $\delta = R^2$ are coupled fields. This will be important later by introducing wave-functions into the formalism.
Moreover, δ does not represent the actual position of the particle. It is rather a fictitious

ensemble that represents our knowledge of the system, and cannot intervene in the motion of the particles. This is due to the fact that δ is defined by several copies of the same particle. In other words, for every solution S, we have different δ.

The important fact here is that the evolution and preservation of the initial distribution density throughout time is guaranteed by Liouville equations.

A summary is the following: although the evolution of the system is actually determined, the ignorance of the actual state of the system requires classical statistical considerations. There are two different sort of possible statistical ensembles, because there exist the possibility to fix either S or the initial conditions:

(a) The ensemble of particle trajectories are the result of specifying different initial conditions among the same solution S.

(4.2)

(b) The ensemble of states corresponding to S are the solutions satisfying only one initial condition or what is the same, only one single trajectory.

Both are different in the sense that represent the ignorance about either the trajectories of the particles and the state of the system respectively.

In the following lines, the formalism of this new approach is formulated, namely BQM.

5 The formalism and structure of Bohm's theory

5.1 The postulates

In the following lines the postulates of BQM[4]are given[2, p.67]:

(a) An individual physical system comprises a wave function propagating in space and time together with a point particle, which moves continuously under the guidance of the wave.

(b) The wave is mathematically described by $\Psi(q, t)$, a solution to the Schrödinger equation.

(c) The particle motion is obtained as a solution $q(t)$ of:

$$\dot{q}(t) = \frac{1}{m} \nabla S(q, t)|_{q=q(t)}, \tag{5.1}$$

where S is the phase of Ψ. To solve this equation we has to specify the initial condition $q(0) = q_0$. This specification constitutes the only extra information introduced by the theory which is not contained in $\Psi(q, t)$ (the initial velocity $\dot{q_0}$ is fixed once we know S). An ensemble of possible motions associated with the same wave is generated by varying q_0.

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[4] These postulates are not part of 1952 papers, but are from QTM interpretation. However as far as one pays attention to these postulates, both QTM and BQM are not controversially different.

(d) The probability that a particle in the ensemble lies between the points q and $q + dq$ at time t is given by

$$R^2(q,t)d^3q, \tag{5.2}$$

where $R^2 = |\Psi|^2$.

The latter postulate shows compatibility with the results of quantum mechanics but is not necessary. Contrary to the Born's postulate, the latter is formulated in terms of the probability that a particle actually is at a precise location at time t, independent of the measurement. It does not represent the probability to find it after a measurement.

5.2 Formulation

In orthodox quantum mechanics the concept of individual particle is not well defined, because only provides probability distributions of several particles. Indeed, the concept of particle together with the concept of trajectory just make sense in the context of measurement, but regardless of the former, the definite outcomes are ultimately unpredictable.

Now, following with [5], the introduction of the concept of particle in quantum mechanics is attempted. At first, Schrödinger equation is discomposed into two real equations containing two real fields S and R by a general complex solution:

$$\Psi = Re^{\frac{iS}{\hbar}} \tag{5.3}$$

(5.3) is given in terms of S with action dimensions. This is because S is precisely the Principal Hamilton Function, and from the Schrödinger equation, the Hamilton-Jacobi equation may be derived by making this identification. One then translates all the conditions that Ψ must satisfy in ordinary quantum mechanics to R and S.

Inserting (5.3) into the Schrödinger equation and separating into real and imaginary parts, the following field equations for the fields R and S are obtained:

$$\begin{gathered} \frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + V = 0 \\ \frac{\partial R^2}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{R^2 \nabla S}{m}\right) = 0 \end{gathered} \tag{5.4}$$

Once these relations are known, Ψ conditions are translated to R and S fields. From quantum mechanics, the conditions for Ψ are the following: uniqueness, continuity, finiteness and single-valuelessness.

(a) *Uniqueness.* To obtain a unique solution we have to specify the initial real functions:

$$R_0(q) = R(q,0) \qquad S_0(q) = S(q,0).$$

These are unique under the Schrödinger equation because whatever one inserts, either multiplicative or additive constants, Ψ_0 will be physically equivalent. Thus Ψ uniquely defines S.

(b) *Continuity and finiteness.* R,S,∇R, and ∇S have to be finite continuous real functions.

(c) *Single-valuableness.* R must be a single-valued function of position. Contrary to the former, the value of phase S at each point is not uniquely fixed, for it is easy to prove that different S which differ by a phase factor give rise to the same wave-function Ψ. However, the difference in phase is due to a discontinuity that may be present just in case $\Psi = 0$. Therefore, from the general solution (5.3), the possibility of S to have different values at each point is translated into the following condition:

$$R = 0$$

This result is very important, because nodal regions, where the phase S would change, exist. Nonetheless, the particles cannot be localized in these nodal regions.

Now, let's interpret (5.4). Looking closely, it is precisely the Hamilton-Jacobi equation apart from the extra term $Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ called the "Quantum Potential". It follows that:

$$\vec{v}(q,t)|_{q=q(t)} = \frac{\nabla S(q,t)}{m}|_{q=q(t)}, \tag{5.5}$$

where the initial position q_0 must be specified. The velocity corresponds to the tangent of a possible particle trajectory passing through a given point at a given time (q,t) for all q and t, from which only one track is actually realized.

As already mentioned in the beginning of this section, (5.5) is introduced by the Hamilton-Jacobi theory obeying the Hamilton equations for the one-particle system. It is postulated and then shown to be consistent with the empirical predictions of quantum mechanics. But it does not follow from Schrödinger equation. In fact, this equation is re-written in a manner that is suggestive for a new physical interpretation.

In BQM, the initial velocity or momentum of the particle is determined by Ψ_0. (5.5) shows that if one knows Ψ_0 and the initial position of the particle, then $\nabla S(0)$ may be obtained, and therefore, the unique initial velocity is known. In this manner, the momentum, once the position is given, does not be specified. In fact, for every wave-function Ψ, that is, for every S field, an ensemble of possible trajectories is obtained, with different initial positions.

5.3 The properties of the amplitude R and the phase S of the wave function

The properties of R and S read:

(a) Both R and S are fields defining an individual physical system. They are always accompanying the particle. Their status is not explicitly proved to be ontological and real (for Holland they are).

(b) The properties of the particles are functions of position, which are evaluated through R and S along the particle trajectory.

(c) S and R are causal agents in the *equation of motion* via the quantum potential Q.

(d) S is the generalized-Jacobi function.

(e) S defines the phase-space through the assignment of the position $\Omega(q,p)$.

(f) Although the phase function S is multi-valued, ∇S is a single-valued function of position. This means that at each point in space at each instant of time(excepting nodal points), there is unique tangent vector associated with ∇S and consequently only one trajectory passes through that point.

(g) Particles cannot pass through nodal regions (where $\Psi = 0$), because the tangent to a curve and ∇S is undefined. Non-nodal regions map into non-nodal regions along trajectories.

Now, in the context of quantum mechanics, a common trajectory does not follow from different solutions S_1 and S_2 with equal initial conditions $\nabla S_{0_1} = \nabla S_{0_2}$, because the whole system is defined by the wave besides the initial positions of the particles. Although two different solutions for the Hamilton-Jacobi equation may be found, both could not be the same solution for the Schrödinger equation. In this manner:

(h) Every field S and R correspond to one unique individual system.

6 The interpretation analysis

Let us focus on the interpretation of BQM. Different interpretations of BQM have been proposed, defining the status and meaning of theoretical entities in a different manner. We know that some of these interpretations are different from Bohm's original proposal. Considering this fact, BQM and Bohm's later works ([4], [7], [9], [10], [11]) will be analyzed to reconstruct a complete and reliable version of his theory.

Given this, let us enumerate the main points of this interpretation:

(a) The particle and the field Ψ are not different and autonomous entities, they form part of one fundamental entity instead. Both define one physical system in which neither one nor the other can be ruled out.

(b) The equation of motion $q = q(q_0, p_0, t)$ is derived from the Schrödinger equation, *once* (5.5) has been postulated. The law of motion cannot be derived. As we know from the previous part, the equations of motion are derived given S. But to express explicitly S in the context of quantum mechanics, we must provide a solution that satisfies Schrödinger equation.

(c) The particles respond to the point-value of the field via the quantum potential Q, but a reciprocal action no longer exists. This is due to the fact that the individual particle motion does not necessarily follow from the evolution of the system given by the Schrödinger equation, where the wave and Q are defined. As pointed out above, the law of motion should be assume before, and besides the evolution of the field, the corresponding initial condition for the particle should be known. In other words, the free specification of the initial position does not have any effect on the temporal development of S and R.

(d) Q is coupled with S and depend on its derivatives. It follows that Q is not a preassigned function of the coordinates and is not independent of the dynamics as it is in the case of classical external potentials. Q always depends on the whole quantum state.

(e) As far as the system as a whole may not be appreciate, but only the particle's motion, Q may be considered as a classical potential V. Regardless of the value of V, the motion of the particles depends on the total quantum state via Q and S. So varying Q and fixing V, several possible motions may be generated by the linear superposition of solutions to the Schrödinger equation.

(f) The influence of the field Ψ upon the particles is independent of the intensity I. This can be shown by simply looking to the amplitude term R in Q, which is invariant under multiplicative constants through the definition $I = R^2$. This has important consequences. *First*, the quantum potential does not become ineffective at larger distances, where the intensity of the field considerably decreases. This implies non-locality. *Second*, The quantum potential and thus the physical effects are invariant through normalization of the wave-function as in usual quantum mechanics.

(g) *Interference* is the effect produced by the the addition of two or more linear wave-functions. Interference produces another wave of different kind, which is different from a superposition, because the later generates an additional term in ∇S. Now, if a wave-function is generated from the product of two or more other independent wave-functions, the total description of a physical system can be achieved with the description of every particle guided by independent wave-functions.

(h) Bohm proposed [11, p.326] Q to be a kind of *active information potential.* This idea is related to particles that move under their own energy, but are guided by Q. In other words, moving particles do not generally take their kinetic energy $(\nabla S)^2/2m$ from the quantum potential (that depends on R) i.e, they are energetically independent on the one hand, but are steered by forces via Q on the other hand. Moreover, *The information that belongs to the wave function is potentially active but it is actually active when this information enters into the activity of the particle.* The quantum potential may be imagined as a *background* field in the sense that it is spread out passively in space encoding at each point information on the whole, and it only becomes active when the particles are placed in there.

(i) Every quantity may be expressed in terms of the position. The procedure is in [2, pp.91-95.].

6.1 Probability in Bohm's theory

If one recalls the role of probability in Bohm's paper [6], the following interpretations can be ascribed to the function R^2:

(I) The first one via the quantum potential, which is the key element to understand new quantum properties such as non locality.

(II) The second one via the probability density.

Bohm interprets this function according to (II), ascribing to probability only an epistemic significance. In fact, BQM does not require the usual assumptions on probability, in particular, the theory does not take probability as fundamental. The probability is epistemic and has been introduced to be consistent with the random predictions of quantum mechanics. In this manner, as far as one regards R^2 as distribution function, representing our ignorance in relation to objective positions, the epistemic status of probability distributions remains.

However, to regard R^2 as distribution function δ can be true once we have assumed this equality in advance. Let us clarify this: as far as no measurement has taken place,

$$\frac{\partial R^2}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{R^2 \nabla S}{m} \right) = 0 \tag{6.1}$$

From here, the equality $\delta_t = R_t^2$ does not follow. It only means that if this equality holds at one initial time t_0, then it holds for all t. The justification of this equality should be provided by the initial condition $\delta_{t_0} = R_{t_0}^2$.

So if R_t^2 corresponds to our ignorance in relation to the position of the particles, the justification of the initial condition $\delta_{t_0} = R_{t_0}^2$ must be addressed by additional knowledge that is practically impossible to obtain. This is why the probabilistic interpretation of R^2 is avoided in the context of BQM, turning out to be a field playing an essential role via the quantum potential. However, two questions remain: what is the cause of our ignorance? Why does the initial condition take this form?

Bohm argued in [6] that the need for such an ensemble arises from the chaotically complicated character of the coupling between the quantum particles and classical systems, at which measurements take place. Indeed, a cuasi-chaotic and randomly motion can be obtained in mostly all systems with one or more particles. Accordingly, this could be translated to the case of the microcanonical probability measure in the H theorem.

However, how exactly it works has remained obscure and no one has yet given a proper proof. Today most Bohmians agree that it has roughly the same justification as the microcanonical probability measure, but the later has not been justified after all!. According to Callender:

> Neither result succeeds in rigorously showing that any initial probability distribution will work. As a result, for more than fifty years the distribution postulate has been the subject of scrutiny, with many bohmians attempting to show that the postulate has essentially the same justification as the microcanonical probability measure does in statistical mechanics.[26, p.5]

Then, in order to justify the micro canonical probability measure,

> Some believe the thermodynamic limit solves the foundational puzzles, others environmental perturbations, others symmetries and ignorance, others mixing dynamics, and others the H-theorem.The answers these theories give are sometimes no more similar than chalk and cheese.[26, p.5]

Whatever its reality, the problem is not to give an account or justification of any distribution postulate, but rather, the meaning of probability in BQM. In fact, one thing is the interpretation of probability (our ignorance) in which we are concerned, and other thing Bohm's point of view about justifying R^2 as probability (justifying the origin of our ignorance). BQM distributions do only have epistemic significance and do not ascribe to the theory any meaningful ontology. This means that no matter what the justification of the distribution postulate is, R^2 represents our ignorance in relation to the behavior of systems in certain situations rather than a principle or fundamental manifestation of nature.

6.2 The hidden variables

Hidden variables are variables that are hidden in quantum mechanics, but are additional entities that help us to ensure that the theory is complete in the sense that every (Physical) entity has its counterpart in the quantum theory. In the BQM picture, since the observed position is what the actual position evolves into, the consideration of positions as hidden variables is misleading. Although particle trajectories cannot be directly observed, they cannot be considered as hidden variables. In fact, they are unobservable due to our actual ignorance. What should rather be considered as hidden variable corresponds to the wave function, because the introduction of such entity provides information of how particles move in a deterministic way. However, from the point of view of BQM, these entities are always guiding particles and cannot be thought as structurally independent.

6.3 Many body systems

To begin with, let us define a *N-body system* in the following manner:

(a) Let $(\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_N)$ be positions of each particle in $\mathbb{R}^3$ at one instant of time. *One* wave-function $\Psi(\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_N)$ is given as $\Psi \in L^2(\mathbb{R}^{3N})$ where $\mathbb{R}^{3N}$ is the configuration space of the system.

(b) A set of N particles are defined, each one with trajectory $\mathbf{q}_i(t)$ in three dimensional Galilean space.

(c) The set of all N-trajectories in Galilean space correspond to one and single trajectory in configuration space.

In the case of a many-body system one wave function is defined together with the set of n-particles. Moreover, one may ascribe to configuration space as much (Physical) reality as we do to Galilean space in the one-body theory, because the wave function together with the particles define one inseparable ontological entity.

Now, the properties of a many-body system are inherited from those defining a one-body system. In fact, the set of functions $\nabla_i S$ form a single-valued function of position in configuration space, and the trajectory expressed in terms of the n-particles do not cross. However, once the single trajectory has been projected to three dimensional space, the trajectories obtained, in general, cross.

The properties of a many-body system are:

(a) *State-dependence.* The evolution of the particles given by the quantum potential is determined by the wave function, which is external to them and only depend on their positions at a given time. Since an infinite set of possible quantum potentials is associated with the same physical situation and Schrödinger equation, this property is essential to BQM.

(b) *Non-Locality.* Looking at the equation of motion of a many-body system, the motion of any particle depends on the coordinates of all other particles *at the same time.* This implies the existence of correlations among their individual motions, such that if part of the system is disturbed in a local region of three-dimensional space, the wave changes in configuration space, and all the particles are then affected. Moreover, there is an action occurring at large distances, because the quantum potential does not depend on the intensity of the wave.

(c) *Distinguishability.* A set of particles are *identical* if the physical intrinsic parameters associated with them are the same. Because the intrinsic properties of the particles respond to associated fields, the latter definition may be translated to the symmetric property of the S field in the presence of the quantum potential. In other words, if one interchanges any two identical particle positions, then R and the derivatives of S, corresponding to the energy and momentum, are preserved.

> Off course in the case of BQM, Ψ should be symmetric or antisymmetric. Although the particles are identical, they are distinguishable by their trajectories, and the former feature has nothing to do with symmetrization. In fact, symmetrization manifests by correlations between particles, given by the quantum potential and determined by Ψ.

Part II

The problem

7 General introduction

Before the main problems are written, an overview of BQM foundational issues should be provided. Furthermore, a line between BQM and other interpretations should be drawn to understand this theory in a better way.

Since the publication of Bohm's 1952 papers [5] and [6], extensive literature has been written, not only to criticize BQM, but also to support the theory based on a critical examination of Bohm's original work. Moreover, BQM has been the focus of many misunderstandings and confusions. For the sake of clarification, the following issues are analyzed:

(a) Differences between BQM and other interpretations to clarify the meaning of the quantum potential, the field and the particles, must be shown.

(b) To explain and justify Bohm's philosophy developed after the 1952 publications.

(c) To respond to some intriguing questions along the lines of Hiley's mathematical approach to Bohm's philosophy.

Let us begin with.

8 Introduction to Bohm's interpretations

8.1 The status of Bohm's theory

In this part, some possible ontologies that prove to be consistent with BQM are explored.

Besides the quantum postulates that are the basis for Bohmian interpretations, a reliable ontology should be assumed to draw a line between BQM and other interpretations at the fundamental level, and to guarantee explanatory power to the theory. If one aims a realistic position, one cannot only rely on empirical knowledge: theories that are empirically equivalent might differ in their ontology and other important features as basis for a suitable explanation.

At first, one could think of a particle ontology in BQM. Nonetheless, the behaviour and motion of BQM quantum particles are different from other theories, besides the quantum potential term appearing in the equations of motion. For example, in classical mechanics correlations between particles are not observed; and in relativity there is not such thing as non-locality. Therefore, another ontology should be considered, which retrieves the empirical predictions of OQM and makes possible an objective description of particles in motion.

Although Bohm succeeded in suggesting a reliable example of hidden variable theories, he hesitated about some BQM foundational aspects that were not appropriate to his natural philosophy. In fact, for him the deterministic and objective description of particles in motion was apparently in contradiction to the behavior of quantum systems. Bohm was convinced that quantum systems could not be described in the same way as classical mechanics, ascribing to both particles and fields the same classical properties. Both were not playing the same classical role anymore, and to conceive them as ultimate and autonomous entities would eventually reveal some inconsistencies in the domain of quantum mechanics. This was the motivation for him to propose another ontology in the context of BQM, which aims consistency with the state-dependence described above 6.3: the wave-particle ontology. However, one question remains: if particles and waves cannot be regarded as different ontologies, what exactly wave-particle means?

To answer this question, some aspects about Bohm's later works should be clarified. After his 1952 publications, Bohm developed a new interpretation of the formalism provided by BQM, which explains and clarifies the significance of this particular ontology. Bohm was not aiming to establish a deterministic and mechanical theory, but was proposing a particular interpretation arising from a new and broad notion of reality that he was developing during those years. Let us explain this in detail.

During the following years after the 1952 publications, Bohm engaged in an extensive research towards a nobel philosophical view. After several discussions and publications with prominent philosophers and physicists, Bohm developed the idea that nature was in fact mediated by universal processes. He gave fundamental priority to changes and processes occurring in nature, over any stable, definite, and absolute ontology defining physical systems.

The way he reaches this conclusion comprises many philosophical aspects that are discussed later. However, Bohm points out the fact that theoretical descriptions have evolved in time, to the extent that have contributed to develop certain paradigms along the history of science, but non has been able to overcome new problems that suddenly appear, as measurement techniques are better. One example provided by Bohm corresponds to the old standard view in science, namely, the mechanistic model of nature. He argues that with the advent of new domains in physics, to which classical mechanics probed to have limited scope, the standard mechanistic description of a system was not appropriate anymore. Physicists believed that any complete description may be achieved by examining the individual parts and the manner in which they interact each other. However, this particular view faced serious difficulties with the advent of electromagnetic phenomena and gravitation and was replaced by other theories such as relativity theory and quantum mechanics, in fact, giving rise to alternative ontologies.

In this manner, in the face of quantum phenomena, where mechanistic descriptions are not appropriate, Bohm is convinced that reality is not composed by autonomous particles that are ultimate and may make up together the whole; rather, he feels confident about the importance of the wave function, and the impossibility to split up the particles from this field.

Likewise, Bohm's philosophical ideas had evolved into a new understanding of quantum theory assuming an holistic position. Accordingly, the quantum state is given by the wave function and the position of the particles. However, although positions define the state of the systems at a given time, the wave function is required to govern the evolution of the system, and is in fact responsible for the non-local quantum effects observed in the experiments. This is the reason to give ontological priority to the configuration space, from which the particles are only three-dimensional projections of a higher dimensional reality. In other words, Bohm is convinced that any particle, field, and any ultimate ontology is a particular and contingent case of a higher dimensional reality, which is, in principle, variable. He claims everything is part of a total process, from which all physical systems and entities defined therein, are approximately stable up to a certain degree. This is explained in detail in the following section (9).

Now, paying attention to BQM and the way is formulated, one may conclude that Bohm's philosophy is somehow implicit in the formalism but is not actually represented therein. The question here arises whether one can have a (Physical) counterpart of the process and wholeness involved in Bohm's philosophy. The answer is no. As it is explained later, Bohm's philosophy cannot account for any kind of ultimate ontology as one cannot split up the particle-wave ontology into independent elements. Yet, Bohm's argument rests on the idea that everything is process and change in such a way that a clear picture about the veracity of a theory cannot be obtained. However, a problem arises here: if knowledge is never true, never false, then there are no good reasons to prefer one theory over the other!

To shed light to these difficulties, a good starting point is to find out the theories that are consistent with the current status of knowledge. In fact. such philosophical position suggests to see BQM as a particular approximation to the total variant process of knowledge, comprising elements that are consistent with current experimental developments, and getting rid of certain scientific paradigms that are not appropriate with the advent of new domains.

8.2 On a reliable interpretation of Bohm's theory

Currently, there are several interpretations about BQM that were developed after the 1952 publications. A brief analysis reveals that they are not on equal footing, because some of Bohm's interpreters missed the central point. In fact, Bohm's original papers [5] and [6], did not aim to develop a fundamental deterministic theory that could stand on a permanent basis. Conversely:

> The author's principal purpose had not been to propose a definitive new theory, but was rather mainly to show, with the aid of a concrete example, that alternative interpretations of the quantum theory were in fact possible. Indeed, the theory in its original form, although completely consistent in a logical way, had many aspects which seemed quite

> artificial and unsatisfactory. Nevertheless, as artificial as some of these aspects were, it did seem that the theory could serve as a useful starting-point for further developments, which it was hoped could modify and enrich it sufficiently to remove these unsatisfactory features.[12, p.75]

When Bohm claims that BQM "had many aspects which seemed quite artificial and unsatisfactory", he refers to some concepts introduced by the theory, which are supposed to be problematic. These concepts correspond to the quantum potential and the wave function, apart from fundamental problems arising from the incompatibility of the theory with special relativity.

In fact, in his book [12] Bohm writes four arguments aiming to establish a self-criticism to his own interpretation: the first two arguments are about how to extend BQM to the spin of the electron and to the case of relativistic electrons. The third argument is about how to clarify the concept of a wave in a $3N$-dimensional space, to which he felt incredulous. Finally, the fourth argument is about the ambiguous definition of a quantum system, which is likely to be quite different from a simple wave or a simple particle.

Although Bohm could not provide a suitable answer to these questions, he was convinced that BQM was indeed the starting point for the development of a new kind of view, taking into consideration his holistic philosophy that was developing during those years.

This is an important issue to remark, because there have been a lot of interpretations that ascribed to the theory several features that Bohm never assumed. According to Hiley:

> His own assessment of his early work seems to have been forgotten and in consequence, the work he published in later papers refining and making new proposals tends to have been completely ignored.[13, p.2]

Of course, interpretations developed by Dürr and others [14] were remarkable, which aimed to explore how quantum mechanics arises from a new fundamental status of particles and fields. However, since these interpretations do not share the same ideas developed by Bohm's later writings and they are ultimately independent from BQM, they cannot be considered as a continuation to BQM.

For this reason, BQM and other interpretations are compared. At first, BQM should be compared with QTM. In fact, although they are related to the same formalism, they have quite sharp differences behind the ontological interpretation of (Physical) states. In other words:

BQM and Holland's interpretation QTM are not fundamentally the same theory.

According to QTM, the particle ontology together with the ontology of the wave field should be regarded alone as independent. In other words, in contrast to Bohm, QTM ascribes to waves and particles a fundamental and independent status. Holland imagines real particles that are guided by a real wave field via another fundamental entity called the quantum potential. In fact,

> Ontologically the wave and particle are on equal footing (i.e., they both objectively exist) [...] The particle is always treated as a point within the wave (regardless of the form of the latter). They are logically distinct ultimates.[2, p.79]

In contrast, BQM ascribes to both particles and fields an inseparable ontology. They can be divided up to a certain degree of approximation, but cannot be regarded as ultimate, autonomous and fundamental entities. It follows that foundational assumptions about the status of particles and fields make a distinction between QTM and BQM. QTM corroborates the ontological status of particles and fields whereas BQM denies so. In this manner, as far as this difference is not appreciated, the rest of both theories are actually similar (we shall call BQM-QTM to appeal to both theories regardless of their differences).

Moreover, there exists plenty of differences between different interpretations with regard to the meaning of the quantum potential. In particular, Bohm argued in his 1974 paper that "Our attitude is that we can sooner or later drop the notion of the quantum potential". Although, it is not clear the meaning of such entity in BQM, we can in fact interpret the theory from the point of view developed after [5] and [6] publications. This will be explain better along the lines of Hiley.

Yet, the ontology of the wave function together with the ontology of the particles have been refuted by prominent philosophers and physicists in recent years. For this reason, the question is here to assume the idea of which theory is the best candidate among all theories that have discussed so far (OQM, BQM, QTM).

8.3 Should we believe in Bohm's theory?

The preference of one over other interpretations has been controversial. In fact, there is always a price to pay when one wishes to support another interpretation of quantum mechanics besides OQM. For example, although some particular problems are solved in the context of BQM, i.e. the measurement problem, other kind of issues are introduced (such as the quantum potential and the point particles), which are sometimes vague. For this reason, the only apparent way out to the problem of theory choice depends on the strength of the arguments and the criteria used to prefer one over the other. Perhaps, the criteria may depend on whether a deterministic theory is preferred or on alternative aspects related to the ontology of the theory.

To compare different interpretations, OQM should be brought about into the debate. As mentioned OQM is contextual: objective descriptions may only be obtained just after measurements have occurred. In the rest part of the process, OQM is just a mathematical device for predictions and whether statistical distributions are epistemic or not, still remains controversial.

Moreover, besides BQM-QTM and Many worlds interpretation [3](or interpretations derived from them), there are no alternative theories capable to explain the same empirical data. However, if one wishes to stand on a realist point of view, no matter if they are deterministic or not, the following theories should be considered: OQM, Many Worlds, modal interpretations, BQM-QTM and other Bohmian interpretations. Now, one question emerges: which is the best theory?

In this thesis, arguments about the modal interpretations and Everett's approach will not be discussed. Although they should be analyzed in further works, the theories that are discussed here are OQM, BQM and all Bohmian interpretations.

At first, some objections against BQM-QTM are discussed, and later this theory is compared with OQM and other Bohmian interpretations. According to Holland, these are the most important objections against BQM-QTM, with corresponding answers [2]:

(a) *One cannot prove that the trajectories are there.* This is indeed true, however, science admits non-observables. The argument in favour of trajectories lies on its ability to make intelligible empirical facts.

(b) *QTM attempts to return to classical physics.* In QTM a different notion of state is invoked, laying beyond the material points of classical physics. Therefore, this theory cannot be part of classical physics.

(c) *There is no reciprocal action of the particle on the wave.* It is true, however, this claim cannot be regarded as a logical compelling deficit. QTM is different from a classical field theory.

(d) *QTM is counterintuitive.* QTM is indeed against classical intuitions. However, the concept of intuition is contextual and historical.

(e) *The price to be paid is non-locality.* It is not comparable to the price we should pay denying an objective view or reality.

Looking closely, mostly all these objections are apparently solved by invoking non-classical arguments. In fact, BQM-QTM is different from old theories, which are not feasible anymore in the face of quantum phenomena. In this manner, some of these objections vanish when one assumes another view differently from classical descriptions. However, the asymmetry of the particle-wave dynamics and multidimensional non-local fields, are thus the intriguing features that we should deal with. Of course, BQM solves the measurement problem by giving an objective description of particles in motion, nonetheless the price to be paid for this resolution is non-locality given by the quantum potential, besides unusual properties belonging to the wave function.

In the following lines, a brief explanation of the importance of non-locality is given, besides the motivation for many Bohmians to prefer this view over other interpretations. In [4] Bohm claimed that BQM was precisely the way to explain through imaginative and intuitive means the behavior of entangled systems of many particles. To give an objective description of particles in motion and to conceive a non-local wave-function explained by mechanical processes seem to be the most picturable way to explain the behavior of such systems. However, to make theoretical descriptions clearer does not mean they are fundamental, in the sense that BQM may describe the truth behind the phenomena. For this reason, the main motivation to prefer BQM over other interpretations lies in its ability to suggest further descriptions that are much more closer to the quantum domain. Looking closely, this criteria, used to give priority to BQM, has been motivated by Bohm's philosophy.

In detail, in BQM-QTM the wave function and the positions of the particles contain the complete information of the system in the presence of correlations. The difference between OQM and BQM-QTM partly lies on the impossibility for the former to explain these correlations through individual and objective processes whenever any measurement has not occurred. In fact, correlations are contextual and definite values of quantities are only known when the wave function has collapsed. In contrast, BQM-QTM explains causally how these correlations come about before, during and after the measurement with the help of the quantum potential. The measurement problem is solved then through the introduction of the particle-wave ontology.

Thus, the new features pertaining to BQM-QTM are determinism, objectivity and non-locality. Neither the many-worlds interpretations nor the orthodox interpretation have such status. In the first case, a new philosophical idea has emerged along the development of branching theories, but in which the concept of many worlds [31], is still problematic. In the second case, ontic statistical considerations have been important but not strong enough to solve the measurement problem. Among these considerations, in the context of BQM, several questions emerge: What are the potentials? Do they exist? Bohm has suggested a solution to the measurement problem but BQM lacks interpretation and meaning about potentials.

However, as we have pointed out, Bohm himself was not convinced about the foundations of his original theory and thought of BQM as a mere intuitive approximation. His essential achievement was for him the contact with quantum theory via the wave-particle ontology, the interpretations ascribed to the quantum potential, and his philosophical approach that Bohm would eventually develop in his late years in collaboration with Basil Hiley.

For the time being, the most important papers written so far will be introduced, and then our own position will be grounded. In fact, our discussion will focus on the meaning of the quantum potential, together with the ontologies considered by different interpretations of BQM. Following the lines of [14], [15], [16], [18], [19], [21], and [26], an overview of one of the the most important interpretations is given, say, 'Bohmian Mechanics' (BM):

8.4 Bohmian Mechanics interpretation

BM is a theory about point particles, compatible with the predictions of OQM. Those particles describe trajectories in three-dimensional Galilean space, described by simple deterministic equations of motion, specifying their positions at any time. In contrast to the case of classical mechanics, the state of a quantum system in BM comprises the configuration of all the particles of the system $Q = (\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_N)$besides one unique wave function determining their motion (Q, Ψ).

Let $q_k(t) \in \mathbb{R}^3$ be the position of the k-th particle of the system at time t, and $Q(t) = (\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_N) \in \mathbb{R}^{3N}$ its configuration. The equations of motion for N particles in BM form a system of two simple equations, one which corresponds to the law of motion determining the motion of the particles ($d\mathbf{q}_k/dt$), and the other expressing the evolution of the wave function ψ by the Schrödinger equation:

$$\begin{aligned} \frac{d\mathbf{q}_k}{dt} &= \mathrm{i}\frac{\hbar}{m_k}\mathrm{Im}\frac{\nabla_k\psi(\mathbf{q}_1,\mathbf{q}_2,\cdots,\mathbf{q}_N)}{\psi(\mathbf{q}_1,\mathbf{q}_2,\cdots,\mathbf{q}_N)} \\ i\hbar\frac{\partial\psi(\mathbf{q}_1,\mathbf{q}_2,\cdots,\mathbf{q}_N)}{\partial t} &= -\frac{\hbar}{2m}\nabla^2\psi(\mathbf{q}_1,\mathbf{q}_2,\cdots,\mathbf{q}_N) \end{aligned} \tag{8.1}$$

The formalism of BM involves the same mathematics as in the usual OQM. The only difference is that the law of motion is not postulated classically, but follows from requiring Galilean and time symmetries in the Schrödinger equation. For this purpose is important to understand the role of symmetries in BM, and in quantum mechanics in general.

8.4.1 Role of symmetries in Bohmian Mechanics

The introduction of symmetries into the formalism of quantum mechanics plays an essential role in the theory, and yet provides an alternative formulation that does not require to postulate the law of motion and to insert the quantum potential term. Therefore, we need to understand the significance of such symmetries in the case of this particular interpretation.

In 1931 Wigner developed a systematic and axiomatic formulation of quantum theory based on the role of symmetries [34]. Indeed, one can show that every physical symmetry (Galilean) under which certain quantities remain invariant (the momenta, angular momentum and so on), acts as a unitary transformation in Hilbert space, and one may obtain the Schrödinger equation as far as no measurement has been performed. Of course, this was known by the time Dürr developed BM. However, they are not at all the same formulation.

To figure out what Dürr did in BM, it is worth it to explain as follows: Dürr starts developing the formalism with both Galilean symmetry and time symmetry, in addition to the Schrödinger equation. On the one hand, the Galilean symmetry (space invariant symmetry), is indispensable if one aims to ascribe an essential role to conservation of momentum, both linear and rotational. On the other hand, the time symmetry is required to introduce reversible equations of motion and energy conservation.

The role of both symmetries do not follow from classical considerations, they are rather general requirements that are essential to determine the equations of motion (and indeed, they prove to be consistent with the evolution of Schrödinger equation. In opposite direction to BQM, in which the classical law of motion has been postulated, Dürr derives (8.1) from basic symmetries (a brief procedure to derive (8.1) is in Appendix 3 (16)).

BM does not use the notion of the quantum potential. The primary ontology corresponds to the particles, which are in fact the only (Physical) constituents of our world.

Now, a much more extended description of this theory is written:

8.5 On the interpretation of Bohmian Mechanics

BM formulates Bohm's theory in a different way and suggests another interpretation in relation to theoretical entities. The purpose here is to investigate the meaning of such entities arising from an examination of their properties and the connection with BQM. A brief discussion about the formalism in BM is given, and then the meaning of probability is provided along the lines of [14]. Finally, the meaning of the wave function is discussed with the help of [21], [22] and [18].

8.5.1 The wave function of the universe

Let us imagine a universe of particles governed by (8.1). The entire system of the universe cannot be a subsystem of a wider system, it is rather ultimate in the sense that contains absolutely all particles of the universe, including the observer. The motion of all particles in the universe is choreographed by the *wave function of the universe*, say Ψ, living in a $3N$-dimensional space, which corresponds to the configuration space of the composite system. In this manner, the wave function of the universe is not at all a meaningless concept in the context of BM, as it influences the motion of the particles according to the law of motion. Besides these properties, the wave function is unique, timeless and cannot be controlled or fixed by any observer. This is in fact argued in detail in [22, p.11].

Moreover, to the extent one refers to fundamental reality, one should think about the universal wave function expressing the laws of the whole universe rather than any other entity expressing the laws of only one part of the world. Indeed, as one can see in (8.1), a complete specification of the motion of the particles can be achieved, if one takes Ψ to be the universal wave function. Otherwise, it would not be a complete description.

For all these reasons, to draw special attention to the special properties of the wave function of the universe is essential. In particular, one of the important features of BM, naturally described by the form of the wave function, and deeply imbedded in the nature of quantum systems, is non-locality. In fact, the configuration space in which the wave function lives, plays a central role in defining the non-local character of BM. Looking at (8.1), the velocity of each particle depends on the position of all the particles, no matter how far away they are. However, the amount of action at a distance depends only on the degree of *entanglement* of the wave function on the configuration space. In the following lines this is discussed:

8.5.2 The wave function of subsystems

A crucial step towards the justification of BM is the empirical equivalence with OQM, where there is no universal wave function. Indeed, in the predictive domain of quantum mechanics, the wave function of a subsystem, say ψ, should be considered, rather than wave function of the universe. The latter follows from foundational elements of the theory but does not add any predictive content. As far as we deal with familiar quantum systems such as the hydrogen atom, Stern-Gerlach experiments, or any other of this kind, only subsystems of the whole universe are considered.

This is how the concept of the wave function of a subsystem arises, because it is required to describe subsystems of the universe up to a certain degree of approximation. In fact, the central idea of BM is to start from the fundamental reality, represented by the wave function of the universe in $3N$-dimensional space, and then simplify all descriptions to subsystems of lower dimension by means of approximations. The behavior of the parts of any system is already determined by the behavior of the whole. This means that one may construct the wave function of subsystems from the universal wave function. In [14, pp.21-29], there is an extensive description of what Dürr called *Conditional wave functions*. In fact, in the context of BM, the conditional wave functions behave exactly the way wave functions are supposed to behave in OQM: they evolve in time according to the Schrödinger equation (when the subsystem is suitably decoupled from its environment), and they collapse according to OQM.

The concept of factorization is essential, because it is the requirement for the wave function of the universe to split up in conditional wave functions. From quantum mechanics one knows that an ensemble of a composite system W is factorizable if and only if complete knowledge of such ensemble is obtained with the knowledge of their corresponding subsystems (described by sub-ensembles w_i). In other words, $W = w_1 \otimes w_2$ only in case W factorizes into a pair of subsystems. Conversely, if w_i is known, and complete knowledge of the current state of the whole system cannot be obtained, then Ψ is not factorizable, and therefore one says it is *entangled* ($W \neq w_1 \otimes w_2$).

The question is whether or not individual subsystems of the whole state W, described by the ensemble $w_i = \sum_j a_j \left|\psi_j\right\rangle \left\langle\psi_j\right|$, are composed of pure states ψ_j with certain probability a_j. If this were the case, an ensemble of composite systems W might be divided up into sub-ensembles or pure states (proper mixtures). An example of an entangled state would turn this clear: a system of two identical particles that are in the same state; in this case, one cannot assign a pure state to the individual particles, although the state of the composite system is pure. They are, in fact, entangled.

Factorization is the essential requirement for the wave function of the universe to split up in conditional wave functions. If W factorizes $W = w_1 \otimes w_2$ or $\Psi = \psi_1\psi_2$, then the splitting $v^{\Psi}(v^{\psi_1}, v^{\psi_2})$ is possible, and in particular, the equation of motion of one of the particles is obtained, as long as factorization occurs.

Moreover, in the context of BM, the form of the quantum states of subsystems are not all on the same footing as in OQM: one of the elements $\left|\psi_j\right\rangle$ in the sub-ensemble w, and only one, is selected, or more precisely supported by the outcome corresponding to j which actually occurs. This is the objective character of the theory. Now the definition of conditional wave functions *at a given time* follows from here:

$$\psi = \Psi(q, Y) \tag{8.2}$$

ψ and Ψ correspond to he wave function of the subsystem and the universe respectively, whereas q and Y are the configuration of the positions of the particles in the subsystem and the *fixed* configuration of the rest respectively. Y is fixed because it refers to the actual configuration of particles of the surroundings.

Now, what is important to the purpose of this thesis is that:

> *Factorization is extremely unphysical*

After all, interactions between system and the environment tend to destroy the factorization. In particular, they occur whenever a measurement is performed in one of the subsystems. Conditional wave functions do not take part of the fundamental character of the theory, they are secondary elements that are defined by approximation and hold up to a certain degree.

8.5.3 Probability as epistemic

If the BM interpretation of probability is discussed, one is facing a similar case as in BQM: BM thinks of probability as epistemic. However, the epistemic character of probability in BM is not as evident as it is in BQM. Distributions are indeed introduced to be consistent with the empirical predictions of quantum mechanics but they cannot be considered as fundamental due to the objective character of the particles in the theory.

In detail, the epistemic character of BM distributions follows from the equality between the distribution of particles with conditional wave functions $|\psi|^2 = \delta$[5], which are approximately derived from the wave function of the universe. Non-fundamental conditional wave functions must be introduced as far as we deal with the statistical predictions, because a distribution function cannot be defined for the wave function of the universe[6]. It follows that when we speak of distributions δ we are actually referring to "conditional empirical distributions", because they are equal to the square of the norm of the conditional wave function that are only relevant in the context of experiments. Thus, conditional probability distributions are given by:

$$P(Q_t \in dq | Y_t) = |\Psi_t q|^2 \, dq, \tag{8.3}$$

where $\psi = \psi_t^{Y_t}$ is the conditional wave function of the subsystem at time t. In particular, this conditional distribution of the positions of particles of a subsystem depends on those of its surroundings only via the wave function. "The entire empirical statistical content of Bohmian mechanics flows from this definition (8.3) with remarkable ease".

On the same footing as in BQM, Dürr departs from the initial distribution postulate (in terms of BM the *quantum equilibrium hypothesis*, in which the initial distribution of the configuration of any subsystem is equal to $|\psi_{t_0}|^2$), and justifies the invariance over time through the equations of motion by "equivariance". In fact, taking into account that $\delta_t = R_t^2$ has been translated to

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[5] Equal to $\delta_t = R_t^2$ when $\psi = Re^{\frac{iS}{\hbar}}$

[6] In [14, p.19], "What physical significance can be assigned to a probability distribution of the initial configurations for the entire universe? What can be the relevance to physics of such an ensemble of universes? After all, we have at our disposal only the particular, actual universe of which we are a part."

$|\psi_{t_0}|^2$, taking $\psi = Re^{\frac{iS}{\hbar}}$, Dürr assumes the initial distribution postulate, and with the evolution of the equation of continuity (6.1), he shows that the distribution postulate holds for all times.

Moreover, similar to the case of BQM, there is a justification of the *quantum equilibrium hypothesis*. This is typicality exposed in [15]. But as we have claimed, our interest here and what is important for us, regardless of the concept of typicality, is the meaning of probability distributions in BM. As far as our discussion is concern, BM has not given any postulate at the fundamental level of the theory. Thus,

> We regard the quantum equilibrium distribution P, at least for the time being, solely as a mathematical device, facilitating the extraction of empirical statistical regularities from Bohmian mechanics and otherwise devoid of physical significance. [14, pp.30-31]

8.5.4 The wave function as nomological

In regard to the meaning of the wave function several questions arise: does it merely describe our ignorance or does it describe reality independent of the observer? Is the wave function epistemic or objective? If it is objective, is it (Physical) or something else? In the case of a non-hidden variable theory such as OQM, the wave function cannot be epistemic. Since the OQM wave function provides complete description of the system, the probability distributions do not represent our ignorance. The probability distribution plays a fundamental role and contributes to give statistical ontological meaning to the wave function. Conversely, in the case of BM, although there is a mutual relation between the wave function and probability distributions, they do not play a fundamental role.

The primitive ontology of BM that connects the theory with (Physical) reality are the positions of the particles. No more. However, the wave function provides complete information of (Physical) systems that are governed by the laws of nature. Therefore, the wave function is not epistemic nor ontological, but has a more actual role to play: it governs the motion of the particles. This is in fact the motivation to think about this entity as nomological.

Moreover, in [22] and in [21], there is an extensive explanation about the meaning of the wave function, and why one should not think of it as ontological. The reasons basically comprise the answers to some of the objections against BQM: the asymmetry of the wave, and the concept of the wave living in $3N$-dimensional space. In regard to the second objection, there is a qualitative relation between the Hamiltonian and the wave function. In comparing both, the Hamiltonian lives in a $6N$-dimensional phase space and has the same form as the wave function in some fundamental equations[7]. However, one knows that in classical mechanics, the Hamiltonian do not exist, it is a convenient device in terms of which the equations of motion can be expressed in simple form. Therefore, there are traces to think about the wave function in a similar manner.

To conceive the wave function as nomological apparently leads to some problems to deal with. An important point that weakens such interpretation is the fact that laws are not supposed to

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[7] See in detail in [22]

be dynamical as the wave function is, as well as they cannot be controlled or set as the wave function does. However, in BM there is only one fundamental wave function: the wave function of the universe, that cannot be dynamical and cannot be controlled. So, as far as the meaning of the wave function is concerned, attention towards the universal wave function should be given, to which the laws of nature are ultimately connected.

> From a fundamental point of view the only genuine Bohmian system in a Bohmian universe—the only system you can be sure is Bohmian—is the universe itself, in its entirety. It cannot be an immediate consequence of that that subsystems of a Bohmian universe are themselves Bohmian, with the motion of their particles governed by wave functions in the Bohmian way.[22, p.5]

To conclude, BM constitutes a theory with the following basic considerations:

(a) *The positions of the particles are the primary ontology.* Particles exist in (Physical) reality. It follows that the configuration of the positions are regarded to be the unique primary and fundamental property. The wave function is rather considered as nomological devoid of (Physical) significance.

(b) *BM is deterministic.* The BM equations of motion are deterministic and one may determine the exact position of every particle at any time.

(c) *BM is a quantum theory without observers.* BM is formulated not in terms of what observers see but in terms of objective events, regardless of whether or not they are observed. Unless positions, any operator is not fundamental. BM rejects realism of operators.

(d) *BM Solves the measurement problem.* BM provides a reliable resolution to the measurement problem. In particular, the collapse of the wave function may be derived.

(e) *Quantum equilibrium* This hypothesis corresponds to the assertion that whenever a system has wave function Ψ, then its configuration is random with probability distribution $|\Psi|^2$. From here, the empirical equivalence between BM and OQM follows. This randomness and the quantum equilibrium hypothesis are actually explained by typicality.

(f) *Absolute uncertainty.* Although probability is epistemic, there is an absolute uncertainty arising from measurements of positions, because we may only randomly determine the values of conditional wave functions, using the quantum equilibrium hypothesis. It follows that the ignorance of the state of the system cannot overcome by any technological innovation.

(g) *Non locality.* In BM the motion of a particle in a multiple particle system depends on the positions of other distant particles. This non-local behavior is not due to any (Physical) entity acting between the particles, but corresponds to a natural feature of our world.

9 Bohm's original work

Now, the content of Bohm's original philosophy is described, namely his book [9].

9.1 An introduction to Bohm's philosophy

Along the development from early ideas to the emergence of modern science, there have been theories in all domains of knowledge, where paradigms have been established during a certain period of time. However, these theories have ultimately proved to be approximations when they are extended to new domains. Indeed, to account for all phenomena from one single picture and to find the ultimate explanation have not been accomplished so far. Concerning an endless process of discovery, there is no way to prove that any particular aspect of our knowledge is absolutely correct. Moreover, any attempt to succeed in such enterprise has proved to be wrong.

For example, the theory of Newton, which held sway for several centuries, was supported by a common belief that nature was actually mechanical and deterministic. Complete knowledge of the system at all times was possible by only determining the momentum and position of the parts at a certain time. Moreover, on the basis of a priori conceptions, Newton's model developed an absolute interpretation of space and time. But with the advent of new theories in other domains such as the electromagnetic theory, the newtonian view became to be inadequate with the discovery of electromagnetic fields, which were continuous and possess different properties compared with point particles. In this manner, another paradigm came to play the role o certainty and veracity in physics, regardless the presence of inconsistencies on experiments that were challenging the current theory. Finally along the lines of Lorentz and Einstein, electromagnetism and statistical mechanics gave way to quantum theory and relativity, where fundamental conceptions had been modified.

From this example, the point of view to which we are concern is clarified: a single unification theory is not expected, where conceptions of causality, locality and determinism are ontologically grounded; rather, we are concerned about a particular view where nature does not have boundaries and where change and processes play a fundamental role. On the basis of this view, some dichotomies have proved to be irrelevant. Indeed, distinctions such as *object-observer* and *essence-appearance* are not feasible anymore.

To think about nature as endless process, is to think of knowledge about nature as variable as well. Knowledge is actually never true, never false. The common conception about knowledge as a set of true or absolute statements must be discarded. However, any claim such as being absolute variable lead to erroneous conclusions, because knowledge brings up absolute variance to itself, what seems a contradiction then. But if one thinks of (Physical) objects together with what we know and perceive of them as a whole process, then both knowledge and reality are abstractions of the unknown totality that cannot be perceived separately. In fact, any distinction between them would demand to know something about both terms, what seems impossible, because the whole unknown process is beyond the limits of our knowledge.

Bohm argued in the third chapter of [9], that the idea of process is much more concerned to the idea that everything is flux. What exists in nature is the process itself to become something, and all definite claims such as the determination of what nature is, together with stablished dichotomies, structures, entities and ontologies, are abstractions coming from the unknown process of motion.

In these lines, no matter how the progress of physics comes about, the knowledge of all physical theories will correspond to such abstractions. And for every step, the development of our knowledge will depend on the unknown wholeness of the process.

> It should be clear from the above that we do not expect to come to the end of this process of discovery (for example, in a form that is currently called the "Theory of Everything"). Rather our view is that nature in its total reality is unlimited, not merely quantitatively, but also qualitatively in its depth and subtlety of laws and processes. Our knowledge at any stage is an abstraction from this total reality and therefore cannot be expected to hold indefinitely when extended into new domains.[10, p.321]

9.2 Against any distinction

9.2.1 Essence-appearence

The quotation above rests on the idea that all new insights and explanations follow from the limited validity of previous paradigms, which later are then recovered only as approximations. This is in fact contradictory to the distinction between 'essence" and "appearance". The view about nature as process cannot maintain such dichotomy, because the facts that were supposed to be essences probe eventually to be appearances. A good example are the atoms that were later found to be composed of electrons, neutrons and protons, and later even these particles composed of quarks. In this manner, everything plays both the role of appearance and that of essence. But if this pattern never comes to an end, then ultimately all our knowledge seems to rest on appearances. What then is the point of making an ontological interpretation of the quantum theory?

Every theory is not only a mere appearance, but also must reflect the whole process of reality within its own domain. The scientific activity aims to step on further appearances that erase mistakes and illusions that follow from old appearances. An example is classical mechanics, which proved not coherent with the result of new experiments and other domains such as electromagnetism. But eventually the ultimate reality is unlimited and unknown, and the only way successive appearances can serve us is to follow a coherent action at a certain domain in relation to this reality.

In this manner, the point of making an ontological interpretation of quantum theory follows from a coherent action with the current status of knowledge and discovery. In fact, a new interpretation should be provided, because the old mechanical paradigm is inconsistent with quantum phenomena.

> Which view is appropriate in a given case will depend both on the unknown totality and on our particular mode of contact with it (e.g. the kinds of experiments we are able to do).

> So as the situation changes, different views are constantly called for. But the unknown and unlimited essence is not restricted to any of these views. Rather it may be thought of as somewhere between them and ultimately beyond them, as indeed it is beyond what can be captured in thought, which is always limited to some abstraction from the totality.[10, p.324]

9.2.2 Object-observer

Now, if one relies on the notion of perception in relation to (Physical) objects, the role of appearances and essences may be clarified. The observer (including knowledge, thought, perceptions and consciousness) and (Physical) reality form part of the same total process. This is the main conclusion that arises from Bohm's philosophy. To make a distinction between them would demand certain assumptions that are not feasible under general domains. In fact, on the basis of an ontology of a total process, any distinction would demand stability and absolute claims in relation to this process.

In detail, the observer is part of this total process, because it cannot be excluded from the whole reality. However, to be part of this process does not mean that knowledge and reality are in perfect correspondence. In fact, the observer and the content of any perception about reality cannot be in perfect correspondence with the whole of this reality, because reality is more than any absolute and stable status of knowledge. The whole process of reality includes all stable material objects, besides knowledge.

In this manner, Bohm regards theories as a particular way of observation, which does not follow from any preferred ontology about separated and autonomous (Physical) entities. In fact, the whole total process is beyond any fragmentation or distinction, where autonomous and elementary parts usually constitute the building blocks of nature. In particular, the mechanistic nature of classical mechanics is just an abstraction of this process that is only reliable in the domain of the Newtonian world. However, although this view has brought numerous effects in the way the world is observed, one should not think that corresponds in fact to an actual description *of* the world. In fact, all descriptions are literally true only when theories are absolute and invariant. Nonetheless, the ways of observation have always changed by the empirical evidence and reality has been represented from one particular perspective.

Thus, object and observer cannot be regarded as independent anymore. Reality is not a process absent of any perception, nor a process of solely perceptions. A realistic point of view is assumed here but where the observer is part of one undivided process. There is actually something that we can describe, the only thing is that reality is a flux of processes and any description is true at a certain approximation. Knowledge contributes to the process of reality but this process cannot be described in a complete form.

> Finally, the view that our theories constitute appearances does not deny the independent reality of the universe as a whole. Rather it implies that even the appearances are part of this overall reality and make a contribution to it. What we emphasize is, however, that the

> content of the theory is not by itself the reality, nor can it be in perfect correspondence with the whole of this reality, which is infinite and unknown, but which contains even the processes that make theoretical knowledge possible. It is our view that this approach provides a more coherent understanding of the relationship of theory to the reality towards which it guides us, than do other approaches that are currently available.[10, p.325]

9.3 In relation to the quantum domain

To be more specific in relation to the content of this thesis, the quantum domain is now considered.

Both objects and observers play an important role in the domain of quantum mechanics. In fact, a complete quantum system is composed by the system under consideration together with the instruments employed to perform any measurement. In different interpretations of quantum mechanics this fact has been assumed, no matter the nature of the interpretation. In the case of OQM, the absence of this distinction follows from the uncertainty relations and the complementary principle. Some operators do not commute besides the fact that they are contextual. In fact, any description of the system (in terms of operators), depends on the system and the measurement device. On the other hand, in the case of BQM, the nature of the wave function contributes to the fact that any measurement device cannot be separated from the system.

Moreover, appearances and essences are in fact within quantum theory. At first sight, appearances seem to correspond to the predictions and observations of the instruments, while essences correspond to the supposed quantum fields and particles that are non observable. However, an ontological status to the distribution function of quantum theory is given by Heisenberg's version of the orthodox interpretation OQM, where the statistical nature of the predictions has proved to be fundamental and represents the nature of reality (this is the case where predictions are essences). On the other hand, BQM is an objective description of particles in motion, and for this reason, the predictions that are statistical represent only ignorance in relation to the particles that are actually there. In this case predictions are only appearances while point particles are essences.

However, if our view is concerned with reality as a total process, then any essence-appearance distinction evaporates. According to Bohm, BQM cannot be absolutely true and cannot account for a primary ontology. BQM is a theory of a particle-field ontology that may be proved to be inconsistent in other domains. In this manner, what seems to be the essence of quantum theory in this case (point particles and fields) are abstractions that are derived from the total process at a certain approximation. Essences become appearances.

An overview of the concepts and definitions employed in [9] and [10] has been given so far, but now the following questions should be answered: what branch of philosophy are Bohm's ideas related to? Why are such distinctions observed? Why do we observe autonomous objects? What is the meaning of the quantum potential?

9.4 Bohm's ontological holism

In this part, the concept of holism and the relation with Bohm's philosophy exposed in [9] are described. In detail, to understand the relation between basically all areas of knowledge, the meaning and definition of ontological holism should be given along the lines of Bohm's approach, in which reality is actually a total variant process.

Holism is defined as the thesis that the whole is more than the sum of their parts. However, we are not interested in the broad philosophical approach, but only in those interpretations that prove relevant to Bohm's approach: ontological holism. This is part of a wider branch 'metaphysical holism', based on the thesis that the nature of the whole is not determined by that of their parts. This area comprises branches different from other holistic approaches. An example of other branch contrary to the metaphysical approach is 'methodological holism' that privileges the understanding and description of a complex system at the level of the whole system, and not at the level of its component parts. In particular, an holistic fundamental behavior does not follow from here in the sense of principles governing the nature of the system, but in this case systems are described in this way only by linguistic or methodological purposes.

Bohm's philosophy is actually just part of metaphysical holism. Indeed, it is not merely an artificial or linguistic model of nature, Bohm rather wishes to drill down to the underlying meaning of things. However, once ontological holism has been assumed, the methodology and descriptions are important to construct a coherent theory that wishes to follow along the lines of this philosophy. In this way, methodological holism is assumed but just as a coherent action in relation to the idea of a total variant process.

9.4.1 Ontological holism and the total process of nature

Ontological holism is defined as the thesis in which there are objects that are not wholly composed of basic (Physical) parts. Indeed, this thesis require an adequate clarification of the notion of a basic physical part, but this definition is contextual and depends on the way one means by parts. A particular example is the one exposed by Bohm in BQM. As Bohm himself claims, BQM provides an example that seeks to prove the existence of non-local hidden variables as basic parts (point particles guided by fields). Although this interpretation is not entirely compatible with his later philosophy, it is supposed to be the starting point to his ontological holism. But before going in depth into the content of BQM ontology, Bohm's philosophy should be clarified in a much more extended way.

After a brief discussion of Bohm's philosophy, the (Physical) fact, including matter and the mind, has been defined as "what has been made". In other words, Bohm suggests that the content of the fact cannot be regarded alone from the way we observe, the way we understand, and the instruments used to employ in quantum experiments. Actually, all these aspects are part of one undivided whole process that is ultimately unknown. Indeed, this framework has been introduced into the reality, into the theories, and in the way we perceive and think about the world. Knowledge, perception and matter cannot be seen fundamentally separated anymore, and such distinction emerges only at a certain approximation when one seeks to describe real elements, independent of our observations. For instance, quantum mechanics holds that this distinction is not feasible anymore, and this theory follows from approximations of underlying theories that are, in turn, approximations of other deeper theories. In this sense, as far as one wishes to explain real facts, to make a complete description in correspondence to this reality has proved to be impossible, because those descriptions are always part of the same reality that they are willing to explain. In accord with ontological holism, any distinction of such nature produces fragmentation in the whole total process, and therefore, must be avoided. This is in fact the link between the idea of a process and holistic realism, which are both behind the underlying foundations of Bohm's philosophy. In the following lines this is clarified.

9.4.2 The perception of order

To start with a coherent discussion about ontological holism, the methodology and the language used in different domains of knowledge have to be discussed, particularly those arising from quantum mechanics. In this context, the most important aspect to be clarified is the perception of *order*.

To every revolution in science, humans have developed different descriptions, that are different in terms of ontological, linguistic and methodological aspects. In particular, new theoretical structures have emerged, considering an harmonic connection between the form of description and the content of the description, or in other words, between the linguistic and the ontological aspect of the theories. In this context, new ways of using language that are appropriate to the communication of these dynamical structures have been relevant, partly to avoid confusions and to be coherent with the advent of new domains of knowledge. Moreover, to understand the perception and communication of new structures is important be able to define ratios and measures, distinguished by these structures.

In fact, science aims to discover universal ratios or reasons in (Physical) reality, which includes not only numerical ratios or proportions in the world, but also general qualitative similarities from the form of description to the world as follows: "As things are related in a certain idea or concept, so they are related in fact". Moreover, to this action of giving attention to relevant similarities, a hierarchic order of high degree generally is involved, where orders of lower degree normally end up with boundaries. For example, cat is to feline as dog is to canine corresponds to a ratio of lower degree if we consider the following: feline or canine is to animal kingdom as pine-tree is to plant kingdom. It follows that the relation between the cat and the dog is limited to the animal kingdom. This is where the measure of certain structures emerge, which has been specified by ratio and proportions.

Both ratio and measure have always changed depending on the language and categories employed in the description (whether we consider natural kingdoms or any more general category), and they has been followed by an stablished conception of *order* that aims to have a coherent counterpart in the (Physical) world. For example, when new orders became relevant in the Greek ancient world, a new usage of language had to be developed for the description of this new order. In fact, arithmetics and geometry were constructed with new forms and ratios, so that, one could describe the detailed orbits of planets implying an underlying conception of order represented by degrees of perfection, which corresponded to the order of distance from the centre of the Earth (perfect circles). Later on, in classical physics the Newtonian order was represented by the cartesian grid, defining a collection of points that were moving according to Newton's laws, essentially part of absolute spatial and temporal conceptions. Indeed, we were supposed to think of a particle as a point in cartesian basis, moving along three normal axises and choreographed in terms of this absolute grid.

At first sight, one may suggest that the conception of order in both cases corresponds to idealizations of the complex behavior and characteristics of nature, not at all in contradiction with the common experience of time. However, even with the advent of new experiments that were challenging the old standard view, the conceptions of order remained unchanged and perceptions were adapted to prevailing orders in spite of the evidence against (Physical) experience. This was carried out to the extent that one had been convinced that laws of nature were described by epicycles determining the motion of planets around circular orbits, and also by trajectories followed by autonomous charged particles traveling in ether, respectively.

This was of course valid to a limited extend, nonetheless, it is clear that revolutionary changes in physics have always involved the perception of new orders and appropriate new languages associated to them. When theories change new orders emerge and they cannot be seen as fundamental or absolute. Thus, there is no standard definition of order. What rather seems relevant to the present purpose is to eventually know which are the current conceptions of order, and to see if they are consistent with the language employed to describe new domains in physics.

At first, the current definition of order as rows or series is not feasible anymore. For example, if we define the axes of a cartesian grid with a series of numbers with a successive order, then there could be also different coordinates (polar coordinates) that share implicit similarities (inertial frames), but they are not cartesian anymore. Another example is Euclidean geometry. Euclidean geometry is a linear approximation of curvilinear geometries where raws and series are not naturally constructed. In the former case, ratios and measures emerge from flat space, such as the congruence between angles of different triangles. But some inconsistencies arise at the moment we wish to apply those congruences to curvilinear surfaces. In this manner, different curvilinear geometries imply another conception of order, where Euclidean geometry is a limiting case in common. Moreover, this new order gives some insights into the foundations of general relativity, where curvilinear coordinates defines a dynamical metric field playing the role of gravitation. Point particles are thus replaced by fields defining continuous coordinates within space-time that is not absolute anymore.

From these examples, it follows that the concept of order does not only correspond to series and raws behind some standard conceptions of strictly particular cases. The notion of order also corresponds to a wider structure of some relevant relations between parts or concepts, pointing out similarities that are to be find in the differences among them. For example, in the case of classical physics, one usually pays attention to similarities in apparent independent cases (the free fall of the moon and the apple), but also one usually discards irrelevant features in different aspects, such as the epicycles of Ptolomy. Thus, one may conclude, along the lines of Bohm, that the conception of order is rather everywhere and is part of the total process of reality. As mentioned above, the fact cannot be distinguished from the form of description, because all they are part of one undivided process of unbroken wholeness. The order, ratio and measure form dynamical structures that define the whole fact, and this is the essential role of ontological holism.

9.4.3 The development of new orders

Physicists have corroborated that the old conception of order (either in classical physics and also in relativity) is not feasible with the advent of quantum mechanics. This is discussed now.

In classical physics there is an essential feature that we must deal with. In the context of Newtonian mechanics, once the initial position and momentum of every particle in a system is known, their past and future values for any given time is determined by the governing Newtonian laws. The system is represented by a set of points within the cartesian grid, from which we can discompose the motion of every particle into their corresponding components along normal axis. Moreover, the phase space generated by the position and momentum of the particles, supplies complete information of the whole system. This means that the sum of information restricted to the degrees of freedom corresponding to every particle determines the evolution and state of the whole system.

In this theory there are interactions between the particles and other external potentials that are governed by deterministic and causal laws, but the essential feature behind these interactions are the stability and independence of the particles, which are distinguishable and possess unique properties. This may be compared to the way machines work, because every component, although in mutual interaction, possesses a relatively independent behavior. This order is usually called mechanical philosophy.

Of course, the mechanistic approach is in fact an idealization, because observable (Physical) objects are not punctual or rigid objects. But, the important fact here is the way a complete system is analyzed in terms of separated components, or in other words, the way parts may in fact contribute to define the whole. This is the essential meaning of the mechanistic order in the context of classical mechanics, because the form of the description is in mutual correspondence with the world in this way. In fact, likewise classical mechanics represents a model with a mechanistic structure determined by the absolute character of the cartesian grid and Euclidean geometry, the nature of the order behind the (Physical) world is said to be of mechanistic kind[8]. In this manner, with the advent of new discoveries such as those observed in Brownian motion,

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[8] To avoid confusions, there is a germ in the nature of this order that we must deal with: mechanical philosophy does not deny the existence of particles or any other part of the whole system. In opposite direction to the thesis of ontological holism, the essential idea behind the mechanistic view is the fact that particles actually exist, but the whole system cannot be defined by the complete knowledge of these parts.

the random behavior of particles suspended in water was interpreted as a natural behavior of millions of moving particles with mutual interactions. The classical notions of order adapted to other contexts such as the case of the Brownian motion. However, the possibility of such adaptation depends on a mere supposition and laws cannot proved to be deterministic in this case. The classical notion or order started to show some difficulties.

Now, attention is given to the foundations of general relativity. With the advent of curvilinear geometries, the absolute nature of space and time change into a unified entity comprising dynamical properties. The conception of rigid objects was replaced by the notion of world-lines, taking into consideration the temporal evolution of three-dimensional objects. Moreover, the notion of forces as external mechanical interactions occurring in absolute flat space, was replaced by the notion of a field, which determines the structure of space-time, and therefore, the distribution and motion of the energy and matter. A new order had emerged from the fundamental status of the gravitational field. However, in relation to the mechanistic order introduced by classical mechanics, some difficulties started to reveal, because objects were not three-dimensional entities comprising unique properties that would eventually determine the whole system. Conversely, the evolution of the system was determined by the field dynamics that in turn would give to the objects their corresponding properties and behavior. The classical idea of material bodies as relatively stable and independent entities was not appropriate to the temporal and dynamical behavior of relativistic objects. In fact, general relativity implied that the primary concepts were not the point particles nor the rigid objects, but the events and processes taking place in space-time. Three-dimensional objects were just abstractions of some invariant pattern of these processes. Thus, the order of processes and events emerged.

Moreover, just after the publication of general relativity, and following along the ideas developed in recent years, Einstein begun to search for a theory of everything. Indeed, taking into account the field dynamics developed in his theory, the primary ontology was given by the field of the whole universe, where the unification of all interactions followed. In this manner, the analysis of systems in terms of fragmentation in autonomous parts was replaced by another conception of order, where the universe was described as an undivided and unbroken total reality. The parts and components of the whole were in fact abstractions and approximate perturbations of the field. Thus, the order of undivided wholeness emerged.

Finally, the classical notion of order is again objected with the emergence of OQM. In this case, the order of an undivided wholeness was taking the same primary relevance. However, in the case of quantum mechanics, this particular order followed by the absence of any distinction between the observer and the real fact. They were considered as part of one and the same unity. Moreover, non-local effects and correlations between distant particles were discovered and would eventually reinforce the usage of this new notion of order. But, although OQM and general relativity share this particular conception of order, the concept of objects are not at all on the same footing, because they are not considered as temporal world-lines and part of a process, where light imposes fundamental limits. Quantum objects are statistical punctual particles instead, with or without possible potentialities to be realized in the world.

Thus, after a long discussion about the development of these new orders, Bohm suggests a different approach beyond any definite theory or interpretation. He embarks on a project of investigating the way one is able to construct new notions of order from which quantum mechanics and relativity are derived only as approximations. This is in fact a resolution to the problem of making both theories compatible.

However, instead of adapting standard notions of order to current situations, an adequate order is derived from the quantum phenomena. In particular, the one from which unbroken wholeness is implied. At the same time as general relativity and quantum mechanics show that the observer and the fact cannot be distinguished, the notions of order that help us to give form to the description and the content of the description cannot be seen separately. In this way, unbroken wholeness is not only implied in the content of the theories but also in the form of description.

In the following lines a new notion of order where unbroken wholeness has been implied is proposed.

9.4.4 The implicate and explicate orders

Bohm suggests that beyond the phenomena that manifest in some particular visual order (for example, the order of three-dimensional objects), there is a more fundamental implicit order everywhere and at all times. In this notion of order, similarities between different autonomous and independent objets are not only considered, but also similarities, ratios and patterns of everything contained in the universe, including all forms of description, the instruments employed in experiments, the mind and so on.

> There is the germ of a new notion of order here. This order is not to be understood solely in terms of a regular arrangement of objects (e.g., in rows) or as a regular arrangement of events (e.g. in a series). Rather, a total order is contained, in some implicit sense, in each region of space and time.[9, p.188]

In fact, Bohm argued that all separate objects, structures, and events in the visual world are relatively autonomous and stable parts abstracted from a deeper implicate order implying a process of unbroken wholeness. Indeed,*the implicate order* plays the role of the notion of a natural order, in which everything is enfolded into everything. In other words, to the extent that observable objects are part of this reality and make a contribution to it, each element of physical reality contains implicit information about the universe rather than just one part of the whole. Any observable object is a particular manifestation and abstraction of a whole undivided process which takes with it the implicate order. For instance, one way to understand this notion with a simple example is the hologram[9]. In this instrument, a unique correspondence between the parts of the object and the parts of the record does not exist, every point in the

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[9] An hologram is a photographic record of interference pattern of light waves that come off from an object, so that every point in the image correspond to the pattern of light waves coming from the whole object.

record is rather related to the whole structure of the object and every part of the object is also related with the whole record. There seems to be an implicate order in the interference pattern that is explicitly shown only with light beams in the form of one picture and one point of view.

> In the implicate order the totality of existence is enfolded within each region of space (and time). So, whatever part, element, or aspect we may abstract in thought, this still enfolds the whole and is therefore intrinsically related to the totality from which it has been abstracted. Thus, wholeness permeates all that is being discussed, from the very outset.[9, p.172]

In contrast to this notion of order, in the *explicate order* things lie independently in their own regions of space and time acting in an autonomous way. When light shines the record of the hologram, the image of the object appears and this is what we call explicate order. The process in which the light waves travel from the object to the hologram will be called *enfoldment* or implication, while the process in which the order in the hologram becomes manifested to the viewer through an image will be called *unfoldment* or explication.

Another example will clarify this: imagine a radio-wave traveling towards a TV. In this case, the implicate order corresponds to the signal that enfolds through the space. Points in the image cannot correspond to the same pattern of points in the wave. However, the TV plays the role of a decoder, in which the image suddenly unfolds into the screen and the explicate order is revealed. However, if we return to the case of the hologram absent of light, the explicate order cannot unfold from the interference pattern before the light shines (that would correspond not to an hologram but to a lens, where there is a one-to-one correspondence between image and object). Although it is just an example, in this particular case as in quantum mechanics (or general relativity), there is a new germ in the notion of order from which ontological holism follows: the fact, the instruments and the forms of description are part of the same total process of unbroken wholeness.

9.4.5 The fundamental status of the implicate order

The explicate orders are particular cases and abstractions of implicate orders from which are ultimately derived. The fundamental and primary element corresponds to the implicate order while the explicate order is just a part of the former. In other words, we must think of the implicate order as the fundamental and primary reality, from which the phenomena derive by unfolding and projection to a lower dimensional order: the explicate order.

Now, the case of the hologram is just a particular example, but eventually we are conduced to think, not in some independent cases or events such as the object together with the light, plate, and the observer, but in the totality of facts occurring in the universe. In this way, one may talk of the implicate order as a total order of nature, from which only one explicate order is derived and manifested to the senses up to a certain approximation. The perception of objects is behind the explicate order, derived by approximation from the *total implicate order*.

It follows that the way we perceive the objects of (Physical) experience is determined by certain laws given by the totality of processes taking place in the universe. This means that mind, consciousness, and matter form part of the same unbroken process, each carrying an implicate order, all of them based on a common explicate order.

Moreover, a notion of natural orders of different degrees has been apparently given. Although a total implicate order encodes the structure, measure and natural patterns of the whole universe, *sub-totalities* have been considered too, such as the hologram and any other sub-system arising from sense experience, in which implicate orders of lower level are implied. The main reason why sub-totalities have been described is the following: as far as the whole universe is considered, a whole meaning and understudying of this absolute system has proved to be limited, because any perception is ultimately part of the same system. But, if these sub-systems are always contained in broader systems, then it seems that the whole content of Bohm's philosophy as comprising ontological holism is lost. From these insights, a question emerges: if we wish to maintain an ontological holism in the form of description, why sub-totalities appear from which implicate orders of higher and lowers levels are implied?

The answer to this question is the idea of process of unbroken wholeness that we have discussed so far. Ontological holism as an approach to knowledge cannot be seen as absolute. I mean that it cannot be seen as comprising any defining truth, in the same way that it cannot account for any complete description of the universe as a whole. It is rather a manifestation of an unknown total process. In this manner, implicate orders of lower levels should be regarded as encoding in each region of space and time some sub-processes of less complexity, while higher levels of implicate orders should be regarded as encoding sub-processes of higher complexity. Now, although there is no absolute in nature, the idea of a a total process may be approximately described in terms of a given language. If we define such thing as total implicate order, then there is a concept behind the idea of a total variable process, that in some way 'carries' this implicit information throughout space and time. This concept is the *holomovement*.

> To generalize so as to emphasize undivided wholeness, we shall say that what 'carries' an implicate order is the holomovement, which is an unbroken and undivided totality.[9, p.191]

In certain cases, particular aspects of the holomovement are abstracted, such as light and particles, but more generally, all forms that take place in the holomovement are inseparable "Thus, in its totality, the holomovement is not limited in any specifiable way at all. It is not required to conform to any particular order, or to be bounded by any particular measure. Thus, the holomovement is undefinable and immeasurable.".

> *In Bohm's view, the fundamental reality corresponds to the holomovement, an infinite-dimensional flux carrying the total implicate order*

Therefore, the most important thing to understand is the fact that implicate and explicate orders are part of the unknown total process, the holomovement. And if we come up with a general notion of implicate order, then the total process taking place in the whole universe flows by enfoldment and unfoldment of a total order implicit in each region of space and time.

Since any relation at this level of description corresponds to the content of all predictive theories, the laws behind sub-totalities are given in the following part.

9.4.6 The laws behind sub-totalities

In relation to the previous discussion about essences and appearances at the level of physical theories, three levels of descriptions are considered: super-systems, systems and subsystems. In fact, this division is not given to provide an analysis into autonomous basic parts, independent of the states of the systems in which they participate. But rather to serve as an abstract basis of description, which does not imply the independent existence of the elements that are distinguished by this description. In fact, subsystems depend ultimately on the systems in which they participate, which in turn depend on super-systems. However, in typical cases, some subsystems are functionally[10] independent but not fundamentally independent. This is because in some particular cases, this description has been used according to the way the senses perceive the phenomena, but at certain point, physical independence is an approximation that in a general context is not valid anymore.

Thus, these levels of descriptions imply that wholeness in the form of description is not compatible with completeness of content. As far as (Physical) reality is described with this form of description (subsystem, system and super-system), a complete description of the whole universe is not possible, because the observer is part of another system. It follows that super-systems and subsystems are only part of the unknown totality of the universe and each level makes an irreducible contribution to the content of the description. In [4], Bohm's ontological holism is outlined as follows:

> Inseparable dependence[11] of the whole universe is the fundamental reality and that relatively independently behaving parts are merely particular and contingent forms within this whole.[4, p.102]

Then, the emergence of sub-totalities is compatible with a reliable notion of holism: they are merely *functionally* autonomous parts that possess certain recurrence and stability. Indeed, 'functionally autonomous' means that sub-totalities cannot be considered as fundamental but they are just relatively invariant dynamical abstractions that are relevant to the way the phenomena is perceived. Of course, the way sub-totalities are described is ultimately determined by the law of the holomovement due to the primacy and influence of this total process. This law operates in such a way that the abstraction of sub-totalities with certain recurrence and stability are allowed.

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[10] The term functional is relevant in the context of usefulness: to say that a system is functionally independent means that subsystems are abstract systems, which is useful to consider them as independent.

[11] The concept of inseparable dependence is taken by an example in quantum mechanics, in which all qualities and relationships of the elements defining a physical system depends on the state of the whole.

Moreover, the laws of these sub-totalities operate under certain conditions as follows: a set of implicate orders of certain degree are given; one single case in common of these implicate orders (the explicate order) is defined; and a law that expresses the necessary connections between elements of these implicate orders is stated in such a way that such law contributes to one common explicate order.

This law depends on a deeper level of description with recurrence and stability with implicate orders of higher order. For example, relatively independent sub-totalities must be seen as an approximation of higher implicate orders when in a particular context the observer and the instruments are removed. But this works at a certain point when everything enfolds into everything as quantum correlations have shown. To make this clearer Bohm writes:

> [...] Everything is to be explained in terms of forms derived from this holomovement. Though the full set of laws governing its totality is unknown (and, indeed, probably unknowable) nevertheless these laws are assumed to be such that from them may be abstracted relatively autonomous or independent sub-totalities of movement (e.g., fields, particles, etc.) having a certain recurrence and stability of their basic patterns of order and measure. Such sub-totalities may then be investigated, each in its own right, without our having first to know the full laws of the holomovement. This implies, of course, that we are not to regard what we find in such investigations as having an absolute and final validity, but rather we have always to be ready to discover the limits of independence of any relatively autonomous structure of law, and from this to go on to look for new laws that may refer to yet larger relatively autonomous domains of this kind.[9, p.226]

9.5 The ontology of Bohm's theory in terms of Bohm's philosophy

The subject of this part is about the ontology of BQM. According to Bohm's philosophy, any theory describing laws governing a sub-totality with definite ontology (just as BQM), is just an approximation valid up to certain domains in relation to this sub-totality, and cannot be regarded as generally true. However, the same conclusion may be drawn with any empirically equivalent theory such as any other interpretation of quantum mechanics. So one question emerges: What is special about BQM?

In relation to Bohm's philosophy, the key to solve this question rests on the possibility to extend such theory to other domains. In fact, BQM is able to explain with an intuitive and imaginative form a new notion of order: the order of unbroken wholeness. The way this fact is clarified follows from interpreting BQM in terms of the Bohm's philosophical approach, and to figure out to what extent the implicate and explicate orders are represented by the theory. Of course, since BQM is formulated at the predictive level of all physical theories in a given sub-totality, and not at the fundamental level of the entire universe, implicate orders of lower levels are considered.

Later on, the form of the equations of motion provided by BQM and the phase space where the dynamics of BQM takes place, are derived as a particular approximation from a formal mathematical structure, namely *Clifford algebras*, to which Bohm's philosophy is consistent. It follows that BQM is not a conventional structure in quantum mechanics but an essential part when discussed in terms of Clifford algebras. Let us begin with.

As applied to BQM, ontological holism is about physical objects that are not wholly composed of basic physical parts. Bohm defines *basic parts* as follows:

> *Objects are basic, relative to a given class of objects subjected only to a certain kind of process, just in case every object in that class continues to be wholly composed of a fixed set of these basic objects.*

The difference between physical parts and *basic parts* should be clarified. According to BQM, a complete specification of the state of the "undivided universe" requires not only a listing of all the positions of the particles, but also of a field associated with the wave-function that guides these particles. In this case, the ontology of the theory is given by particles and waves, comprising an undivided unity. On the other hand, QTM considers both particles and waves as autonomous and independent entities. However, particles are not *basic parts* in the sense stressed above, because to make up the whole structure and composition of the system, a wave function must be provided. In fact, the motion and the properties of every object of a given class is actually determined and guided by a non-local quantum potential and wave-function of higher dimension. It must be recalled that in BQM and QTM, the wave-function acts not as external to the particles but as an inner part of the whole structure of the system. Particles are rather entities perceived as"parts" but underlying a deeper structure, in which the wave-particle ontology plays the central role.

In this manner, particles are not *basic parts* in the sense that they are independent constituents of matter or interacting building blocks of nature, from which all objects of a given class are composed.

> All that is important here is that one finds, through a study of the implications of the quantum theory, that the analysis of a total system into a set of independently existent but interacting particles breaks down in a radically new way. One discovers, instead, both from consideration of the meaning of the mathematical equations and from the results of the actual experiments, that the various particles have to be taken literally as projections of a higher-dimensional reality which cannot be accounted for in terms of any force of interaction between them.[9, p.182]

To suggest a new order of unbroken wholeness, BQM is interpreted in terms of two assumptions: the first one corresponds to the non-fundamental character of the quantum potential, and the second one deals with the special meaning and preference of this version of non-local framework.

9.5.1 The interpretation of particles and the quantum potential in terms of Bohm's philosophy

Along the lines of Bohm's philosophy, Bohmian positions are actually part of the explicate order. Although the trajectories of the particles are hidden and cannot be observed, their positions correspond to the manifested level of reality, as many experiments have confirmed. For example, in the double-slit experiment, the particles travel along trajectories guided by the wave function, and finally reach the screen where the positions of the particles are recorded. These particles are manifested by their positions and are supposed to be abstractions or projections from an implicate order of higher dimension.

Besides the positions, the wave function, living in the configuration space, provides information to the particles by enfoldment and unfoldment, and contains the complete knowledge of the system. In accordance to the notions of order defined above, the implicate order corresponds to the information of the whole system contained in the wave function, which is eventually communicated to every particle through the quantum potential.

This has been clarified by the concept of active information. Particles move under their own energy and do not generally take their kinetic energy from something external. They are just steered by forces via the quantum potential in the sense that are only guided by some kind of implicit information. This information belongs to the wave function and remains potentially active, but it becomes actually active when enters into the activity of the particle.

In this manner, the meaning of the quantum potential is ultimately related to the notion of information. There have been plenty of papers that seek to explore the philosophical and physical aspects regarding the notion of information in the domain of quantum mechanics. Although this subject is beyond the limits of this thesis and will not be discussed, there is one important aspect related to the status of the quantum potential: in the context of quantum mechanics, is the information (Physical)?

The question here is whether the concept of information may be considered as an object or thing, and part of (Physical) reality. After giving answer to this question, the status of the quantum potential in the context of Bohm's philosophy may be clarified. In this regard, Christopher G. Timpson [35] points to the fact that 'information' is an abstract noun and cannot be considered as a thing; rather, it is ultimately defined as a process:

> The genuine question we face is: what are the physical processes that may be used to transmit information? Not the (obscure) question 'How does information behave?'.[35, p.613]

Timpson defines 'information' as it appears in 'active information' as the action of "giving form to". On the same footing, Basil Hiley claims in [36] that information is the activity of shaping or putting form into a given objective process that result from a dynamics constrained in an irreducible way by the environment. In this manner, the quantum potential is said to be an information potential, in the sense that it is a potential that gives form to the possible trajectories associated with the particles, provided they have their own energy. It follows that it does not make sense to say that a thing called 'active information' is transferred; rather:

> 'Information' here refers to a particular action—the giving of a form to something—and an action is not a thing that can be moved.[35, p.610]

Thus, with 'active information' understood in this way, all that can be said is that an action has being performed by the quantum potential, not that something has been transferred.

Timpson's argument contributes to understand the correspondence between the implicate order and the information provided by the wave function. This is because the former has been defined not as a thing or substance with relative independence and stability, but as a physical process contained, in some implicit sense, in each region of space and time, and encoding the variable relations and the structure of the whole.

However, although the physical processes described by BQM reliable tell us how information is transmitted, the action performed by the quantum potential does not represent the fundamental role provided by the implicit *total* order of the whole universe, but only the role of the implicate order of a given sub-totality. As any mathematical object describing physical processes in a given theory, the quantum potential should rather be regarded as a genuine representation of lower implicate orders defining sub-totalities, which are relatively stable and functionally independent systems.

It follows that the quantum potential, compared to the total implicate order, is of non-fundamental character and only makes a relative contribution to the whole total process.

> *The quantum potential is not fundamental in Bohm's philosophy.*

The notion of unbroken wholeness has been introduced in some way into BQM. However if it is introduced at the fundamental level, it cannot be represented by any definite ontology. Fundamental reality corresponds, in principle, to the holomovement, flowing throughout configuration space, where the total implicate order propagates. This is why higher levels of implicate orders are suggested by Bohm, besides those introduced in BQM (comprising only the quantum potential). In other words, he claims that the quantum potential is itself organized and guided by a "super-quantum potential", representing a second implicate order, or super-implicate order. In accord with his philosophy, Bohm envisages that infinite hierarchies of implicate orders may be abstracted, some of which organize the lower ones, which in turn influence the higher ones.

9.5.2 The interpretation of non-locality in terms of Bohm's philosophy

Now, to come up with an intuitive picture of Bohm's philosophy in the context of BQM, non-locality must be discussed.

Besides the interpretation given to the quantum potential in BQM, a new notion of unbroken wholeness is described in terms of non-locality. However, plenty of non-local theories have been proposed, some of which do not play an important role here. So, what is so special about BQM in relation to non-locality?

As far as only the concept of non-locality is discussed, OQM is apparently similar to BQM. Correlations in EPR experiments are described by entangled wave functions besides certain fundamental assumptions about physical reality. Indeed, in the context of distant particles, an experimentalist located on each particle performs a measurement of a given physical quantity, where the corresponding outcomes seem to influence each other. One question emerges here: do outcomes imply a kind of influence between the particles?

Non-local theories describe correlations between particles regardless their separation. These correlations cannot be reproduced by any local realistic theory, where counterfactual definiteness and the principle of locality holds (the first correspond to the ability to speak meaningfully of the 'definiteness" of the results of measurements that have not been performed, while the second is about states that are influenced directly and only locally).

Considering this, since the collapse of the wave function (of non-local character) is required to describe the measurement process, OQM violates local realism. Operators do not have a pre-existing value before any measurement. The wave-function describes the evolution of the system regardless of any measurement and is just a mathematical tool used for statistical predictions.

On the other hand, in the context of BQM, correlations are described by non-local hidden variables that violate the principle of locality. Contrary to OQM, counterfactual definiteness holds because a realistic and deterministic framework is formulated. Explicitly, the hidden variables are the wave fields. They are introduced besides the particles to have an objective and deterministic description of quantum processes.

In detail, the introduction of the quantum potential as a non-local causal mediator between the particles and the wave function implies the violation of the principle of locality, clearly exposed by Holland in his interpretation (QTM). In fact, one can say that BQM is non-local theory thanks to the quantum potential. According to Hiley, the quantum potential:

> [...] was a key element in gaining insights into what could underlie the quantum formalism. It was his deeper analysis of this aspect of the approach that convinced him that the theory could not be mechanical... ...interconnectedness could not ultimately be reduced to nothing more than an interaction between some a priori given fundamental entities which compose the system.[13, p.4]

Therefore, it seems that the non-local character of OQM is due to the collapse of the wave function (that includes the measurement postulate), while in BQM, non-locality follows from the character of the quantum potential described in terms of causal interactions.

In brief, as far as the notion of unbroken wholeness follows from non-locality, BQM is preferred over OQM, because in the former the measurement problem has been solved. This preference cannot be grounded by the causal structure of Bohmian interpretations, nonetheless, non-locality is described without introducing additional postulates and measurement-like troubles.

10 The development of a new order: Basil Hiley

Basil Hiley is one of the proponents that has been engaged in an extensive research, aiming to explore fundamental ideas introduced by the late Bohm. In 1993 they published their last work together, “The Undivided Universe” [10], the product of a long-time collaboration. Along the lines of Bohm, Hiley rejects the model where the waves and the particles are independent and autonomous entities. This is the motivation for Hiley to radically change the meaning of the classical point particle to a new notion of order that aims to describe the “interconnectivity” of events behind natural processes: the order of unbroken wholeness. In collaboration with Bohm and Peat, Hiley claims:

> What we concluded was that this potential enabled the global properties of quantum phenomena to be focussed on the particle aspect, but in doing this we must remember the ‘particle’ is not independent of the background. Furthermore it is the quantum potential that contains the effect of this background. This implies that the particle and quantum potential form an indivisible whole.[13, p.6]

Although wave-particle has proved to be the most satisfactory ontology for BQM, there are plenty of questions that have not been solved so far. In fact, the motivation to investigate the meaning of theoretical entities defined in BQM naturally arise, in particular, the meaning of the quantum potential. Of course, Hiley and Bohm have made a considerable effort in developing a consistent interpretation, but fundamental questions still remain unanswered. One of these questions reads as follows: if the properties of particles are defined in BQM by a whole interconnected structure, how to explain our observations of the world in separated and individual objects and particles?

According to Hiley, the translation to talk about ‘particles’ and ‘waves’ into a ‘localized kinetic energy‘ and a ‘global quantum potential energy‘ respectively is a suitable choice to pursue this goal [13, p.9]. In this case, both are regarded as secondary elements arising from one single entity described in terms of ‘energy´. For example, in the case of interference phenomena, the classical behavior and interactions between autonomous and independent particles form a particular case when all the energy is kinetic and the quantum potential energy vanishes. However, although Hiley is able to describe autonomous objects in this way, the initial problem remains unanswered: why does energy divide into these kind of energies in the first place? why the existence of particles is assumed?

According to Hiley, the separation of these kind of energies is conceived as an approximation arising from one single ontology, and cannot be said just they contribute at the fundamental level of BQM. However, as far as these foundational aspects are not considered, the explanation of the non-local behavior of BQM has been awarded to the quantum potential while the solution to the measurement problem has been provided by the introduction of particles and waves.

Although BQM solves many puzzles in relation to quantum theory, the division between particles and fields has not been explained by the same theory, because they are only postulated to be coherent with a deterministic and objective description of particles in motion, consistent with the predictions of OQM. In other words, the origins of the quantum potential and therefore the ultimate origins of non-locality haven not been explained in terms of basic principles in physics. Non-locality and hidden variables are rather *adhoc* and phenomenological features that have not been yet justified in physics with satisfactory answers. This will be explained later in a more extended manner, when the formulation of BQM is analyzed and the way the quantum potential has been introduced.

Perhaps the answer may be provided by what have been discussed so far. In fact, an interpretation of BQM in terms of Bohm's philosophy has already been given. But a mathematical derivation of both potential and particle terms has to be taken seriously. In this manner, a new mathematical theory must be proposed, where the notion of process in the context of Bohm's philosophy is formally introduced. And indeed, this formalism has been developed by Hiley and collaborators in recent years! A brief review of this theory (HOL) is discussed:

10.1 The emergence of Hiley's approach and the connection with Bohm's philosophy

Contrasted to the mechanical view of current physical theories and following with Bohm's philosophy, Hiley starts from a primitive notion of process. Under this perspective, flux, activity, and motion are taken as fundamental basis, from which quantum phenomena in particular emerge. One should expect that dynamical properties of particle and field interactions within a given space-time manifold arise from these basic processes. Moreover, in addition to such expectation, Hiley suggests a new way to derive BQM in terms of Clifford algebras and formulates quantum theory in a different manner compared to previous interpretations. The main content of HOL is clarified in the following lines.

To start with a mathematical description, Hiley labels the start A and the end B of one indivisible process. He defines 'extensions' as finite and oriented processes $[AB]$, which could be in turn organized in a multiplex of processes. Translated this to what have been seen here, the total process made up by the totality of all extensions in the universe corresponds to the holomovement. In the same manner, they alone denote the activity of one region of space transforming into another, which is a single abstract process taking place within the variant structure of the whole.

Now, and idempotent is defined as $[AA]$, interpreted as a dynamic entity essentially invariant, yet continually transforming into itself. In particular, idempotents possess the property that $[AA][AA] = [AA]$. Any stable and autonomous entity in (Physical) reality should be thought as a process continually transforming into itself. In this manner, as long as relatively stable sub-processes are observed, relative idempotents have to be considered.

Hiley assumes that the sum of all these processes (*extensions*) form an algebra over a real field in the following sense: let Ω be an algebra over a real field $\mathbb{R}$.

(a) $\lambda[AB] \in \Omega$, if $\lambda \in \mathbb{R}$ and $[AB] \in \Omega$. In fact, λ denotes the strength of the process.

(b) $[AB] = -[BA]$. The process is oriented.

(c) $[AB] \cdot [BC] = \pm[AC]$ defines an associative inner product in Ω. This defines the order of succession.

(d) $[AB] + [CD] \in \Omega$, defines the sum of processes.

The algebra Ω forms a *Brandt groupoid*[12], and this groupoid can be identified with another type of algebra by doing operations between *extensions*. In fact, with the help of this primitive algebra Hiley makes connection with *Clifford algebras* (Pauli, Dirac, and Schrödinger's). The algebras developed by Schrödinger, Pauli and Dirac, form a hierarchy of Clifford algebras, that correspond to non-relativistic particle without spin, non-relativistic particle with spin and relativistic particle with spin, respectively.

At first, Hiley concentrates in the formulation of orthogonal 2-dimensional space-time symmetries (rotations) in terms of *extensions* of the Ω algebra. To do so, he identifies Ω with the Quaternion Clifford algebra by isomorphisms from *extensions* of the former to elements of the later e, arising from the same inner product structure[13]. Having established that the essentials of the quaternion Clifford algebra lies at the heart of Ω, Hiley uses the known properties of this algebra to generate rotations through the Clifford group. This is achieved by inner automorphism on the elements of the algebra e by the elements of the Clifford group $g(\theta)$:

$$e_1' = g(\theta)e_1g^{-1}(\theta) = cos(\theta)e_1 + sen(\theta)e_2 \tag{10.1}$$

It follows that to construct a vector space from the group of *extensions* is possible indeed, rather than the usual reverse procedure (we just induce rotations in a vector space by identifying the elements of the algebra with the vector basis). In the same spirit, Hiley derives 3-dimensional symmetries from Ω (Pauli Clifford algebras). In fact, so far special attention has been confined to a very simple case where two degrees of freedom are considered. However, this procedure can be generalized and applied to three basic *extensions* to produce the Pauli Clifford, which in turn produces a 3-dimensional vector space.

Later on, Hiley introduces the notion of time as process as well. In detail, he shows that from Ω we can derive some of the usual properties of Minkowski space-time by identifying elements of Dirac algebra with the cone structure. In this case the basic processes are *spatial or polar extensions* $[P_0, P]$ and *temporal extensions* $[P_0, T]$.

Hiley considers the properties of the product in Ω with the requirement that $[T, T] = -1$. The minus sign will ultimately emerge in the signature of the metric when constructed. Now, ordering the multiplication of elements of spatial and temporal extensions, this structure is identified with another Clifford algebra (Dirac algebra) via an isomorphism, which in turn has direct connection with the cone structure. Lorentz transformations and properties of Minkowski space-time are derived from one-dimensional Dirac algebras (see [11, pp.8-11]). In detail, special

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[12] The proof can be found in [11, p.13]

[13] This can be found in [11, p.6]

relativity arises by identifying elements of Clifford Dirac algebras with the structure of light-cones. This ensures that the underlying manifold so far constructed is Poincare invariant, and thus, HOL is consistent with special relativity.

To summarize, a light-cone structure is provided by orthogonal Clifford algebras, and it can be abstracted from the algebra without the need to start from an a priori given vector space. In other words, processes are considered as basic, and from there, light-cone structures are abstracted. This has been achieved by generating a variety of spinor structures (in particular, the light-cone structure) from primitive idempotents (different time frames), which are chosen arbitrarily from the total process. The idempotents define equivalence classes of observers, namely Lorentz observers.

Moreover, besides the latter result, quantum mechanics may be reproduced without the need to introduce the Hilbert space and the wave function. One element of the Clifford algebra $[A, B]$ must be factorized into a pair of bras $\langle A|$ and kets $|B\rangle$ (in Dirac notation). This is achieved by identifying the minimal left ideals of the Clifford algebra with the kets and bras used in quantum theory. One way to generate such minimal left ideals is to choose a suitable primitive idempotent ϵ as in the case of the light-cone structure (the details are in [11, pp.19-20]).

Eventually, elements of the Clifford algebra are generated from particular primitive idempotents. The structures of special relativity and quantum mechanics are constructed from these elements, depending on the particular choice of idempotents. For example, choosing a particular time-invariant frame, the desired structure of special relativity is given, whereas choosing a particular invariant-parameter ϵ, a quantum ket is given. Therefore, both physical theories are derived from the minimal left ideals of Clifford algebras. Moreover, in addition to those that have been considered, a richer structure is obtained from sets of primitive idempotents. Additional structures are generated by primitive idempotents, where an insight to the implicate order introduced by Bohm has been suggested.

According to HOL, any element of clifford algebra has been identified as an *extension*, which was defined as a labeled process. Process is taken as basic, and abstract structures (such as those of special relativity and quantum mechanics), are abstractions of this process. On the same footing, the structures of space and time are derived as well from this basic processes, because they depend on the time frame chosen. For this reason, space and time are not apriori concepts. In fact, there is no space or time, there is only a pre-space, where the elements of the Clifford algebra live. Processes are not to be thought of as occurring in space-time, but rather the properties of space-time are to be abstracted from this pre-space. In connection to Bohm, space-time is merely a "shadow manifold" constructed from the so-called holomovement, in which the whole total process evolves. In this manner, the holomovement or total implicate order follows from the total sum of processes taking place in some abstract pre-space, and the shadow manifolds are then the sets of explicate orders projected from the implicate order.

10.2 The connection between Hiley's approach and Bohm's theory

Considering what has been discussed above, the connection with BQM is straightforward. To factorize one element of the Clifford algebra $[A, B]$ into the pair of elements $\langle A|$ and $|B\rangle$, the minimal left ideals of the algebra should be identified with pure quantum states (wave functions). Now a question emerges: How BQM equations of motion are derived?

Suppose a shadow manifold of a Clifford algebra is the base-space of a given Clifford bundle. From the abstract of [11]:

> [...] By constructing an orthogonal Clifford bundle with a Dirac connection, we make contact with quantum mechanics through the Bohm formalism which emerges quite naturally from the connection, showing that it is a structural feature of the mathematics.[11, p.1]

To introduce particle dynamics into the shadow manifold, Hiley shows the equivalence between a particular connection constructed in the shadow manifold (the Dirac derivative), and the momentum operator in quantum mechanics, which is interpreted as the usual derivative in BQM.

Two types of derivatives are considered, one operating from the right and the other form the left, in accordance with the algebra. Hiley shows that if two types of Hamiltonians are considered (corresponding to the minimal left and right ideals), the dynamics of the system is described in terms of a pair of equations, which are exactly the Hamilton-Jacobi Equation and the continuity equation appearing in BQM (The details are in [11, pp.36-40]).

Therefore, the foundation and formulation of BQM can be derived from Clifford algebras, taking into consideration the dynamics in the Clifford bundle. BQM, and in particular, the quantum potential term, arises as a structural feature of the mathematics, when processes as primary ontologies are considered.

10.3 Remarkable conclusions

Along the lines of Bohm's late publications and Hiley's collaboration, a new mathematical formulation has been written. Clifford algebras are the starting point to describe the notion of total process in terms of mathematics. In other words, an abstract pre-space with this special algebraic structure is supposed to contain the sum of processes we can see in nature. From pre-space one is able to identify a particular shadow manifold, where the configuration space of BQM is identified. This is in fact the demonstration that BQM reduces from HOL, and is actually the only one to do so.

In view of this correspondence, the quantum potential corresponds to the physical interpretation of implicate orders of lower degree and part of the explicate order. The quantum potential is not fundamental as any entity defined within a shadow manifold. Moreover, the non-local behavior of quantum systems and the non-fundamental character of the quantum potential arise as a manifestation of this new order of unbroken wholeness. Indeed, with the development of HOL, the origin of the quantum potential and therefore the ultimate origin of non-locality is explained, in the same way that the origin of apparent autonomous and fragmented entities is justified by the primary relevance of the holomovement. These principles are indeed mathematical, where the notion of process and ontological holism are deeply connected.

Thus, the introduction of the non-local quantum potential and the objective character of particles do not correspond to an alternative way to interpret quantum mechanics, but as essential features that follow from Bohm's philosophy. Moreover, intriguing difficulties are solved in relation to relativity theory. HOL constructs an appropriate and particular fourth-dimensional space-time identifying Dirac clifford algebras in the so called pre-space.

This is a great result but not the ultimate one!

Part III

Our solution

Facing different interpretations of BQM that hold distinct views and explanations, one should ask whether they are able to answer fundamental issues, in particular those questions regarding the meaning of the quantum potential.

An alternative interpretation of BQM, different from Bohm's original thoughts, has been given [2]. This theory has been compared with other interpretations that are equivalent at the empirical level but are fundamentally different. In particular [14] formulates quantum mechanics in a different way, removing the concept of the potential and deriving the equation of motion using symmetries and the Schrödinger equation.

Different trends have developed after BM: those who think of the quantum potential as (Physical), and those who have challenged their ontology.

Besides all these important elements, our position will be given. We do not fully agree with the whole-body of only one interpretation, because there are important aspects that are not feasible under this view, or even elements to consider in other interpretations. The purpose of this thesis will be discussed in the following lines:

11 Our main purpose

We will engage in a project of investigating among all interpretations discussed so far the status of the quantum potential. Specially we shall conclude:

> *The quantum potential introduced by BQM is not (Physical).*

Since BQM-QTM are formulated in terms of the quantum potential (which also plays an essential role in both interpretations), the (Physical) status of this concept seems plausible. However, some difficulties appear when one seek to interpret this concept in detail. Indeed, one shall conclude that:

(a) *The quantum potential ontology cannot be interpreted non-classically.*

This fact suggests a serious debate about the fundamental meaning of the quantum potential. To compare BQM with BM may be a good starting point, nonetheless, although the quantum potential has been removed from the formalism, BM introduces another primary ontology, say, the ontology of *basic parts*[14], where fundamental status to the positions of the particles is assumed. However, there are some ambivalent features that one must deal with.

The criteria used to justify the existence of *basic parts* in BM as the primary and essential ontology has not been given properly. Although this ontology may be proved to be consistent with the formalism of quantum mechanics,

(b) *The criteria used to assume the ontology of basic parts are not natural and does not reflect the (Physical) facts*

This statement is the second fact we shall corroborate.

BQM has been derived from a particular abstract mathematical structure proposed by Bohm and developed by Hiley (HOL), to which Bohm's philosophy is essentially compatible. Along these lines, if one relies on a theory that is strictly linked to ontological holism, then the fundamental status of the quantum potential is not required, and one must derive BQM from a deeper theory. Therefore, in the final part of this thesis some heuristic insights will be suggested: to decide which theory is best candidate beyond the empirical terrain. Besides the fact that they share common features, both HOL and BM think of the quantum potential as not fundamental, but due to the ambivalent character of BM, HOL will be preferred.

12 Methodology

The way to achieve our purpose is carried out in two separated parts. The first one called "Why am I not convinced of the ontology of the quantum potential?" (13.1), discusses some arguments against the ontology of the quantum potential and ends up by defining our own position after giving proof to part (a) above; the second one called "Bohmian Mechanics (BM) without the corpuscular ontology of *basic parts*" (13.2), discusses the status of particles in BM and gives proof to part (b) above. Finally both (a) and (b) will give hints to prefer HOL instead of BM.

In the second part (13.2), an introduction is given, wherein the *ontology of basic parts* is defined. This definition is important, because the meaning of elementary particles or basic parts have not been written in explicit form. To answer (b), four premises are given in (13.2.1), which are written in italics. In the next part (13.2.2), the proofs of all these premises are

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[14] Recall the definition of basic parts: objects are basic, relative to a given class of objects subjected only to a certain kind of process, just in case every object in that class continues to be wholly composed of a fixed set of these basic objects

developed in detail. The corresponding connection between the premises and the proofs are labelled by letters (A, B, C, D). Finally, the conclusion arising from the premises is drawn in (13.2.3).

The premises are:

(A) Momentum in particular and self-adjoint operators in general are not fundamental properties in BM.

(B) Dynamic fundamental positions do not in general imply distinctions between basic parts

(C) Un-(Physical) factorization does not lead to the *ontology of basic parts.*

(D) The nomological interpretation of the wave function of the universe does not reflect the (Physical) facts, when basic parts are assumed.

Let us begin with:

13 Against the ontology of the quantum potential

13.1 Why am I not convinced of the ontology of the quantum potential?

> Our attitude is that we can sooner or later drop the notion of the quantum potential (as we can drop the scaffolding when a building is ready) and go on to radically new concepts, which incorporate the wholeness of form–which we feel to be the essential significance of quantum descriptions.[4, p.105]

The main concern of the thesis is to investigate the meaning of the quantum potential. For this purpose, Holland's formulation QTM and BQM will be criticized, and then, our own position will be drawn. Let give proof to statement (a):

(a) *The quantum potential ontology cannot be interpreted non-classically*

Referring to [2], if the equation of motion for QTM is postulated,

$$\dot{x}(t) = \frac{1}{m}\nabla S_{\mathrm{QM}}(x,t)|_{x=x(t)},$$

that is, to identify the phase of the wave function S_{QM} with the Hamilton's principal function of velocity $S_{\mathrm{classical}}$ (which obeys the classical law of motion):

$$S_{\mathrm{QM}} = S_{\mathrm{classical}}(\dot{x}),$$

then, quantum theory and classical mechanics are linked. The Schrödinger equation may be written in terms of $S_{\text{classical}}$, so one obtains the classical Hamilton Jacobi equation (H.J.) in addition to the quantum potential term:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S_{\text{classical}})^2}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + V = 0.$$

Finally, one interprets this equation in terms of classical mechanics, considering particles that are guided by ‘quantum forces’ via the quantum potential.

The classical law of motion has been postulated and then proved to be consistent with quantum mechanics. In this context, the classical role played by the quantum potential arises from postulating this law and writing the equation of motion in terms of the classical H.J. formulation. However, one question emerges: may the quantum potential be interpreted classically?

The meaning of the quantum potential is still uncertain. Although Holland claims that the potential formulation is not an attempt to return to classical physics, arguments against this framework have been provided by prominent physicists and philosophers, but no consensus has been drawn so far. These arguments are summarized along the following lines:

In [29], Dickson claims:

> The potential is only required to explain the deviation of particles from Newtonian trajectories.[29, p.7].

The equations of motion are the result of what would have been obtained classically if the particles were acted upon through the force generated by the quantum potential in addition to the usual forces. On the same footing, Goldstein claims in [20]:

> Bohm’s rewriting of Schrödinger equation via variables that seem interpretable in classical terms does not come without a cost.[20, p.158]

According to Goldstein, the resulting cost is increased complexity. Moreover, if the existence of the quantum potential is assumed, there would rather be in nature what appears to be an ad-hoc additional force term, because this concept emerges rewriting quantum mechanics in classical terms. Besides these arguments, Goldstein argues:

> The point is that Bohmian mechanics is not classical mechanics with an additional force term. In Bohmian mechanics the velocities are not independent of positions, as they are classically, but are constrained by the guiding equation.[20, p.158]

Of course, there are explicit differences between the dynamics of classical mechanics and quantum mechanics, so the status of the quantum potential cannot be justified by reading the equations of motion. According to Goldstein, the Schrodinger’s equation and the guiding equation determine the dynamics, and there is neither need nor room for any further axioms involving the quantum potential. This is one of the key arguments from which another formulation emerges: Bohmian mechanics (BM).

Many questions arise here regarding the meaning of the quantum potential. Goldstein formulates another theory free of the quantum potential term, and concludes that such entity is devoid of (Physical) significance, in the sense that it is neither fundamental nor an object of (Physical) reality.

Of course, that a new theory may be formulated without such entity does not necessarily mean that it is not fundamental, because there are no empirical arguments to prefer one theory over others. Our objection is of foundational kind and should be clarified.

Before ascribing any status to the mathematical elements of BQM equations, a wave function is expressed in terms of two real functions R and S (5.3): $\Psi = Re^{\frac{iS}{\hbar}}$. Then, this wave function is assumed to be a solution to the Schrödinger equation, so the following equation is obtained (see (5.4)):

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + V = 0 \tag{13.1}$$

Up to now, any meaning to S and R has not been given. Now, if the phase of the wave function S_{QM} is identified as Hamilton's principal function $S_{\text{classical}}$, then a generalized Hamilton-Jacobi equation (HJQM) is obtained, containing an extra element. This element turns out to be what is called 'the quantum potential'. A question emerges here: if S_{QM} is identified with $S_{\text{classical}}$, what exactly this means?

At first sight, the equations HJQM and H.J. are almost equivalent, regardless of the quantum potential. So, the only possible interpretation here arises by classical approximation, assuming that $\hbar$ goes to 0, and obtaining the classical H.J. equations. But to conclude something by just looking at the equations is misleading, because the overall solution of the wave function is contained in a pair of coupled partial differential equations (5.4), where the fields S and R codetermines one another.

So, Bohm suggests in BQM that the potential acting upon the particles should be thought as a generalized potential, including the classical and the quantum terms. In this way, HJQM can still be regarded as the Hamilton-Jacobi equation for the ensemble of particles and ∇S can still be regarded as the particle's momentum, because the quantum potential does not include S. In this manner, quantum effects may be explained by the "appearance" of this quantum potential.

As far as the formalism is considered, rewriting the Schrödinger equation in this form does not bring about any problem. But what remains is whether one may use classical assumptions in this interpretation. Although there is no difference in terms of mathematics, classical mechanics might not be fundamental at the level of quantum mechanics. One should be aware of the limits of classical mechanics, because one cannot reach any kind of fundamental statement from a basis which is neither fundamental nor general. In other words, there is no logical constraints to write the solutions of the Schrödinger equation in the same form as in the H.J. formulation. However, problems might arise when a classical framework is assumed via the ontological identification of the field S_{QM} with the Hamilton's principal function. Let us go in depth:

If the well-known classical law of motion and the Schrödinger equation are considered as basis for quantum mechanics, then the quantum potential appears naturally in the formalism as a result of the expectation of classical trajectories. Of course, from a classical point of view, the quantum potential might be seen as an extra force acting upon the particles and therefore as an object of (Physical) reality, in analogy to electromagnetic potentials.

However, if descriptions are extended to other domains, not necessarily classical, then some problems arise *once* the equations of motion have been interpreted classically. For instance, in the presence of many particles, the quantum potential possesses a non-local behavior besides the fact that it is an entity living in a 3-N dimensional configuration space. Moreover, the particles do not act as sources of such a field and the analogy with the electromagnetic potentials in classical mechanics vanishes.

So, from a broader perspective, the mechanical nature of the quantum potential in classical mechanics seems relevant at a certain point but cannot account for these non-classical features pertaining to the domain of quantum mechanics. Since in principle, the quantum potential has been introduced into the equations to read them classically, the problem arises when one interprets this entity as fundamental *and* non-classically. In other words, to read quantum mechanics in terms of classical mechanics does not come without a cost; a non-classical quantum potential must be introduced. But this cannot be regarded as fundamental because it cannot be interpreted in terms of the same classical considerations used to read the equations of motion.

An analogy proposed by Dürr is the role of the Hamiltonian in classical mechanics. This entity has been introduced in the framework of the Hamilton formulation to derive the classical law of motion from certain principles. However, this entity does not possess classical properties and these are similar to the wave function in quantum mechanics. Thus, although it is introduced to give an axiomatic and simpler account for classical mechanics, it cannot be regarded as fundamental.

Thus, there are two problems here:

(I) First, the actual role played by any theoretical entity must be determined and interpreted in the context of its underlying theory. In fact, if the Schrödinger equation has been written in terms of classical mechanics, introducing with it the quantum potential term, then this term should be interpreted as a fundamental entity, from the point of view of classical mechanics. But if descriptions are extended to other domains, then there cannot be quantum potential term, because the Schrödinger equation is described in other way compared to (5.4). In this context, the non-classical properties of this entity do not make sense, as far as it is regarded as fundamental. In other words, the quantum potential ontology cannot be interpreted non-classically.

(II) Second, there is a confusion here between the perception of nature and how nature actually is. If an arbitrary theory has been formulated in classical terms, one is trying to understand elements in the best picturable way, but sooner or later, such picture will not be consistent with the advent of other domains in physics. One should try to change the descriptions according to the current status of knowledge, and of course, elements that were part of earlier descriptions should be replaced by others. In regard to the quantum

> potential, there is an essential difference between regarding such entity as part of our perceptions and fundamental.

Along these lines, one may have two possibilities: either to assume classical mechanics as our main guide and to try to interpret the quantum potential in this classical way that could be consistent with the advent of quantum mechanics. Or to simply regard such entity as non-fundamental and to try to interpret quantum mechanics from a new framework. Of course, after showing the existence non-classical features in the quantum potential term, the second option is reliable.

Besides these arguments, Dürr in [14] has proposed a different interpretation essentially different to BQM. Positions of particles play a fundamental role in the theory, where the wave function is devoid of any (Physical) significance and the quantum potential has not been introduced. Thus, a problem of theory choice emerges: whether one should accept a corpuscular ontology or introduce the quantum potential term as an essential mathematical element in the theory.

To provide a suitable answer, the suggestion now is to go in depth in the formalism of BM and to discuss the fundamental status of positions.

13.2 'Bohmian Mechanics' without the corpuscular ontology of *basic parts*

13.2.1 A revision of our premises

The purpose of this part is to prove that the foundational criteria used to assume the ontology of basic parts in BM is not natural and does not reflect the (Physical) facts. Since an ontology of basic parts may be defined in different ways, it is important to clarify the meaning of particles according to Dürr.

Definition: the ontology of basic parts. Contrary to ontological holism, the mechanistic view in physics normally speaks of real point particles, which takes on a reductionist meaning of *basic* and structureless parts. In other words, systems *are* wholly composed of basic (Physical) parts. Many of these parts might be different, i.e. all electrons have different characteristics from the rest of the particles. However, this view is not concerned about whether there are different species of basic parts, but about the reductionist and discontinuous nature of physical systems, where the behavior of these parts contributes to define the systems in a complete form.

In general, *fundamental properties* are the basic properties that *basic parts* possess. In this manner, they are autonomous basic parts in the sense that they actually possess certain basic properties. Formally, fundamental properties have one or more degrees of freedom, corresponding to the space coordinates of each *basic part*. The total number of degrees of freedom defines the phase space of the system, in which all possible states of a system are represented. In this view, the analysis of a composite system means the understanding of each of these *basic parts* taken as real and part of (Physical) reality. This is in fact what is called the *ontology of basic parts*.

Although, the *ontology of basic parts* arises from the notion of order developed by Newtonian mechanics, the interpretation of *basic parts* in this theory is different compared to other theories. For example, in Newtonian mechanics basic parts are like billiard balls. Since all these parts possess independent dynamic *fundamental properties* (momentum, position and so on), they are dynamically independent from each other. In other words, the dynamic fundamental properties of one particle do not depend on those of the others.

On the other hand, a counterexample to the Newtonian approach is given by classical electrodynamics: fields are included and are produced by sources, which correspond to particles of certain kind. Since the particles are guided by external mechanical fields generated by other particles, the velocity and evolution of the particles are not independent of the positions of the others. Thus, the notion of *basic parts* discussed in classical electrodynamics should be assumed to be different compared to the Newtonian.

Now, the status of the relation between the *ontology of basic parts* and the BM theory will be drawn. At first sight, BM seems to hold this view as it is stated by Dürr. In fact, "particles" means particles, in other words, particles are fundamental[15] entities:

> For the non relativistic theory that we have been discussing, the primitive ontology is given by particles described by their positions.[16, p.28]

BM apparently holds that there are *basic parts*, in the sense that there *are* autonomous particles, which contribute to know together the composite system in a complete form. However, the *fundamental properties* of BM particles are not the same as in Newtonian mechanics, and for this reason, the corresponding quantum properties are discussed in the context of BM.

According to Newtonian mechanics, the *ontology of basic parts* implies not only fundamental positions, but also fundamental significance to all primary properties from which these *basic parts* are composed. For example, a system of N particles in Galilean space defines a collection of points in phase space in terms of independent coordinates corresponding to the total number of degrees of freedom in the system. This phase space is built up in terms of all components of position *and* momentum, which are ultimately fundamental. Thus, momenta is fundamental just as position. In contrast, this not the case in BM.

In the case of BM, the state of the system is given by (Ψ, Q), where Ψ and Q correspond to all degrees of freedom (including, of course, the degrees of freedom corresponding to each dimension). In this manner, the rest of dynamic properties are derived from the wave function and the position of the particles.

In "naive realism about operators" [15], Dürr introduces momentum and other operators within the wave function, because they are not primary (Physical) observables and are only derived as secondary elements from the wave function. However, as it is stated, the wave function in BM is not (Physical) and is merely nomological. Thus, contrary to the newtonian view, all dynamic properties ultimately depend on the positions of the particles. Under these

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[15] By fundamental here I mean to be part of (Physical) reality but also that they are actually the primary and elementary ontology from which the world is made up.

circumstances, although positions do not offer a complete description of a quantum system and this description should be provided by a 'nomological' wave function as well, positions form the primary dynamic property and particles form the essential ontology in the quantum world. In simpler words, *basic parts* possess positions and the wave function is not "so important".

In the context of BM:

(A) *Momentum in particular and self-adjoint operators in general are not fundamental properties in BM.*

A proof of this claim is given in (A) of (13.2.2). Thus, the *ontology of basic parts* has been clarified in the context of BM. In this manner, BM seems to assume that there *are basic parts* in the sense of structureless point particles, which contribute to know together the composite system in a complete form, and that these only possess positions as dynamic property.

Moreover, one should know that the *ontology of basic parts* is not logically derived from the formalism of quantum mechanics; rather, this ontology has been assumed and then proved to be consistent with the foundational and mathematical structure of the formalism. Of course, as any other interpretation, the connection between this particular ontology and the formalism of quantum mechanics cannot be decided experimentally and the outcome might depend on foundational elements. In fact, this connection is not unambiguously decidable.

This may become clearer when one tries to give certain ontological distinctions to *basic parts*, as particles belonging to different species i.e., electron points may be different from quark points or photon points. As in the situation of any ontological claim, the impossibility of corroborating experimentally these distinctions is a fundamental limitation of science. A choice can only be based on theoretical considerations. Let us first explain why:

At first, position (the unique real dynamic quantity in BM), is a vague property. All point particles possess positions, because any particle, no matter if they are divisible or not and if they belong to certain species or not, may be assigned punctual coordinates (positions). Following the same line of thought, to say that only positions are dynamic fundamental properties implies that any punctual entity is part of the *ontology of basic parts*. In fact, [17] shows:

(B) *Dynamic fundamental positions do not in general imply distinctions between basic parts.*

The proof of this statement is in (B) of (13.2.2).The way to prove this follows from the fact that the evidence available by the predictions of BM is insufficient to identify which ontology holds about that evidence. Indeed, there is a problem of underdetermination, because the description in terms of species of particles cannot be determined by empirical evidence. The solution to this problem of underdetermination has remained always tentative and any answer would have to be grounded on purely foundational, possibly philosophical criteria.

Besides this problem of underdetermination, it is not clear the criteria used by Dürr to assume the *ontology of basic parts*[16]. The following lines will be written to clarify this question.

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[16] In other words, it is not clear the criteria to assume that any composite system is wholly composed of *basic parts*.

Particularly, one should justify these criteria when the BM quantum state (Ψ, Q) is defined in terms of *basic parts*, which solely possess positions, besides a nomological wave function.

If one assumes the *ontology of basic parts*, then this fundamental assumption should have a counterpart in the formalism, and this formalism should represent this ontology in a consistent way as much as possible. Nonetheless, the definition of this ontology arises from Newtonian mechanics as stated above, and one expects that when quantum mechanics is considered, some foundational problems might possibly emerge, no matter if the formalism has proved to be consistent with this ontology. In this context, one may prove that there *are* problems already, and that the criteria used to assume *ontology of basic parts* are not natural. This is in fact the second purpose of this thesis (11). In other words, to make the *ontology of basic parts* consistent with the formalism of BM has proved to be no natural, in the sense that it requires additional ad-hoc assumptions that do not reflect the actual (Physical) situations occurring in nature. One eventually realizes that these assumptions lie behind the role of the wave function and they are the following:

(1) The (Physical) manifestation of the factorization of the wave function.

(2) The nomological interpretation of the wave function.

To give suitable reasons to point out these criteria, the meaning of the wave function of the universe should be analyzed. In detail, the connection between the properties of the wave function in the formalism and the (Physical) counterpart of these properties, must be drawn.

In regard to (1), one knows that in the predictive domain of quantum mechanics, one must concern about the wave function of a subsystem rather than the wave function of the universe. The former is only required to describe subsystems of the universe up to a certain degree of approximation, while the latter is regarded as part of the foundations of the theory but does not add any predictive content. In this manner, if one aims to describe subsystems of a wider system, or even *basic parts* in the case they exist, one must consider conditional wave functions.

The key to derive conditional wave functions from the fundamental one is given by factorization, because any subsystem has been defined as long as factorization occurs. For example, let W be a 2-particle system. Then, the equation of motion of both particles may be obtained and all statistics may be given, if the wave function defining W is factorizable.

Now, *basic parts* are represented by subsystems. In this manner, to be able to describe each individual *basic part* in the context of quantum mechanics, real factorization should be possible (see in [17]). Factorization makes indeed the connection between the formalism and this particular ontology. However, the nature of 'factorization' of the wave function has proved by Dürr himself to be contingent in the sense that occurs only in ideal cases. This is due to the fact that the contribution of the environment cannot be neglected and the secondary dynamic properties of each particle (spin, momentum, and so on), ultimately depends on the positions of other particles (including the measurement device).

So, if factorization is un-(Physical), then the description of the composite system in terms of each individual *basic part* is impossible, as far as this description is not approximative and is fundamental. Probably one might suspect that these parts do not exist and are only methodological approximations, but one must remember that the positions of the particles are actually observed in certain experiments (double-slit experiment). So, the only reliable conclusion that one might claim is that any composite system is not wholly composed of these *basic parts*. *Basic parts* are perhaps stable and sometimes detectable point objects, which depend on something more fundamental. There is no way to describe subsystems (*basic parts* in particular), from wave functions that are not factorizable. To clarify this, a proof to the following statement is given in (C) of (13.2.2):

(C) *Un-(Physical) factorization does not lead to the ontology of basic parts.*

In this manner, a fundamental description in terms of point particles cannot be given. Assuming that our descriptions are fundamental and reflect the (Physical) reality[17], the description of a whole system in terms of the description of the individual parts is impossible. The system *is not* wholly composed of these parts. Given this, the essential part of the *ontology of basic parts* vanishes.

Moreover, in regard to (2), the wave function corresponds to the second degree of freedom of the system. In particular, it cannot be regarded neither as a (Physical) object nor a fundamental property of the system. The wave function is part of laws of nature and the essential properties implicit within its mathematical structure reflects all quantum features. Along these lines, is the nomological interpretation of the wave function reliable?

We think that the answer is no, because it has been assumed only to bring about an ad-hoc and appropriate foundational basis for the *ontology of basic parts*. In fact, to renounce to the strange and non-standard multidimensional world (where the wave function lives), and to give preference to the mechanistic and reductionistic picture (arising from a Galilean world), are enough reasons for Dürr to support this interpretation.

However, if one knows that the nomological wave function of the universe brings about some difficulties when *basic parts* are assumed, then this interpretation becomes less natural. In fact these difficulties arise from the fact that:

(D) *The nomological interpretation of the wave function of the universe does not reflect the (Physical) facts, when basic parts are assumed.*

A proof of the following statement is given in (D) of (13.2.2). Let us begin with the proofs:

13.2.2 Proofs

Proof. **(A)** *Momentum in particular and self-adjoint operators in general are not fundamental properties in BM.*

Dürr claims that in contrast to any other dynamic property, positions are real, primitive, natural kinds and non-contextual;[18] in other words, only positions are fundamental.

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[17] Of course, if one does not assume this, then there is no sense to give an ontology to BM

[18] This is exposed on [15, p.12]

> Bell has said that (for Bohmian mechanics) spin is not real. Perhaps he should better have said: “Even spin is not real”.[15, p.10]

A question here emerges: if any other dynamic property besides position is not fundamental, what they might be then?

Dürr thinks all dynamic properties, with the exception of positions, are not primitive. This means that besides position vectors, no actual degrees of freedom are added to the equations of motion. The operators corresponding to these dynamic properties are only part of the wave function.

Moreover, if the outcome of any operator A is defined in terms of the map $F_A(Q, \Psi)$, unique values cannot be assigned to F with the arguments defining the state (Q and Ψ). This is because different measurement setups may contribute to different wave functions of the composite system (measurement setup and system), and this may lead, in turn, to entirely different maps F.

An example, in relation to the component of spin σ_z of the electron, is the following: if Ψ and a magnetic field have an appropriate reflection symmetry with respect to a plane between the poles of an Stern-Gerlach magnet, and if the magnetic field is reversed, then the sign of $F_{\sigma_z}(Q, \Psi)$ will be reversed: for both orientations of the magnetic field the electron cannot cross the plane of symmetry. If the electron is initially above from the symmetry plane it remains above, while if it is initially below it, then it remains below it. But when the field is reversed so must be the calibration, and F changes sign with this change of experiment.

It follows that any dynamic property, with the exception of position, is contextual, and depends on the arrangement of the experiment, the surroundings of the system and on any other feature implicitly contained in the wave function. The corresponding operators are mathematical and abstract entities that are ultimately part of the wave function, which only have meaning in the measurement process.

At this point, these operators should be interpreted in the context of measurements, and besides this, they are related to the wave function at the moment of the measurement. In order to make this explanation clearer, the particular case of the momentum operator is explained.

BM equations of motion read:

$$\mathbf{v}(\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_N) = \frac{d\mathbf{q}_k}{dt} = \mathrm{i}\frac{\hbar}{m_k}\mathrm{Im}\frac{\nabla_k \psi}{\psi}(\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_N) \tag{13.2}$$

It follows that the momentum operator $\mathbf{P} = m\mathbf{v}$ only depends on the wave function, and particularly on the phase of the wave function, given a general solution $Re^{\frac{\mathrm{i}S}{\hbar}}$. This functional dependence arises from primary considerations behind the equations of motion: insofar as first derivatives are simpler than higher derivatives, the simplest possibility would appear to be that the wave function determines the velocity field. So the relation between the momentum and the wave comes up from the fact that Bohmian mechanics,

> [...] is a first-order theory, in which it is the first derivative of the configuration with

> respect to time, rather than the second, that the theory directly specifies. And the role of the wave function in this theory is to generate the vector field.[21, pp.7-8]

In this case, like the case of all other operators, the velocity field depends on the wave function, the latter specified by the system of interest together with the surroundings (measurement devices). Insofar as the term of the devices can be neglected, reproducible definite trajectories are obtained to the extent that momentum is regarded as a dynamic (Physical) property of the particles in motion. But unfortunately this never occurs. In fact, one may only performs measurements of positions of a pointer as an indication of the values of any other dynamic property. This is the reason why Dürr prefers to regard any operator (regardless position), as an abstract entity specified by the experiment rather than by any (Physical) quantity.

The essential argument here arises from the fact that evidence provided by any contextual measurement cannot be fundamental. This is in principle the argument Dürr develops in his paper. However, such an argument is not entirely obvious and should be justified. The justification Dürr provides follows from associating certain operators with particular experiments and then prove they are not well defined in the context of other experiments. On the basis of this argument is the fact that fundamental elements are ontologically grounded and their existence cannot be contingent and subjected to only certain experiments.

The above justification has been given in [16], introducing a general definition of operators. First, let us think of an idealization and approximation restricted to the domain of reproducible experiments: as far as one deals with measurements that yield the same outcome as originally obtained if immediately repeated, self-adjoint operators are associated with observables. In other words, self-adjoint operators represent actual (Physical) variables of a certain quantum state.

However, although BM self-adjoint operators are only observables in reproducible experiments, one may prepare other experiments that are not reproducible (such as particle's time decay), where this kind of operators are not appropriate. For this reason, a more general definition of operator is required to hold for any kind of experiment: normalized positive-operator-valued measure (POV)[19].

Operators in BM may only be considered (Physical) observables as far as one performs reproducible experiments. Thus, since any dynamic fundamental property cannot be contingent and subjected to only certain experiments, BM dynamic properties are not fundamental, regardless of the positions. Proving (A) in this way, it is time to go ahead to the following problem. □

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[19] The basis of the generalization lies on the equivalence between self-adjoint operators and orthogonal projection-valued-additive set functions. The generalization follows from the fact that orthogonal projections are particular examples of positive operators. With the exception of positions, Davies proves that any other observable arises from a natural association POV between an experiment and a bilinear function. This function maps the wave function of the system and apparatus to the distribution of configurations of the system (the outcome). In this 'extremely natural´ way one defines any operator from this corresponding association, depending on the type of experiment that has been performed. In particular, the momentum operator in OQM is defined as a particular case of certain reproducible experiments. See [16] to read this in detail.

Proof. **(B)** *Dynamic fundamental positions do not in general imply distinctions between basic parts.*

If one aims to assume the *ontology of basic parts*, where (Physical) reality is actually represented by systems that are wholly composed of autonomous parts, one requires additional assumptions besides the complete knowledge of the system in terms of positions.

Positions are defined as punctual coordinates restricted to Galilean space, where all (Physical) events occur. These punctual coordinates may represent points such as particles belonging to species of different kind. Likewise, all these objects may be different in their properties. But whatever they are, the positions of these elements does not generally imply distinctions between them. For example, think of electrons and quarks: although positions may be assigned to these entities, modern discoveries have proved that they are part of different particle species, in fact, to hadrons and leptons, respectively.

Now, given these insights, detailed proof should be provided to the following statement: dynamic fundamental positions do not in general imply distinctions between basic parts. The proof is given as follows:

1. First, a new formalism consistent with BM to distinguish between *basic parts* is provided.
2. A problem of underdetermination in relation to this formalism and the original one is given.

In the particular case of BM, the state of the system is only defined with Q and Φ. Moreover, all dynamic properties, except positions, are then recovered from the wave function but they are not fundamental. Positions form the primary property of the theory. In this manner, one imagines that there is a limitation to describe the variety of particle species in (Physical) reality solely in terms of positions and non-dynamical properties (mass, charge and so on). It seems that the decision as to whether a given particle is an electron or a muon is based on its dynamic behavior under certain conditions imposed, such as external fields.

Now, although the formalism of BM does not distinguish between points, Dürr introduces in [17] a modified version of BM where the formalism is modified in a limited extent to do so. Without changing the essential structure of the theory, Dürr introduces distinguishable particles and provides one way to recognize a variety of particle species with common properties, e.g. whether a given particle is an electron or a muon.

However, Dürr himself argues in [17] that the question about the preference of this formulation and the original one, compatible with BM, can never be decided experimentally. In fact, in the context of BM, one cannot obtain empirical knowledge of any dynamical *fundamental property* beyond the positions of material points. Thus, to know if particles of certain species have labeled or unlabeled world lines is underdetermined by the evidence. Such requirement must be grounded by purely theoretical, possibly philosophical considerations. In fact, the particles in BM might be just points without any labelled structure: they could be any punctual object. In [17] Dürr provides a detailed proof of this particular case of underdetermination.

So an overall summary of the arguments discussed so far reads as follows: one wishes to prove that to distinguish between different *basic parts* is possible. In order to do so, Dürr formulates other version of BM. However, the preference of the original and modified version of BM was proved to be underdetermined by the evidence.

In this manner, whether a given particle is an electron or a muon, must rather be grounded on purely foundational and philosophical considerations consistent with the mathematical structure of the theory. □

Proof. **(C)** *Un-(Physical) factorization does not lead to the ontology of basic parts.*

As far as the formalism of BM is concerned, the state of the system at a given time is specified by the positions of the particles and the evolution of the system governed by the wave function, without any emphasis on the existence of *basic parts.*

However, if one assumes the *ontology of basic parts*, the behavior of these parts should have a counterpart in the formalism provided by the positions *and* the wave function. In fact, aiming to establish a complete description and evolution of *basic parts*, certain "corpuscular" behavior must be provided by the wave function.

Contrary to the case of quantum mechanics, the classical *fundamental properties* (with their corresponding coordinates), form the total number of degrees of freedom actually present in the system besides positions, say momentum. To know both properties at a given time determines the evolution of the system and defines the (Physical) process occurring therein.

Nonetheless, in the case of BM, momentum is not a fundamental dynamic property. As shown in (A), any other dynamical *fundamental property* besides positions do not exist, but are abstract and mathematical constructions derived from the wave function. In this manner, the wave function is also responsible for the emergence of *basic parts*, when assumed, determining the evolution of these parts in time. In other words, if one assumes this ontology, then each *basic part* would correspond to a point particle possessing position as a dynamical *fundamental property*, and moving according to other contextual dynamical properties, naturally determined by the wave function.

However, the individual evolution of each *basic part* possessing these dynamic contextual properties may only be expressed in the formalism in terms of factorization. Let us explain why.

If one single wave function defining a composite system of particles factorizes into different conditional wave functions defining sub-systems, then the composite system is wholly composed of these sub-systems, as long as the contribution of the environment may be neglected. Moreover, if the composite system is wholly composed of sub-systems, defined by pure states and represented by *basic parts* (ensembles of pure states), then effective wave functions describe the evolution of these *basic parts.*

An example given in (8.5.2) may clarify this. Suppose we have a system of two particles. From the formalism of BM one can show that if W factorizes into $W = w_1 \otimes w_2$ or $\Psi = \psi_1\psi_2$, then the splitting $v^{\Psi}(v^{\psi_1}, v^{\psi_2})$ is obtained, and in particular the equation of motion of both particles is obtained as long as factorization holds.

In fact, factorization of the wave function of the composite system defines the momentum and other secondary dynamic properties of the particles, responsible for the evolution of their corresponding trajectories. This fact leads to consider factorization as the primary cause of our dynamical fragmented world in *basic parts* and the evolution of these parts.

However, according to Dürr, factorization can only hold in certain extremely unphysical cases (see 8.4). This fact reveals problems when the *ontology of basic parts* is assumed, because any ontology cannot emerge from arguments grounded on contextual and contingent bases.

Let us clarify this: if one imagines a classical system made up of several particles, then knowledge about the motion of every particle is eventually sum up in one single system, comprising the complete information of the system (a classical physical state). In this case, knowledge about the parts contributes to the complete knowledge of the whole. However, this is not the case in BM. In EPR experiments, non-local correlations between particles may occur, so the pure states corresponding to every particle cannot wholly define the physical state of the composite system. The dynamic properties of every particle depend on a non-local wave function, which depends in turn, on the positions of the rest of the particles. In this manner, the wave function of the whole system cannot be factorized into pure states, each having complete information of the dynamic properties and motion of every particle. Lack of knowledge about these properties suggests that the *ontology of basic parts* leads to an incomplete picture of objective particles in motion.

Summarizing, in BM one can factorize the wave function as far as we deal with unphysical approximations (in the case we can neglect the contribution of the environment), but this division will never be of fundamental character. In this manner, if the only apparent way to introduce the *ontology of basic parts* into the formalism of BM is by factorization, non-(Physical) factorization does not lead to the *ontology of basic parts*. □

Proof. **(D)** *The nomological interpretation of the wave function of the universe does not reflect the (Physical) facts, when basic parts are assumed.*

From the arguments discussed so far, one knows that the wave function of the universe has been devoid of (Physical) significance in the sense that it is not an object of (Physical) reality. It is rather part of the laws of nature and this is why it is nomological. Dürr has provided some arguments to support this interpretation, some of which read as follows:

On the one hand, the wave function of the universe has similarities with the classical Hamiltonian that cannot be interpreted as (Physical). On the other hand, the nomological interpretation solves the intriguing puzzles that arise when one introduces an objective description of real particles, and only particles. However, there are some objections against this interpretation. In this manner, to bring about these puzzles, the wave function of the universe (the only one that is fundamental) should be analyzed.

This wave function lives in a 3-N dimensional space, is static, timeless, and cannot be considered as a classical field because it does not have sources. Moreover, the structure of such field is of non-local character, in the sense that it depends on the instantaneous positions of all particles at a given time. From these quantum properties, some inconsistencies with the *ontology of basic parts* arise due to the following reasons:

Point particles live in Galilean space, are dynamical in space and time, and are the sources of (Physical) fields. Moreover, they are described as autonomous entities in the sense that each of them possesses properties defining different quantities, and finally, the composite systems are described in terms of the the sum and the behavior of each of these particles.

The solution Dürr provides to solve these puzzles is really simple: the wave function is not an object of (Physical) reality but a theoretical entity that describes how natural laws come about. Let us go in depth into this solution:

Dürr avoids the controversial debate about the (Physical) meaning of the wave function, and argues that the (Physical) manifestations of the wave function are not produced by any (Physical) object, but are governed by natural laws arising from the formalism of BM. However, factorization has been considered as an operation in the formalism of BM that cannot be considered part of these (Physical) manifestations. So if one assumes the *ontology of basic parts*, one knows from (C) that there is no choice but to introduce an element described by the theory indispensable for this ontology (a factorizable wave function), that does not have a counterpart in (Physical) reality. But to say that the factorizable wave function is nomological does not really solve the puzzle, because natural laws are not in contradiction with any (Physical) fact. In fact, the contribution of the environment cannot be neglected and this fact is similar to the so called fundamental non-separability provided by the natural laws and described in this thesis above, which is represented by the non-local character of the wave function.

This is exactly the objection against such ontology in the context of BM. One may formulate quantum mechanics in terms of *basic parts*, provided by the factorization of the wave function. Nonetheless, such operation is not (Physical).

It follows that Dürr's solution is not natural in the sense that it is an apparent way out to the problems arising from the introduction of *basic parts*. In fact, the formalism of BM is a mathematical description of the motion of points comprising elements one cannot see. But, the existence of *basic parts* follows from the formalism as far as factorization is a (Physical) process, which is almost impossible, as Dürr proved. Conversely, such ontology has been brought about by additional fundamental requirements (such as simplicity), aiming to provide a causal and mechanical explanation of quantum processes.

It is then clear that the criteria used to assume a mechanistic, deterministic and objective point of view (the *ontology of basic parts*), either real factorization and the nomological interpretation, bring about some difficulties in accordance with the (Physical) facts. Arguments of such nature are *ad-hoc*. Of course, to interpret a factorizable wave function as nomological is fine as far as one may neglect the contribution of the environment in real and (Physical) cases. However, the (Physical) facts deny so, and one should point out that the non-separability of quantum systems occurs naturally, arising from the new notions of unbroken wholeness developed above. □

13.2.3 The conclusion arising from our premises

The conclusion that follows from these four premises is that the criteria used to justify and assume the *ontology of basic parts* in BM are not at all in correspondence with the (Physical) facts. These underlying foundational criteria that justify this ontology, such as the factorization and the nomological interpretation of the wave function, are not natural, and there are plenty of problems that arise when *basic parts* are assumed as primary ontology. Dürr assumes the *ontology of basic parts* with criteria that comprises subjective elements and *adhoc* assumptions. All these aspects weaken BM, because the existence of basic parts cannot be tested experimentally and the ontological basis of the theory cannot be derived from general principles, where the formalism of BM is compatible with (Physical) reality.

In this manner, to remove the *ontology of basic parts* is the best choice and the foundational basis of BM should be of another kind, in accordance with the essential properties of the wave function. To remove this ontology we do not mean particles do not exist, but that any reductionist description should not be regarded as fundamental. The wave function is so important that it does not make sense to focus on the parts, which fully describe the whole. There is something beyond these particles.

In the meantime, Bohm's philosophy has already provided a new notion of unbroken wholeness, where the non-separability of process arises in a clearer and natural way. In this manner, this discussion suggests that to focus on the parts, which compose the whole system, generates problems in accord to the reliability of the theory. It seems that the fundamental reality is not in the particles but in configuration space, where the wave function lives and contributes to the non-local effects observed in the quantum phenomena. After providing some arguments that weakens BM, what follows then?

A criticism to BM has been given, which is the only theory that removes the quantum potential from the Bohmian approach. Moreover arguments aiming to criticize the status of BQM have been proposed. Along these lines, it seems that Bohmian approaches cannot give a complete and consistent picture of quantum processes.

However, all arguments written so far have already given hints to assess some theories and to choose among them a reliable ontology, in particular the status of the quantum potential. From this discussion one feels that the essential element in Bohm's approach has been removed or forgotten. According to Bohm's original view, although it seems that the ontology of the quantum potential is of great importance and fundamental to some extent, there are plenty of reasons to say that it is not part of (Physical) reality. These reasons do not emerge from BM, but from other general theory already discussed, which has contributed to develop essential features from the later Bohm: HOL.

Conversely to BM, HOL has already provided what is expected. It is indeed a mathematical theory from which the status of any theoretical entity (including the quantum potential, the particles and the wave function), are justified and explained in a consistent way. The quantum potential has proved to be the element that connects BQM with Bohm's philosophy. Although it is an essential mathematical element that has being considered to express the theory in terms of implicate and explicate orders, it is devoid of any (Physical) meaning as it is the case for explicate orders or implicate orders of lower levels.

Moreover, in contrast to the case of BM, which lacks of strong reasons to support a mechanical view, in HOL one finds a coherent philosophical and mathematical justification of BQM arising from the idea o processes taking place in nature, and formalized along the lines of Clifford algebras.

Now, a new mathematical structure has been proposed, one that reflects the process of reality as a whole, and reduces BQM as a mere approximation to such process in the domain of non-relativistic quantum mechanics.

Part IV

Conclusions

Is the quantum potential fundamental in quantum mechanics?

The conclusion is no, in the sense that it is not an object of (Physical) reality. In both 'Bohmian Mechanics' (BM) and Bohm's theory (BQM), the quantum potential is devoid of (Physical) significance. In BM there is no quantum potential; in BQM the non-fundamental status of such entity is considered in the context of Bohm's general holistic philosophy. Bohm's philosophy provides the basis for a new understanding of reality, where a new order of unbroken wholeness emerges in quantum phenomena. Moreover, Hiley's approach (HOL) erases the line between BQM and Bohm's philosophy and contributes to understand quantum mechanics from a new fundamental way, where the role of the quantum potential is interpreted in terms of new notions of order developed by Bohm later in his career. Finally, one concludes that there is no quantum potential in (Physical) reality, but there is at least some arguments explaining the appearance of such term into the equations, besides the relevance of BQM over other interpretations.

BQM and BM may be seen as two similar accounts: both use the same formalism of quantum mechanics as far as the mathematical structure is concerned; both are empirically equivalent so that one cannot draw a line in regard to the evidence they predict; yet, both obey a particular equation of motion together with the Schrödinger equation; and both regard probabilities as epistemic. However, the difference between them lies on their ontology and their connection with HOL. In fact, HOL reduces to the wave-particle ontology (BQM) by postulating the classical law of motion within the formalism of quantum mechanics, while BM only assumes an ontology of particles in contradiction with HOL.

Moreover, HOL has been introduced to solve some problems and objections against BQM, following the lines of Bohm's philosophy. At first, the quantum potential is required not only to make plausible a causal and mechanical description of particles in motion, but to provide a suitable ontological basis, consistent with current (Physical) observations in the quantum domain. Likewise, HOL emerges quite natural from deeper mathematical principles, arising from holism and the notion of process. The quantum potential is therefore the counterpart of Bohm's implicate orders in the theory and explains the relevance of the wave function living in configuration space and the ultimate origin of non-locality.

This thesis proposes that there is a variety of advantages of HOL over BM, because the latter cannot succeed to explain the *ontology of basic parts* from certain criteria that are not at all in correspondence with the (Physical) facts. It seems that this ontology has been introduced to explain quantum mechanics in terms of the Newtonian view. In fact, the *ontology of basic parts* is formulated from a mechanistic point of view arising from Newtonian mechanics, and one expects that when quantum mechanics is considered, then this ontology cannot be assumed naturally. In order to assess the criteria used to support the *ontology of basic parts* one should point out the contingent nature of factorization of the wave function, provided by Dürr. If factorization is not (Physical), there is no way to describe the dynamics of the systems in terms of *basic parts* with definite trajectories. Besides this, to introduce the *ontology of basic parts*, an additional assumption is required beyond the formalism of the theory: a nomological interpretation of the wave function, which behaves very different compared to the classical fields. Thus, BM provides a theory about *basic parts*, where the adaptation of the old notion of order to current domains has brought about several problems. Furthermore, there has been notorious work in trying to make BM compatible with relativity without success, whereas HOL reduces to Einstein's theory.

Therefore, after a long discussion, this thesis suggests to prefer HOL instead of BM.

Part V

Appendix

14 Appendix 1

Let us show how the solutions to the Hamilton-Jacobi equations are solutions of the equations of motion in classical mechanics:

Let $L = L(q, \dot{q}, t)$ be the Lagrangian with q and $\dot{q} = \frac{\partial q}{\partial t}$ the generalized coordinates and their first derivatives respectively.

The canonical momenta relative to the coordinate q_i is defined as:

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

Now, the Legendre transformation of L (where the generalized coordinates q are replaced by the canonical momentum p), introduces a new function H, the so called Hamiltonian:

$$H(q,p,t)=\sum_i p_i\dot{q}_i - L(q,\dot{q},t)$$

Now, expressing in explicit form the action integral $I=\int Ldt=\int\sum_i p_i d\dot{q}_i - Hdt$, and using the Hamilton's principle, the Hamilton's set of $2n$ firsr-order differential equations are obtained:

$$\dot{q}_i=\left.\frac{\partial H}{\partial p_i}\right|_{q_j=q_j(t),p_j=p_j(t)} \qquad \dot{p}_i=-\left.\frac{\partial H}{\partial q_i}\right|_{q_j=q_j(t),p_j=p_j(t)} \tag{14.1}$$

Let us now consider coordinate transformations in phase space by replacing the variables q and p by $Q(q,p,t)$ and $P(q,p,t)$ respectively. The task is to find transformations such that the form of the canonical equations is preserved, the so called canonical transformations. Consequently, both must obey Hamilton's principle to retain this particular form with $K=K(Q,P,t)$, as the new Hamiltonian:

$$\dot{Q}_i=\left.\frac{\partial K}{\partial P_i}\right|_{Q_j=Q_j(t),P_j=P_j(t)} \qquad \dot{P}_i=-\left.\frac{\partial K}{\partial Q_i}\right|_{Q_j=Q_j(t),P_j=P_j(t)}, \tag{14.2}$$

but to satisfy the Hamilton's principle in both coordinates, the integrands from the integral action of both coordinates must differ by a total time derivative:

$$\sum_i P_i\dot{Q}_i - K(Q,\dot{Q},t)=\sum_i p_i\dot{q}_i - H(q,\dot{q},t)-\frac{dF(q,p,Q,P,t)}{dt}, \tag{14.3}$$

where $F=F(q,p,Q,P,t)$ is a total-differentiable function called *Generating function*[20].

To solve the dynamical problem from (14.3), it is important to notice that the evolution of the system in time, is precisely the continuous unfolding of a canonical transformation, i.e, a shift in the time origin $Q(t)=q(t+\Delta t)$ and $P(t)=p(t+\Delta t)$. In fact, considering the infinitesimal transformations M under p and q:

$$\begin{aligned}\dot{Q}_i &= M_{ij}\dot{q}_j \quad \text{or} \quad Q_i=q_i+\delta q_i\\ \dot{P}_i &= M_{ij}\dot{p}_j \quad \text{or} \quad P_i=p_i+\delta p_i,\end{aligned}$$

where $M_{ij}=\frac{\partial Q_i}{\partial q_j}=\delta_{ij}+\epsilon A_{ij}$ is the infinitesimal transformation in Jacobian form, and A an element that is defined. In order for M to be canonical, one must impose that the new elements obey Hamilton's equations under the same transformation.

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[20] It is called Generating Function of a canonical transformation because it is uniquely defined by this transformation. In other words, the new and old Hamiltonian are related via F. Moreover, F is not a function in phase space but maps two sets of coordinate systems in that space.

Let $\vec{x} = (q_1, q_2, ..., p_1, p_2, ...)$, so $\vec{\dot{x}_i} = J_{ij}\frac{\partial H}{\partial \vec{x_j}}$, where J is called the symplectic form. Then, if $\vec{X} = (Q_1, Q_2, ..., P_1, P_2, ...)$:

$$\begin{aligned}\dot{\vec{X}} = MJ\frac{\partial H}{\partial \vec{x}} &= MJM^T\frac{\partial H}{\partial \vec{X}} \\ &= J\frac{\partial H}{\partial \vec{X}}\end{aligned}$$

By the relation $M = \mathbb{I} + \epsilon A$, the transformation is canonical iff the infinitesimal coefficient of ϵ vanishes, i.e. $J = MJM^T = (\mathbb{I} + \epsilon A)J(\mathbb{I} + \epsilon A^T) = J + \epsilon(AJ + JA^T) + \emptyset(\epsilon^2)$. But for the coefficient to be zero, A must be expressed in terms of a symmetric matrix. But the symmetric form is guaranteed if $A = JG$, where $G_{ij} = \frac{\partial \mathbb{G}}{\partial x_i x_j}$ is symmetric and $\mathbb{G}$ is a particular *Generating Function* in phase space that obeys $\frac{\partial \mathbb{G}}{\partial x_i x_j} = \frac{\partial \mathbb{G}}{\partial x_j x_i}$. Therefore:

$$\begin{aligned}M &= \mathbb{I} + \epsilon J \mathbb{G}_{,x_i,x_j} \\ X &= x + \epsilon J \mathbb{G}_{,x}\end{aligned}$$

From the definition of J, the following relations hold:

$$\begin{aligned}Q &= q + \epsilon G_{,p} \\ P &= p + \epsilon G_{,q}\end{aligned}$$

Now, consider the case where $\epsilon = dt$, a small time interval, and $\mathbb{G} = H$, the Hamiltonian. One can easily derive $\delta q_i = dq_i$ and $\delta p_i = dp_i$, where the infinitesimal transformation generated by the Hamiltonian corresponds to the physical change of the generalized coordinates and momenta during the time interval dt.

Therefore, one may conclude that the actual motion during a finite time interval of any physical system, governed by Hamilton's equations, may be expressed as a continuous canonical transformation. So, the following canonical transformations are indeed equations of motion:

$$q = q(q_0, p_0, t) \qquad p = p(q_0, p_0, t) \tag{14.4}$$

Once the previous result is obtained, the equations of motion are derived from (14.3), and thus, from the generating function, defining the canonical transformation. But first, a solution to F must be found.

Suppose one inverts (14.4) to give constant (in time) coordinates, i.e, $\dot{Q} = \dot{P} = 0$, where $Q = q_0$ and $P = p_0$ at t_0. These coordinates are expressed in terms of the variable coordinates q and p at time t (by taking $K = 0$ in the Hamilton's equations (14.2)). Let us denote $F = S$ the generating function[21]. Then, S must be a function of q, Q and t, and from (14.3) one

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[21] In view of the identity of the action function with the form of the generating function.

obtains:

$$\begin{aligned} p_i &= \frac{\partial S}{\partial q_i} && (14.5) \\ P_i &= -\frac{\partial S}{\partial Q_i} && (14.6) \\ K &= H + \frac{\partial S}{\partial t} = 0 && (14.7) \end{aligned}$$

It is important to remember that Q and P are constants in time, while q and p are variables.

Then from (14.5) and (14.7) one obtains a fist-order partial differential equation in the $(n+1)$ variables $(q_1, q_2, ..., q_n, t)$ the so called Hamilton-Jacobi equation:

$$\frac{\partial S(q,Q,t)}{\partial t} + H\left(q, \frac{\partial S(q,Q,t)}{\partial q}, t\right) = 0 \tag{14.8}$$

The function S is called Hamilton's principal function, which generates via Hamilton's equations, a canonical transformation to coordinates and momenta that are constant along trajectories. This equation is solved by deriving a complete solution for S, in terms of the variables q and t, besides the constants in time Q.

In general, integration constants $\{\alpha_i\}$ depending on Q and P are obtained, because S is not in (14.8). So, the new constant coordinates from the transformation are expressed in terms of α by $Q_i = \gamma(\alpha_1, ..., \alpha_n)$. Equation (14.6) then becomes:

$$P_i = -\frac{\partial S(q,\gamma,t)}{\partial \gamma_i} = \beta_i \tag{14.9}$$

This is called *The Jacobi Law of Motion.*

Once the solution of $S = S(q,\gamma,t)$ is given, one must express β and γ in terms of the actual initial conditions $q = q_0$, $p = p_0$ and $t = t_0$. If one evaluates (14.9), and the transformation equation (14.5) by $q = q_0$ and $t = t_0$, a system of two equations with solutions γ and β are obtained in terms of q_0 and p_0. Therefore, the equations of motion (14.4) are derived.

It is important to mention that *Jacobi Law of motion*, or simply *Law of motion*, is introduced by the Hamilton-Jacobi theory obeying the Hamilton equations for the one-particle system. This theory has not been introduced in the context of quantum mechanics. Therefore, the *Law of motion* is postulated by requiring a wave-particle formulation and then shown to be consistent in the quantum framework. It cannot be derived from Schrödinger equation.

15 Appendix 2

To suggest a possible connection between the H.J. formalism used in classical mechanics with the one used in quantum mechanics, a geometric correspondence is given. First, suppose an scalar external potential of a one-particle system is given:

$$m\dot{q} = \nabla S \tag{15.1}$$

Now, consider constant solutions S as surfaces in euclidean space $S(q, \alpha, t) = c$. At $t = t_0$, the surface S may be initially concentrated at the point $q = q_0$ or in a certain initial surface $S_0 = S_0(q_0, \alpha, t_0)$, in the case it depends on α. During the motion of the particle, the surface S propagates in euclidean space and $\nabla S(t)$ represents a normal vector to the surface at time t, where the trajectory of the particle is locally defined. In other words:

> *The family of trajectories that are solutions of the equations of motion, are normal to some constant surface for all* t *and* q.

This is valid in both classical and quantum mechanics. Note that the previous statement is different from the fact that two different solutions (surfaces S_1 and S_2), sharing a tangent vector in common for certain initial conditions $\nabla S_1(t_0) = \nabla S_2(t_0)$, share a common tangent vector for all time $\nabla S_1(t) = \nabla S_2(t)$, i.e, this particular trajectory will be common. The statement above states that a family of trajectories, identified by their initial positions q_0, are always and everywhere normal to constant surfaces. But this does not imply the fact that different constant surfaces, sharing a common normal vector, define a unique trajectory during the motion of the particle.

The second claim holds for classical physics but not for quantum theory. In fact, in quantum mechanics different solutions with fixed initial conditions do not imply an unique common trajectory. Rather, there exists a one-to-one relation between S-dynamics and particle dynamics: every solution S, with certain initial conditions, corresponds to only one trajectory, because the wave (where S belongs), *and* the position of the particles form a complete physical system.

16 Appendix 3

Let us derive the BM equations of motion.

In detail, one wishes to maintain the simplest formulation by requiring functional dependency of the velocity upon the wave function. To arrive at an explicit form of the velocity $v^{\Psi}(\mathbf{q})$, symmetry should be assumed as our main guide, and the quantum state should be given by ($\mathbf{q}$, ψ).

Consider a free particle with mass m, whose wave function satisfies the free Schrödinger equation and whose velocity obeys the following equation:

$$\begin{aligned} i\hbar\frac{\partial\psi}{\partial t} &= -\frac{\hbar}{2m}\nabla^2\psi &(16.1)\\ \frac{d\mathbf{q}}{dt} &= v^{\psi}(\mathbf{q}) &(16.2)\end{aligned}$$

If the velocity $v^{\Psi}(\mathbf{q})$ is expressed in such a way that both equations (16.1) and (16.2) remain invariant, under Galilean (rotations and boosts) and time-reversal transformations, then one obtains (8.1).

Part VI
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