# Standing Sound Waves in a Bose-Einstein Condensate 

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#### Abstract

We study two different subjects related to ultra-cold Bose gases. First we study the influence of the trap and the background pressure on particle loss and temperature change for trapped atom clouds. We find the number of particles to exponentially decrease over time due to the imperfect vacuum, while the rate of temperature change differs between a magnetic trap and a far off resonance optical trap. Secondly, we study the properties of a standing sound wave induced in a Bose-Einstein condensate. Phase contrast imaging is used to image the condensate in order to observe the density profile as a function of time. We find that there is coupling between the standing wave and other collective modes. There is a clear relation between the frequencies of these collective modes, and also between the frequencies of these modes and the frequency used to induce the modes. We also find that the amplitude of the standing wave decays over time.


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## Chapter 1

## Introduction

In the beginning of the 20th century, Bose and Einstein predicted a state of matter in which a macroscopic fraction of the particles would be in the ground state $[1,2]$. This phase occurs only for sufficiently large density combined with sufficiently low temperature. The transition to this phase is called Bose-Einstein condensation. In Utrecht, we make the largest Bose-Einstein condensates in the world, composed of a few hundred million sodium atoms $[3,4]$. As a consequence, our condensates are in the hydrodynamic regime: the mean free path of the atoms is less than the size of the condensate [5]. This gives us the opportunity to investigate several properties of condensates, for example density excitations. Our condensates are cigar-shaped (as shown in fig 1.1), with a length of a few millimeters in the long direction. Their transition temperature $T_{c}$ is of the order of $1 \mu \mathrm{~K}$ [4]. The concept of Bose-Einstein condensation and the process of making them will be discussed in chapter 2.


Figure 1.1: An image of one of our condensates. The image's size is 0.16 mm by 2.68 mm .

In order to make condensates, a cloud of particles has to be trapped in vacuum inside a potential well. However, a trap and the vacuum cannot be perfect, therefore a cloud inside a trap will undergo particle loss and temperature change [6]. It is important to know how these imperfections influence the cloud inside the trap, as that allows one to include these effects when analyzing measurements. This subject will be discussed in chapter 3 .

Bose-Einstein condensates are superfluids, which have two different sound modes: first and second sound [7]. One of these modes corresponds with density fluctuations, while the other corresponds with temperature fluctuations. We can measure the properties of these modes by making images of a condensate at different points in time.

By periodically squeezing the condensates radially, we managed to induce a highly excited mode inside the condensate that closely resembles a standing sound wave. This is a very remarkable feature, considering the condensates do not have sharp edges. The mode itself has interesting properties and we want to discover the origin of the mode, what its properties are and how these properties relate to each other. This specific mode is the main subject of this report, being discussed in chapter 4.

## Chapter 2

## Bose-Einstein condensates

### 2.1 Bose-Einstein condensation

In order to perform research on Bose-Einstein condensates, one first has to understand the concept of Bose-Einstein condensation and its consequences. This is the aim of this chapter.

### 2.1.1 Bose distribution and condensation

To understand the concept of Bose-Einstein condensation, we first need to understand the statistics that describe bosons. The methods used here can also be found with more detail in Blundell [8]. Classically, particles satisfy the Gibbs distribution:

$$
\begin{equation*}
P_{i}=\frac{e^{\beta\left(\mu N_{i}-E_{i}\right)}}{\sum_{i} e^{\beta\left(\mu N_{i}-E_{i}\right)}}=\frac{e^{\beta\left(\mu N_{i}-E_{i}\right)}}{\mathcal{Z}} \tag{2.1}
\end{equation*}
$$

Here, $P_{i}$ is the probability of finding a system in a specific microstate $i, \mu$ is the chemical potential (the energy difference when adding one particle to an ensemble), $\beta$ equals $1 / k_{B} T, k_{B}$ is the Boltzmann constant, $T$ is the temperature, $\mathcal{Z}$ is the grand partition function and $N_{i}$ and $E_{i}$ are the number of particles and the energy of microstate $i$. If we look at one state with energy $E_{i}=N_{i} E$, where $N_{i}$ is the amount of particles in this one state and $E$ is the energy associated with that state, we can write the average amount of particles in this state as:

$$
\begin{equation*}
\left\langle N_{i}\right\rangle=\frac{\sum_{N_{i}} N_{i} e^{\beta N_{i}(\mu-E)}}{\sum_{N_{i}} e^{\beta N_{i}(\mu-E)}}=-\frac{1}{\beta \mathcal{Z}} \frac{\partial \mathcal{Z}}{\partial E}=-\frac{1}{\beta} \frac{\partial \ln \mathcal{Z}}{\partial E} \tag{2.2}
\end{equation*}
$$

Exchange symmetry allows bosons to have many identical particles in the same state. Therefore, we have to take $N_{i}$ from 0 to $\infty$ when calculating $\mathcal{Z}$ :

$$
\begin{equation*}
\mathcal{Z}=\sum_{N_{i}=0}^{\infty} e^{\beta N_{i}(\mu-E)}=\frac{1}{1-e^{\beta(\mu-E)}} \tag{2.3}
\end{equation*}
$$

Here we used the fact that the sum is a geometric series. In order for the series to be finite, the argument $\left(e^{\beta(\mu-E)}\right)$ must be smaller than 1 , so $E$ must always be larger than $\mu$. If we assume the ground state energy is zero, we get that
$\mu<0$. If we now replace $\mathcal{Z}$ from eqn 2.3 inside eqn 2.2 , we get that the average occupation of a state takes the following form for bosons:

$$
\begin{equation*}
f(E)=\left\langle N_{i}\right\rangle=\frac{1}{e^{\beta(E-\mu)}-1} \tag{2.4}
\end{equation*}
$$

This equation is called the Bose distribution.
With this, we can calculate the total number of particles in a fixed volume and with a fixed temperature, namely with $N=\sum_{i} N_{i}$. To calculate this sum, we can use the concept of density of states to transform the sum into an integral:

$$
\begin{equation*}
N=\sum_{i} N_{i}=\int_{0}^{\infty} g(E) f(E) \mathrm{d} E \tag{2.5}
\end{equation*}
$$

For uniform ensembles in 3D, the density of state takes the following form:

$$
\begin{equation*}
g(E)=\frac{(2 S+1) V}{(2 \pi)^{2}}\left(\frac{2 m}{\hbar^{2}}\right)^{3 / 2} E^{1 / 2} \tag{2.6}
\end{equation*}
$$

Here $S$ is the total spin per particle (which is an integer for bosons), $V$ is the volume available to the particles, $m$ is the particle mass and $\hbar$ is the reduced Planck constant. Using this $g(E), N$ becomes:

$$
\begin{equation*}
N=\left[\frac{(2 S+1) V}{(2 \pi)^{2}}\left(\frac{2 m}{\hbar^{2}}\right)^{3 / 2}\right] \int_{0}^{\infty} \frac{E^{1 / 2} \mathrm{~d} E}{z^{-1} e^{\beta E}-1} \tag{2.7}
\end{equation*}
$$

Here $z$ is the fugacity, defined as $z=e^{\beta \mu}$. This integral has a well known solution for the total number of particles:

$$
\begin{equation*}
N=\frac{(2 S+1) V}{\lambda_{\mathrm{th}}^{3}} \mathrm{Li}_{3 / 2}(z) \tag{2.8}
\end{equation*}
$$

The thermal de Broglie wavelength ( $\lambda_{\mathrm{th}}$ ) equals $\lambda_{\mathrm{th}}=\frac{h}{\sqrt{2 \pi m k_{B} T}}$, where $h$ is the Planck constant. The Li function in this relation is a polylogarithm with known values. More specifically, $\mu<0$ in order to get physical results, so $z$ must be between 0 and 1 . Within this range, the polylogarithm can only have a maximum value of 2.612 . This means that, according to this relation for $N$, we apparently have an upper limit for the density $n=N / V$ when the temperature is fixed. This limited density is not a physical result. Let us now separate the total number of particles $N$ in the number of particles in the ground state $\left(N_{0}\right)$ and the number of particles in excited states $\left(N_{1}\right)$. The solution for the problem of getting a limited density lies in the fact that a macroscopic amount of particles occupies the ground state $N_{0}$ below the temperature $T_{c}$ where eqn 2.8 fails. This phenomenon is called Bose-Einstein condensation. We can rewrite eqn 2.8 with the maximum value for the polylogarithm to get the value for $T_{c}$ :

$$
\begin{equation*}
k_{B} T_{c}=\frac{2 \pi \hbar^{2}}{m}\left(\frac{n}{2.612(2 S+1)}\right)^{2 / 3} \tag{2.9}
\end{equation*}
$$

Below this temperature, the amount of particles in excited states $N_{1}$ is still given by eqn 2.8 , but the polylogarithm is now close to its maximum value 2.612 . The


Figure 2.1: Fraction of particles in the ground state as a function of T.
total density below $T_{c}$ is given by:

$$
\begin{equation*}
n=\frac{2.612(2 S+1)}{\left[\lambda_{\mathrm{th}}\left(T_{c}\right)\right]^{3}} \tag{2.10}
\end{equation*}
$$

With this, we can calculate the fraction of particles in the ground state when we are below $T_{c}$ :

$$
\begin{equation*}
\frac{n_{0}}{n}=\frac{n-n_{1}}{n}=1-\left(\frac{T}{T_{c}}\right)^{3 / 2} \tag{2.11}
\end{equation*}
$$

As is clear from this equation, a growing fraction of particles occupies the ground state as the temperature decreases below $T_{c}$, which is Bose-Einstein condensation. Note that this condensation is purely a result from exchange symmetry and has nothing to do with particle interactions. In fact, we have only considered non-interacting particles so far.

### 2.1.2 Gross-Pitaevskii equation

We will now consider interacting particles. The ground state of a system of identical bosons is given by the Gross-Pitaevskii equation $[9,10]$ :

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m} \nabla^{2} \psi(\mathbf{r})+V(\mathbf{r}) \psi(\mathbf{r})+U_{0}|\psi(\mathbf{r})|^{2} \psi(\mathbf{r})=\mu \psi(\mathbf{r}) \tag{2.12}
\end{equation*}
$$

Note that this equation is a nonlinear Schrödinger equation. This equation comes from minimizing the term $E-\mu N$ for fixed $\mu$. This procedure is equivalent to minimizing $E$ for fixed $N$, using $\mu$ as a Lagrange multiplier. The first two terms in the equation are the kinetic energy and the potential energy, which are the same as in the usual Schrödinger equation. The term with $U_{0}$ is the term that includes particle interactions. The constant $U_{0}$ equals $U_{0}=4 \pi \hbar^{2} a / \mathrm{m}$, where $\hbar$ is the reduced Planck constant, $a$ is the scattering length of the particles and $m$ is the particle mass. The final term is similar to the $E \psi(\mathbf{r})$ term of the Schrödinger equation, but now the constant is the Lagrange multiplier $\mu$, found to be equal to the chemical potential. Note that the $\psi(\mathbf{r})$ in the equation is the usual quantum mechanical wave function times the number of particles (since eqn 2.12 describes the sum of all particles in the ground state), so $|\psi(\mathbf{r})|^{2}=n(\mathbf{r})$.


Figure 2.2: Density profile of large BECs inside a harmonic oscillator potential.

### 2.1.3 Thomas-Fermi approximation

The Gross-Pitaevskii equation is very hard or even impossible to solve in most cases. The approximation used here is the Thomas-Fermi approximation: for sufficiently large clouds, one can neglect the kinetic energy (the first term in eqn 2.12) when calculating the ground state energy, because for a large number of particles, the potential energy is large compared to the kinetic energy. Doing this, the Gross-Pitaevskii equation can be solved [11,12]:

$$
\begin{equation*}
n(\mathbf{r})=|\psi(\mathbf{r})|^{2}=[\mu-V(\mathbf{r})] / U_{0} \tag{2.13}
\end{equation*}
$$

In our experiments, Bose-Einstein condensates (or BECs) are trapped inside a potential approximated by a harmonic oscillator potential with radial and axial trap frequencies:

$$
\begin{equation*}
V(\mathbf{r})=\frac{1}{2} m\left[\omega_{r}^{2}\left(x^{2}+y^{2}\right)+\omega_{z}^{2} z^{2}\right] \tag{2.14}
\end{equation*}
$$

Here $x, y$ and $z$ are the position coordinates with respect to the center of the potential, $m$ is the particle mass and $\omega_{x}=\omega_{y}=\omega_{r}$ and $\omega_{z}$ are the characteristic angular frequencies of the harmonic oscillator potential. With this potential, we get a density profile that is an inverted parabola.

We demand that the density is non-negative because a negative density would not be physical, so the density is zero when $V(\mathbf{r})>\mu$. This implies a boundary of the condensate given by $V(\mathbf{r})=\mu$; the condensate only exists within this boundary. The radius at which $V$ equals $\mu$ is called the ThomasFermi radius $R$. This also allows us to measure the chemical potential $\mu$, as we only need to measure the size of the condensate with a known potential.

### 2.2 Cooling to BEC

The process of making a BEC is first explained in short, after which the different steps are discussed in more detail. An atomic beam is created by an oven, after which they are slowed in the Zeeman slower. This consists of a counter-propagating laser beam that slows down the particles emitted by the oven. At the end, they are caught in a magneto-optical trap (or MOT), where they are cooled further. After a certain time, a large amount of the particles is transferred to the magnetic trap (or MT), where forced evaporative cooling
will cool the remaining particles to BEC.

The different steps of making a BEC are now discussed in more detail. We start with a large flux of sodium particles, coming from an oven as a hot atomic beam. They are sent into a counter-propagating resonant laser beam. This part is the Zeeman slower. As the laser beam is resonant with the particles, the photons from the laser beam can transfer momentum to the particles. First the particles absorb the counter-propagating photons. After absorption, the photon is emitted in a random direction. On average this results in momentum transfer from the photons to the particles. The momenta of the laser beam and the particles are in opposite direction, so the transferred momentum slows down the particles [3,4]. However, the required frequency of the laser beam depends on the speed of the particles due to Doppler-shift. To compensate for this, there is a magnetic field that reduces in strength towards the end of the Zeeman slower. With the magnetic field, the Zeeman effect is utilized to shift the frequency to the appropriate value $[3,4]$.

At the end of the Zeeman slower, the atoms are slowed enough to be caught in the MOT. To trap and cool the atoms, the MOT consists of three orthogonal pairs of off-resonant laser beams and a magnetic field. The magnetic field causes a Zeeman shift that radially increases in strength, giving atoms with high velocities and atoms that are far away from the center a higher probability to be slowed. Since particles with high velocities are slowed more often than particles with low velocities, the slowing effect results in cooling. Together this results in a trap that causes cooling, where the particle cloud is kept at the intersection of those six beams [13]. There is a possibility the particles fall back to a state that is not trapped, requiring an additional repump beam to keep the atoms in the cooling cycle [3,4]. However, this cooling is limited by the recoil energy the atoms gain by scattering the photons.

After being cooled to $\sim 10^{-4} \mathrm{~K}$ by the MOT, the particles are transferred to the MT. The magnetic field of the MT closely resembles a harmonic oscillator potential near the center. The potential itself is given by $U=-\mathbf{m} \cdot \mathbf{B}$, where $\mathbf{B}$ is the magnetic field and $\mathbf{m}$ is the magnetic dipole moment. The alignment of the magnetic moment is directly related to the spin state of a particle. This results in high-field seekers (in our case atoms in the $3{ }^{2} S_{1 / 2}$ ground state with $\left|F=1, m_{F}=1\right\rangle$ hyperfine component), low-field seekers (atoms with $\left|F=1, m_{F}=-1\right\rangle$ hyperfine component) and magnetically neutral atoms that are not influenced by the trap (atoms with $\left|F=1, m_{F}=0\right\rangle$ hyperfine component) [3,4]. Only low-field seekers are trapped, which we use to selectively remove the particles with the highest energy, called forced evaporative cooling [14]. A radio frequency (or rf) field is applied with such a frequency that it only flips the spin of the particles with the highest energy, which are then untrapped. After removing the particles with the highest energy, the cloud re-thermalizes to a lower average energy and thus a lower temperature. The frequency is then lowered so that once again the particles with the highest energy are removed, until the point where we reach our aimed temperature. This technique requires a specific rf sweep, because doing it too fast does not give the system time to re-thermalize (resulting in loss of too many particles), while doing it too slow increases the amount of particles
lost to the background pressure. Using this technique, we can reach $T_{c}$ of Bose-Einstein condensation, which is around one microKelvin for our trap geometry and amount of particles.

## Chapter 3

## Trap loss and heating

### 3.1 Theory

A trapped atomic cloud can be influenced by several processes. We will look at two different processes: particle loss and temperature change. We investigated those processes in two different types of traps, namely the magnetic trap (a very deep magnetic potential well) and an optical dipole trap (the FORT, or Far Off Resonance Trap). The FORT is an intense infrared laser beam, used to generate an optical force strong enough to trap particles independent of spin state. Because of the far off resonance frequency of the laser beam, not many particles are excited to higher energies by random scattering.

### 3.1.1 Lifetime

Particles can be lost from the trap. One of the causes is that the clouds are not contained in a perfect vacuum, so there is a background pressure. If particles from the cloud collide with hot particles from the background pressure, the kinetic energy that is transferred is large enough for trapped particles to escape from the trap. This process exists in both the MT and the FORT. This process is exponential: the more particles there are, the more particles are lost due to this process, resulting in exponential decay. This exponential decay introduces a lifetime $\tau$ for the cloud inside the trap. So when limiting ourselves to background losses, the number of particles as a function of time will be:

$$
\begin{equation*}
N(t)=N(0) e^{-t / \tau} \tag{3.1}
\end{equation*}
$$

Another reason for particle loss is three-body recombination [15]. If two particles recombine close to another particle, that last particle gains the energy from the recombination process and is able to leave the trap. This process mostly occurs at large density, because two particles cannot recombine if no third particle is present to carry the energy away. This loss process effectively limits the density one can achieve. Three-body losses are most important when cooling the particles, because the density is increasing during the cooling and three-body losses have to be avoided in order to cool effectively. It is less important when looking at decay of a stationary cloud, because the cloud had time to reach an equilibrium, which would not be an equilibrium if the amount
of three-body losses are still high. However, it is a process to consider.

Particles trapped by the FORT are subject to random photon scattering, which is a type of interaction that also happens with off-resonant photons [16]. This process acts on all particles, resulting in another source of exponential decay. Together with the background losses, an atomic cloud inside the FORT has a shorter lifetime than a cloud inside a MT. Moreover, the FORT has a much smaller trap depth than the MT, causing the particles with the highest energies to escape, which is a form of passive evaporative cooling and also an extra source of particle loss inside the FORT.

### 3.1.2 Heating and cooling

The fact that the FORT has a finite trap depth causing passive evaporative cooling results in temperature decrease. This gives the FORT a form of cooling that does not occur in the MT. The cloud will exponentially cool inside the FORT, converging to a final temperature determined by the trap depth. Apart from this, clouds in both the MT and the FORT may experience heating processes due to small fluctuations in the trap [6]. For the MT, there may be small fluctuations in the magnetic field, modulating the trap potential and therefore adding energy to the system. The FORT may have similar fluctuations. Most other (random) processes, like interactions with particles from the background pressure and random scattering, have no influence on the temperature, as they do not change the width of the velocity distribution.

### 3.2 Experiment

In order to investigate how the number of particles and temperature change over time in the different traps, we performed several measurements. For each measurement, we made absorption images at different points in time and then fitted the images to find the total number of particles in the cloud and its temperature. Note that absorption images are destructive: making a single image causes many interactions between the cloud and the resonant laser beam used for imaging, basically destroying it. Therefore every image had to be a new sample. From the images, it is possible to derive the number of particles by looking at the width, length and and center absorption value (directly related to density) of the cloud. The temperature can be found by turning off the trap, using the expansion rate to determine the average velocity of the particles and thus the temperature of the sample.

For the MT measurements, a measurement series at several different starting temperatures is made, controlled by how far the rf sweep in the evaporative cooling process goes. We varied the final rf sweep value from 1.85 MHz to 3.3 MHz relative to the bottom of the MT, giving us varying starting temperature of a few microKelvin. We also varied the density of the cloud by using a "tight" axial trap frequency ( 16 Hz ) and a "loose" axial trap frequency $(1.3 \mathrm{~Hz})$ for different measurements. The radial trap frequency was kept at 144 Hz . For the FORT measurements (radial trap frequency of 850 Hz and axial trap frequency of 3.5 Hz ), we only looked at the effects of different starting temperatures.

### 3.3 Results

### 3.3.1 Particle loss and heating in the MT

Looking at the MT with the axial frequency of 16 Hz . Fig 3.1 shows a typical example of the number of particles and the temperature as a function of time. We see an interesting result. The temperature increases linearly, but the heating rate changes at around 11 s . The top plot does not show this behavior because that particular point is likely before the first measurement point of that series (which is at 20 s ). This change in heating rate appears in several measurement series. The heating rates found were around $0.2 \mu \mathrm{~K} / \mathrm{s}$ for the second part (after the change in heating rate), while the first part has a heating rate of about $0.8 \mu \mathrm{~K} / \mathrm{s}$.

While the temperature shows unexpected results, the number of particles behaves as expected: it decreases exponentially. From these measurements, we found a lifetime of 80 to 150 seconds. This is lower than what we usually find ( 200 to 300 seconds). This is likely because of a bad vacuum during these particular measurements.

We now compare these results with the results from the trap with an axial frequency of 1.3 Hz , see fig 3.2 . While there was no clear difference in lifetime, the temperature shows different behavior. The difference apparent here, is


Figure 3.1: Number of particles (left) and temperature (right) as a function of time for a MT with an axial trap frequency of 16 Hz . The top plots are over a longer period of time, while the bottom plots are over a shorter period of time. Top and bottom are different measurement series with similar conditions, but have different starting temperatures and number of particles.


Figure 3.2: Number of particles (left) and temperature (right) as a function of time for a MT with an axial trap frequency of 1.3 Hz .
that the temperature hardly changes over time. We found heating rates of $(0 \pm 4) \mathrm{nK} / \mathrm{s}$; some measurements show slight heating, while some show slight cooling, but on average there is no clear temperature change. The density is much lower inside this decompressed trap, allowing us to cool much further without encountering three-body losses. More specifically, decompressing the trap yielded starting temperatures of below $3 \mu \mathrm{~K}$. This starting temperature actually does not deviate much from the starting temperatures with the compressed trap, but in the compressed trap there was rapid heating at that temperature, while in the decompressed trap the temperature was almost constant.

In conclusion, we found that a compressed trap results in high heating rates before a certain temperature or point in time (in this case after 11 s ), after which the heating rate decreases, but remains high. A decompressed trap, on the other hand, hardly has a heating rate at all. This suggests the heating rate has something to do with the density or the gradient of the $\mathbf{B}$-field (a compressed trap has a larger slope in the axial direction than a decompressed trap), as those are the main differences between the two situations. Even though the density is likely related to the heating rate, three-body losses do not seem to be responsible, as there is no clear difference between particle loss rates, and three-body losses should be random so they should not influence temperature either way.

### 3.3.2 Particle loss and cooling in the FORT

For the FORT, we have two measurement series: one starting slightly below $T_{c}$ ( $\sim 1.5 \mu \mathrm{~K}$, so with a small condensate inside the cloud) and one starting slightly above $T_{c}(\sim 2 \mu \mathrm{~K})$. The results can be seen in fig 3.3. Before discussing the behavior of the temperature, we first note that the number of particles is very similar in those two measurements. The lifetime from the measurements just above $T_{c}$ is 34 s , while the lifetime from just below $T_{c}$ is 36 s . Note that the number of particles from the measurements below $T_{c}$ is only the number of particles in the thermal cloud; the number of condensed particles is not shown here, because the number of condensed particles was relatively low ( $\sim 5 \%$ of the total number of particles) and it did not show any clear time dependence.


Figure 3.3: Number of particles (left) and temperature (right) as a function of time inside the FORT just above $T_{c}$ (top) and just below $T_{c}$ (bottom). The red and the blue line in the upper left plot are not continuous because the transition area was not taken into account when fitting.

As mentioned before, we have identified three main sources of particle loss inside the FORT: background pressure, random scattering and passive evaporative cooling. From earlier measurements, we know the lifetime from background pressure is 200 to 300 seconds. As we found a total lifetime of 34 s and 36 s from these measurements, we assume the particles lost to the background pressure are negligible compared to the losses from random scattering and evaporative cooling. The measurements with starting temperature just above $T_{c}$ show a constant temperature after the first few data points. By excluding those first few data points, we can isolate the effects of random scattering. This was already done when fitting the upper left plot from fig 3.3 (blue line), resulting in the previously mentioned lifetime of 34 s . If we instead remove all but the first few points, we find a lifetime of 12 s (red line). Subtracting the random scattering lifetime of 34 s (using that $\left.\frac{1}{\tau_{e v}}+\frac{1}{\tau_{s c}}=\frac{1}{\tau_{\text {total }}}\right)$, we find a lifetime of 19 s caused only by evaporative cooling. We cannot separate these different cooling times for the measurement series with starting temperature just below $T_{c}$, because that data set does not contain a region with stable temperature where evaporative cooling stopped.

One can calculate the theoretical random scattering rate and compare it to the experimental results, using [16]:

$$
\begin{equation*}
\Gamma_{s c}(\mathbf{r})=\frac{3 \pi c^{2}}{2 \hbar \omega_{0}^{3}}\left(\frac{\omega}{\omega_{0}}\right)^{3}\left(\frac{\Gamma}{\omega_{0}-\omega}+\frac{\Gamma}{\omega_{0}+\omega}\right)^{2} I(\mathbf{r}) \tag{3.2}
\end{equation*}
$$

Here $\Gamma_{s c}(\mathbf{r})$ is the scattering rate, $\mathbf{r}$ is the position, $c$ is the speed of light, $\hbar$ is the reduced Planck constant, $\omega_{0}$ is the resonance frequency of the sodium atoms, $\omega$ is the frequency of the laser beam used by the FORT, $\Gamma$ is the damping rate and $I(\mathbf{r})$ is the field intensity. Approximating $\Gamma_{s c} \simeq \frac{1}{\tau_{s c}}$, we find $\tau_{s c} \simeq 27 \mathrm{~s}$. For this approximation we assumed that all atoms subject to random scattering are untrapped, which does not have to be true, resulting in a lower value for the random scattering lifetime. Still, the random scattering lifetime value of 34 s found from experiments is similar to the theoretical approximation, supporting the explanation of random scattering.

As explained earlier, one expects the passive evaporative cooling to cool the cloud inside the FORT, to a specific temperature that depends on the trap depth. The temperature indeed decreases exponentially to an asymptotic temperature in both measurements. However, the cooling time when starting above $T_{c}(\tau \sim 1.6 \mathrm{~s})$ is much lower than the cooling time when starting below $T_{c}(\tau \sim 5.5 \mathrm{~s})$. The temperature they converge to is also different: $0.9 \mu \mathrm{~K}$ and $1.1 \mu \mathrm{~K}$ respectively. This small difference in temperature may be due to a small difference in the FORT itself, as the measurements were done on different days. In order to support these results, we can make a theoretical approximation for the temperature the atomic cloud should converge to. We approximate the speed distribution of the atomic cloud to be equal to the Maxwell-Boltzmann distribution:

$$
\begin{equation*}
f(v)=\frac{2}{\pi}\left(\frac{m}{k_{B} T}\right)^{3} v^{2} e^{-\frac{m v^{2}}{2 k_{B} T}} \tag{3.3}
\end{equation*}
$$

Here $f(v)$ is the probability density for an atom to have a speed $v=|\mathbf{v}|, m$ is the particle mass, $k_{B}$ is the Boltzmann constant and $T$ is the temperature. One can calculate the temperature required for all $\sim 5 \cdot 10^{7}$ atoms to have an energy less than the trap depth of $30 \mu \mathrm{~K}$. This gives us an asymptotic temperature of $T \simeq 1.8 \mu \mathrm{~K}$, which is higher than the temperatures found from the experiments. This difference can be explained by the fact that we used the Maxwell-Boltzmann distribution to describe the speed distribution. The trap potential of the FORT is wider than a harmonic oscillator potential, allowing more atoms to have high speeds. This means that the actual value for the asymptotic temperature is lower than $1.8 \mu \mathrm{~K}$, supporting our experimental results.

A possible explanation for the high cooling rate from the measurements with starting temperature above $T_{c}$ may be that there is a larger difference between the starting temperature and the asymptotic than with the measurements with starting temperature below $T_{c}$, so particles are on average further away from the bottom of the trap. This bigger difference may be the reason more particles are thrown away by the passive evaporative cooling, resulting in faster cooling and disposal of more particles, agreeing with the upper plots of fig 3.3.

## Chapter 4

## Standing sound waves

### 4.1 Theory

During the first half of the twentieth century, the concept of second sound was first observed, for example in liquid helium-2 [17]. Second sound is the name of a sound mode that exists in hydrodynamic superfluids in addition to the usual first sound mode. In BECs in the hydrodynamic regime we also see these two different sound modes. Here follows a short explanation for the existence of these sound modes. The full derivation can be found in Pethick \& Smith [11].

### 4.1.1 First and second sound

The condensates used in our experiments are large enough to be in the hydrodynamic regime: the mean free path of the atoms is less than the size of the condensate, giving the condensate fluid properties [5]. As with every system of particles in the hydrodynamic regime, we can employ the conservation laws. Here, we can use mass and momentum conservation. Neglecting friction and external potentials, we can combine them to the following relation:

$$
\begin{equation*}
\frac{\partial^{2} \rho}{\partial t^{2}}-\nabla^{2} p=0 \tag{4.1}
\end{equation*}
$$

Here $\rho$ is the density and $p$ is the pressure, and since the pressure is a function of both density and temperature, this is a relation between density and temperature. Next, we use the fact that a BEC is a superfluid, so we can write down the superfluid velocity:

$$
\begin{equation*}
m \frac{\partial \mathbf{v}_{s}}{\partial t}=-\nabla \mu \tag{4.2}
\end{equation*}
$$

Here $\mu$ is the chemical potential, $m$ is the particle mass and $\mathbf{v}_{s}$ is the superfluid velocity: the velocity a superfluid flows with. For systems in thermal equilibrium, thermal physics gives us the Gibbs-Duhem relation:

$$
\begin{equation*}
N \mathrm{~d} \mu=V \mathrm{~d} p-S \mathrm{~d} T \tag{4.3}
\end{equation*}
$$

In this relation, $N$ is the number of particles, $V$ is the volume of the ensemble, $S$ is the entropy and $T$ is the temperature. Combining eqn 4.1, 4.2 and 4.3 gives
the relation between the normal velocity and the superfluid velocity:

$$
\begin{equation*}
\frac{\partial \tilde{s}}{\partial t}=\tilde{s} \frac{\rho_{s}}{\rho} \nabla \cdot\left(\mathbf{v}_{s}-\mathbf{v}_{n}\right) \tag{4.4}
\end{equation*}
$$

Here $\tilde{s}=\frac{S}{N m}$ is the entropy per unit mass, $\rho_{s}$ is the density of the superfluid, $\rho$ is the total density and $\mathbf{v}_{s}$ and $\mathbf{v}_{n}$ are the superfluid and normal velocities respectively. From these relations an equation for the speed of sound can be derived:

$$
\begin{equation*}
\left(v^{2}-c_{1}^{2}\right)\left(v^{2}-c_{2}^{2}\right)-v^{2} c_{3}^{2}=0 \tag{4.5}
\end{equation*}
$$

Here $v$ is the speed of sound, $c_{1}$ is the speed of density waves at fixed temperature, $c_{2}$ is the speed of temperature waves at fixed density and $c_{3}$ is the coupling between density and temperature variations [18]:

$$
\begin{equation*}
c_{1}^{2}=\left(\frac{\partial p}{\partial \rho}\right)_{T}, c_{2}^{2}=\frac{\rho_{s} \tilde{s}^{2} T}{\rho_{n} \tilde{C}_{V}}, c_{3}^{2}=\left(\frac{\partial p}{\partial T}\right)_{\rho}^{2} \frac{T}{\rho \tilde{C}_{V}} \tag{4.6}
\end{equation*}
$$

Here $\rho_{n}$ is the normal density, $T$ is the temperature and $\tilde{C}_{V}$ is the specific heat per unit mass at constant volume. Eqn 4.5 has two solutions for the speed of sound $v$. Those two solutions are called first and second sound:

$$
\begin{equation*}
v^{2}=\frac{1}{2}\left(c_{1}^{2}+c_{2}^{2}+c_{3}^{2}\right) \pm\left[\frac{1}{4}\left(c_{1}^{2}+c_{2}^{2}+c_{3}^{2}\right)^{2}-c_{1}^{2} c_{2}^{2}\right]^{1 / 2} \tag{4.7}
\end{equation*}
$$

The solution with the plus sign is defined as first sound, while the solution with the minus sign is defined as second sound. Close to $T_{c}$ first sound corresponds to density fluctuations in the thermal (non-condensed) part, while second sound corresponds to density fluctuations in the condensate. When looking at even lower temperatures, the coupling $c_{3}$ causes an avoided crossing of the two solutions, where the first sound solution becomes second sound and the second sound solution becomes first sound. Fig 4.1 shows the behavior of the two solutions as calculated in the Landau model (solid line) and the ZGN model (dashed line) [19].

In our experiments the temperature is close to $T_{c}$, so the second sound solution is the solution that corresponds to density fluctuations in the condensate. The sound speed of these density fluctuations is given by the Bogoliubov speed of sound [20]:

$$
\begin{equation*}
v=\sqrt{\frac{U_{0} n_{c}}{m}} \tag{4.8}
\end{equation*}
$$

Here $U_{0}$ is the constant for particle interactions from the Gross-Pitaevskii equation (eqn 2.12), and $n_{c}$ is the condensate density. If we make an approximation and assume the waves only travel axially, we can take the radial average density along the $z$-axis (which is the long axis of our cigar-shaped condensates). Using eqn 2.13 together with eqn 2.14, we can calculate this average density by integrating over the radial direction. The result gives us a relation between $v$ and $n_{c}(0,0, z)[19]$ :

$$
\begin{equation*}
v(z)=\sqrt{\frac{U_{0} n_{c}(0,0, z)}{2 m}} \tag{4.9}
\end{equation*}
$$



Figure 4.1: Avoided crossing of first (I) and second (II) sound speed below $T_{c}$, calculated in two different models. The inset shows the same temperature range on a larger vertical scale [19].

### 4.2 Experiment

### 4.2.1 Phase contrast imaging

In order to measure sound, we have to measure the density profile of a condensate over time. To get accurate measurements, we measure the density profile of the same condensate at consecutive points in time. This requires non-destructive imaging. The solution lies in PCI, or Phase Contrast Imaging [21]. For PCI measurements, an off-resonant laser beam is sent through a condensate. Due to the difference in refractive index between the background and the condensate, the coherent laser source accumulates a difference in phase. This accumulated phase shift depends on the density of the sample. The accumulated phase can be converted to an intensity. This is done by placing a phase spot (a small transparent object that blocks the part of the laser beam that was diffracted by the condensate) to make the intensity phase dependent through interference. The CCD (or Charge-Coupled Device) camera then images the intensity (and thus the accumulated phase difference), so we have an image that can directly be translated to a density profile. Note however that phase is $2 \pi$-periodic, so if the density is large enough for the phase to go through $2 \pi$, the intensity of the signal decreases with increasing density. This makes it possible to get dark spots although the density is large.

Figure 4.2: PCI image of a condensate. Note how the dark center of the condensate corresponds with a large density. The image's size is 0.16 mm by 2.68 mm .

### 4.2.2 Measurements

On a condensate, several modes can be induced. One would expect to see lower order modes, like a condensate's position oscillating inside a trap, known as the dipole mode. However, we saw a quite spectacular mode: density peaks appear at periodic locations in the axial direction of our cigar-shaped condensates, closely resembling a standing wave. This is a highly excited mode that is not easy to understand right away. Even inducing this mode is quite an extraordinary accomplishment if one considers the fact that condensates do not have sharp edges. We are looking at the origin and properties of this very unusual mode. Now recall that the trap potential is a harmonic oscillator potential (eqn 2.14). The highly excited standing wave mode is indirectly induced by taking a typical cigar-shaped condensate and then periodically modulating the radial trap frequency at its own frequency. During the driving, we see three collective modes appear. Firstly, we see a collective oscillation of the condensate in the radial direction. We call this the dipole mode. The reason we see the dipole mode is because there are always small perturbations present in the MT. If the condensate is out of the equilibrium position, driving the collective modes will also result in driving the dipole mode. Secondly, we see the condensate's radial width oscillating as a direct result from modulating the radial trap frequency. This is called the quadrupole mode, which is the mode we are driving most directly, oscillating at twice the radial trap frequency [22]. During the driving, a third mode appears, which is the mode that so closely resembles an axial standing sound wave.

In order to gain more understanding about this remarkable feature, we performed several measurements. Measurements consist of a series of thirty to fifty PCI images of the same condensate, with 3.2 to 3.6 milliseconds in between the images. Fig 4.3 shows a typical measurement series and contains all three modes: indeed, the condensate moves as a whole, the width changes periodically and it contains the standing wave pattern, which's amplitude is also changing periodically. We took several measurement series, varying the radial trap frequency (and also the frequency with which we are driving the modes) and the point in time at which series are started. The frequencies were varied between 39.6 Hz and 108.9 Hz , with both big and small variations in driving frequencies, in order to accurately determine how the properties of the collective modes depend on the driving frequency. Varying the starting time of a measurement series is done so that we can investigate how the modes behave over an extended period of time by combining several measurement series. This cannot be done by making one long series, because PCI is not completely non-destructive.


Figure 4.3: A typical measurement series, in this case at a radial trap frequency of 39.6 Hz . These are all the same condensate, but with 3.6 milliseconds in between each image. The upper image is the first one in this series, with its point in time defined as $t=0$.

### 4.2.3 Analyzing the standing wave pattern

To gain understanding why the standing wave pattern appears, we investigated the following:

- How do the frequencies and phases of the three different modes relate to each other and to the radial trap frequency?
- How does the speed of sound depend on the radial trap frequency?
- How exactly does the spacing between the peaks, or the wavelength of the standing wave, behave in the axial direction?
- How does the standing wave pattern behave over an extended period of time?

To extract data from the images, we fitted all images and derived relevant quantities from the fit parameters. These quantities would be fitted over time. Now recall eqn 2.13 and eqn 2.14. We propose that the wave pattern in the data can be approximated by the condensate density $n_{0}$ (given by eqn 2.13), multiplied by $(1+a \cos [k(z) z])$ to account for the standing wave pattern. This density profile should then be enclosed by a sinusoidal function to take care of the fact that the images are made with PCI.

Let us look at the first part of the fit function, taking care of the condensate density $n_{0}$. Our data images are 2D CCD images of a 3D profile. To account for this fact, we need to integrate $n_{0}$ along one of the axes, say the $y$ axis. We take this integration from $-R_{y}$ to $R_{y}$, where $R_{y}$ is the radius of the condensate in the $y$ direction, solved from eqn 2.13 and 2.14 with $V\left(x, R_{y}, z\right)=\mu:$

$$
\begin{equation*}
\left[\mu-\frac{1}{2} m\left(\omega_{r}^{2}\left(x^{2}+R_{y}^{2}\right)+\omega_{z}^{2} z^{2}\right)\right] / U_{0}=0 \tag{4.10}
\end{equation*}
$$

With this, we can calculate the $n_{0}$ we have to use for 2 D images:

$$
\begin{equation*}
n_{2 D}=\int_{-R_{y}}^{R_{y}} n_{0}(x, y, z) \mathrm{d} y=\frac{4}{3} \sqrt{\frac{2 U_{0}}{m \omega_{r}^{2}}}\left(n_{0}(x, 0, z)\right)^{3 / 2} \tag{4.11}
\end{equation*}
$$

For simplicity, we will use normalized densities:

$$
\begin{equation*}
\tilde{n}_{0}(x, y, z)=1-\left(\frac{x-x_{0}}{R_{x}}\right)^{2}-\left(\frac{y-y_{0}}{R_{y}}\right)^{2}-\left(\frac{z-z_{0}}{R_{z}}\right)^{2} \tag{4.12}
\end{equation*}
$$

Here $x_{0}, y_{0}$ and $z_{0}$ are the positions of the center of the BEC in all three dimensions, and $R_{x}, R_{y}$ and $R_{z}$ are the widths of the condensate in these directions. If we now also use a normalized $n_{2 D}$, we get that:

$$
\begin{equation*}
\tilde{n}_{2 D}(x, z)=\left(\tilde{n}_{0}(x, 0, z)\right)^{3 / 2} \tag{4.13}
\end{equation*}
$$

For the second part of the fit function $(1+a \cos [k(z) z])$, we need a relation for $k(z)$. We assume we can use the dispersion relation $v=\frac{\omega}{k}$, so that $k=\frac{\omega}{v}$. We


Figure 4.4: A PCI image (top) and the result from fitting this image (bottom).
already know $v$ from eqn 4.9 making the approximation that the standing wave is purely axial. This results in:

$$
\begin{equation*}
k(z)=\omega \sqrt{\frac{2 m}{U_{0} n_{0}(0,0, z)}} \propto\left(\tilde{n}_{0}(0,0, z)\right)^{-1 / 2} \tag{4.14}
\end{equation*}
$$

However, our experiments do not quite agree with this power of $-1 / 2$. In order to find out what is missing in the theory, we added this power as a fit parameter. We can now combine everything to form a fit function for the density at fixed time $t$ :

$$
\begin{equation*}
f(x, z)=p_{1}\left(\tilde{n}_{0}(x, 0, z)\right)^{3 / 2}\left(1+p_{6} \cos \left[p_{7}\left(\tilde{n}_{0}(0,0, z)\right)^{-p_{8}}\left(z-p_{3}\right)\right]\right) \tag{4.15}
\end{equation*}
$$

Here, $p_{1}$ is the center value that can be converted to the non-excited density at the center of the condensate, $p_{2}, p_{3}, p_{4}$ and $p_{5}$ are respectively $R_{z}, z_{0}, R_{x}$ and $x_{0}$ from eqn 4.12, $p_{6}$ is the amplitude $a$ of the standing wave pattern, $p_{7}$ is $k_{0}$ at the center of the condensate and $p_{8}$ is the power that was mentioned previously. Fig 4.4 illustrates how well this function fits to the data. The only regions where the fit function does not seem to be completely accurate, are the edges. However, the edges are also less clear in the data because of the low density. Table 4.1 shows the results from a measurement series at radial trap frequency of 39.6 Hz . Note that the center value is not the density. Instead, the center density can be obtained by approximating the chemical potential $\mu$ to be constant in the condensate by solving $V\left(R_{x}, 0,0\right)=V\left(0,0, R_{z}\right)=\mu$, then replacing this $\mu$ in eqn 2.13.

With this fit function, we proceeded to fit several measurement series, giving us many data points. Some of these parameters were then plotted and fitted over time, namely the amplitude of the standing wave, $x_{0}$ (to analyze the dipole mode) and $R_{x}$ (to analyze the quadrupole mode). As the only information we wanted at the time are the frequency, phase and the amplitude, using a sine function was sufficient to retrieve the information. A sine may not always be fully correct in some cases, but it is sufficient to extract the frequencies, phases and amplitudes.

| Time $(\mathrm{ms})$ | Center value (AU) | $R_{z}(\mathrm{~mm})$ | $z_{0}(\mathrm{~mm})$ | $R_{x}(\mu \mathrm{~m})$ | $x_{0}(\mu \mathrm{~m})$ | $a(\mathrm{AU})$ | $k_{0}\left(10^{3} \mathrm{~m}^{-1}\right)$ | Power $(\mathrm{AU})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3.6 | $1.173(8)$ | $0.939(5)$ | $1.263(1)$ | $37.2(3)$ | $117.4(1)$ | $-0.17(1)$ | $75.5(5)$ | $0.18(2)$ |
| 7.2 | $1.420(8)$ | $0.797(5)$ | $1.249(1)$ | $48.6(3)$ | $85.9(1)$ | $0.12(1)$ | $78.0(5)$ | $0.20(2)$ |
| 10.8 | $1.182(8)$ | $0.829(5)$ | $1.253(1)$ | $60.1(3)$ | $85.0(1)$ | $0.33(1)$ | $79.3(5)$ | $0.14(2)$ |
| 14.4 | $2.443(8)$ | $0.726(5)$ | $1.256(1)$ | $29.1(3)$ | $64.0(1)$ | $0.25(1)$ | $79.4(5)$ | $0.12(2)$ |
| 18.0 | $1.384(8)$ | $0.807(5)$ | $1.227(1)$ | $41.0(3)$ | $42.4(1)$ | $-0.05(1)$ | $15.8(5)$ | $2.16(2)$ |
| 21.6 | $1.148(8)$ | $0.783(5)$ | $1.228(1)$ | $56.9(3)$ | $31.8(1)$ | $-0.23(1)$ | $80.1(5)$ | $0.08(2)$ |
| 25.2 | $1.162(8)$ | $0.800(5)$ | $1.266(1)$ | $49.6(3)$ | $36.9(1)$ | $-0.27(1)$ | $78.5(5)$ | $0.19(2)$ |
| 28.8 | $2.944(8)$ | $0.718(5)$ | $1.262(1)$ | $24.2(3)$ | $58.6(1)$ | $-0.15(1)$ | $79.0(5)$ | $0.13(2)$ |
| 32.4 | $1.286(8)$ | $0.822(5)$ | $1.228(1)$ | $46.8(3)$ | $85.8(1)$ | $0.15(1)$ | $79.9(5)$ | $0.12(2)$ |
| 36.0 | $1.220(8)$ | $0.841(5)$ | $1.229(1)$ | $52.8(3)$ | $93.5(1)$ | $0.30(1)$ | $78.1(5)$ | $0.16(2)$ |
| 39.6 | $2.651(8)$ | $0.715(5)$ | $1.187(1)$ | $27.4(3)$ | $72.8(1)$ | $0.10(1)$ | $79.2(5)$ | $-0.02(2)$ |
| 43.2 | $1.319(8)$ | $0.819(5)$ | $1.240(1)$ | $41.3(3)$ | $43.2(1)$ | $-0.06(1)$ | $62.1(5)$ | $1.05(2)$ |
| 46.8 | $1.169(8)$ | $0.791(5)$ | $1.196(1)$ | $54.1(3)$ | $23.2(1)$ | $-0.17(1)$ | $78.9(5)$ | $0.12(2)$ |
| 50.4 | $1.343(8)$ | $0.811(5)$ | $1.195(1)$ | $43.2(3)$ | $26.2(1)$ | $-0.21(1)$ | $77.9(5)$ | $0.11(2)$ |
| 54.0 | $2.526(8)$ | $0.757(5)$ | $1.191(1)$ | $23.7(3)$ | $54.1(1)$ | $-0.11(1)$ | $76.1(5)$ | $0.14(2)$ |
| 57.6 | $1.316(8)$ | $0.840(5)$ | $1.230(1)$ | $45.8(3)$ | $91.3(1)$ | $0.08(1)$ | $70.9(5)$ | $0.44(2)$ |
| 61.2 | $1.218(7)$ | $0.852(5)$ | $1.2243(5)$ | $49.1(3)$ | $103.2(2)$ | $0.220(9)$ | $75.3(2)$ | $0.160(7)$ |
| 64.8 | $2.462(6)$ | $0.747(3)$ | $1.2197(5)$ | $27.9(1)$ | $77.13(6)$ | $0.133(5)$ | $74.7(3)$ | $0.090(7)$ |
| 68.4 | $1.281(8)$ | $0.807(2)$ | $1.234(1)$ | $40.9(3)$ | $39.1(1)$ | $0.05(1)$ | $40.2(6)$ | $1.41(2)$ |
| 72.0 | $1.093(7)$ | $0.770(2)$ | $1.195(1)$ | $50(1)$ | $12.5(6)$ | $-0.04(1)$ | $56.2(9)$ | $1.56(1)$ |
| 75.6 | $1.239(8)$ | $0.845(6)$ | $1.219(1)$ | $38.6(5)$ | $17.8(2)$ | $-0.09(1)$ | $75.2(6)$ | $0.15(2)$ |
| 79.2 | $2.241(9)$ | $0.769(3)$ | $1.2153(5)$ | $23.7(1)$ | $53.97(6)$ | $-0.168(8)$ | $75.6(3)$ | $0.133(7)$ |
| 82.8 | $1.163(7)$ | $0.880(6)$ | $1.203(1)$ | $44.0(3)$ | $99.1(2)$ | $0.08(1)$ | $80.1(7)$ | $-0.02(2)$ |
| 86.4 | $1.189(7)$ | $0.910(6)$ | $1.1711(5)$ | $43.3(3)$ | $110.7(2)$ | $0.23(1)$ | $74.3(2)$ | $0.200(8)$ |
| 90.0 | $2.496(7)$ | $0.732(3)$ | $1.2052(5)$ | $27.0(1)$ | $77.91(6)$ | $0.115(5)$ | $70.8(3)$ | $0.348(8)$ |
| 93.6 | $1.145(7)$ | $0.858(3)$ | $1.209(1)$ | $41.5(3)$ | $31.7(2)$ | $-0.06(1)$ | $76.2(7)$ | $0.91(2)$ |
| 97.2 | $1.05(1)$ | $0.842(9)$ | $1.196(1)$ | $53(2)$ | $0.1(9)$ | $-0.12(1)$ | $72.4(6)$ | $0.31(2)$ |
| 100.8 | $1.217(8)$ | $0.904(6)$ | $1.1920(8)$ | $34.1(5)$ | $13.5(3)$ | $-0.15(1)$ | $70.7(4)$ | $0.38(1)$ |
| 104.4 | $1.402(1)$ | $0.913(7)$ | $1.165(2)$ | $27.0(2)$ | $59.1(1)$ | $0.07(1)$ | $75.0(9)$ | $-0.12(3)$ |
| 108.0 | $1.087(7)$ | $0.919(7)$ | $1.200(1)$ | $43.7(3)$ | $109.2(2)$ | $0.11(1)$ | $70.1(5)$ | $0.30(2)$ |
|  |  |  |  |  |  |  |  |  |

Table 4.1: Results from a measurement series with radial trap frequency $\omega_{r} /(2 \pi)=39.6 \mathrm{~Hz}$.

### 4.3 Results

### 4.3.1 Frequencies and phases of the modes

We want to know how the frequencies and phases of the different modes correlate. In order to investigate these properties, we looked at measurement series with radial trap frequencies of approximately $39.6,86$ and 108.9 Hz . Fig 4.5 shows the results of the measurement series of 39.6 Hz , showing the amplitude of the standing wave (blue), the dipole mode (position of the condensate, red) and the quadrupole mode (width of the condensate, green). Here we used a sine function to fit the data. These measurement series are taken at a point after all three modes have started, so $t=0$ does not correspond with the start of one of the modes, but instead is defined as the starting time of the measurement series. As one data set covers only a short period of time, we ignore any damping effects when investigating the frequencies and phases. Damping effects will be discussed in section 4.3.4.

In the lower right corner of fig 4.5, all three fits are plotted in one figure in order to compare the phases of the different modes, with each plot retaining its color. Fig 4.6 shows the same, but now with the information from the 86 and 108.9 Hz measurement series. From these three series, we found interesting results: the dipole mode and the standing wave pattern have the same frequency as their radial trap frequency, while the quadrupole mode oscillates at twice that frequency. This agrees with theoretical analysis [22]. Table 4.2 shows the values we found for the frequencies.


Figure 4.5: Time plots of the amplitude of the standing wave pattern (upper left, blue), the dipole mode (upper right, red) and the quadrupole mode (lower left, green) at $\omega_{r} /(2 \pi)=39.6 \mathrm{~Hz}$. The lower right plot shows them all combined in order to compare them.


Figure 4.6: Amplitudes of the modes at 86 Hz (left) and 108.9 Hz (right).

We are also interested in how the phases of the modes correlate. However, as one can see in fig 4.5 and 4.6 and table 4.2, the different data sets all have different phase relations between the modes. This makes it impossible to draw definitive conclusions, as we need more information to do so. There will be measurements at more different radial trap frequencies in order to determine how the modes correlate, mostly with small radial frequency steps in order to see more details. However, we still see a few things. Firstly, it looks like the standing wave and the dipole mode are either approximately in phase or almost exactly out of phase. Secondly, it seems that de-phasing occurs. For example, for 108.9 Hz the standing wave and the dipole mode are almost in phase at the start, but near the end they are almost exactly out of phase. This may be due to the fit error, as the frequencies found by fitting are not exactly the same and the plot contains many cycles, causing slow de-phasing.

As one can see in fig 4.5 and 4.6 , the peaks of the standing wave pattern follow shortly after extrema of the quadrupole mode most of the time. This can also be seen in table 4.2: in all cases, the phase of the standing wave is slightly higher than the phase of the quadrupole mode (after adding $\pi$ in case of 86 Hz and 108.9 Hz ). This suggests that the quadrupole mode drives the standing wave. However, we do not have enough data to determine the exact relation between these two modes. The only conclusions are that the standing wave and the dipole mode oscillate with approximately the radial trap frequency, while the quadrupole mode oscillates twice as fast as the other modes, and that all three modes have phase relations (that are to be determined in more detail).

|  | Radial trap frequency | Standing wave | Dipole mode | Quadrupole mode |
| :--- | :--- | :--- | :--- | :--- |
| Frequency | 39.6 Hz | $(39.4 \pm 0.4) \mathrm{Hz}$ | $(39.6 \pm 0.3) \mathrm{Hz}$ | $(80.4 \pm 0.5) \mathrm{Hz}$ |
| Phase |  | $-0.36 \pm 0.15$ | $0.05 \pm 0.11$ | $-1.51 \pm 0.19$ |
| Frequency | 86 Hz | $(86.0 \pm 0.6) \mathrm{Hz}$ | $(88.0 \pm 0.1) \mathrm{Hz}$ | $(173.9 \pm 0.7) \mathrm{Hz}$ |
| Phase |  | $2.99 \pm 0.23$ | $-0.28 \pm 0.02$ | $-0.24 \pm 0.27$ |
| Frequency | 108.9 Hz | $(104.4 \pm 0.8) \mathrm{Hz}$ | $(106.4 \pm 0.2) \mathrm{Hz}$ | $(209.9 \pm 0.3) \mathrm{Hz}$ |
| Phase |  | $2.37 \pm 0.29$ | $3.21 \pm 0.06$ | $-1.25 \pm 0.10$ |

Table 4.2: Frequencies and phases of the different modes at different radial trap frequencies.

### 4.3.2 Accurate determination of the sound speed

The speed of sound is a fundamental property of a hydrodynamic system of atoms and we are interested in its value. Using the dispersion relation $k=\frac{\omega}{v}$, all we need to know is the frequency of the standing wave pattern (which equals the radial trap frequency, as found in section 4.3.1) and the wave number $k$. In this case, we look at the sound speed at the center of the condensate $v_{0}$, so we also have to use the wave number at the center of the condensate $k_{0}$. Referring back to eqn 4.15 , this center wave number $k_{0}$ is a quantity we get directly from the fit, as the density inside the cosine is normalized to be one at the center. Fig 4.7 shows the $k_{0}$ found from each image of one series, in this case a series at 39.6 Hz . For this series in particular, we found $k_{0}=(75.7 \pm 0.4) \cdot 10^{3} \mathrm{~m}^{-1}$. We found a sound speed of $3.3 \mathrm{~mm} / \mathrm{s}$ by using $v=\frac{\omega}{k}$ with $\omega_{r} /(2 \pi)=39.6 \mathrm{~Hz}$.

The first thing we notice is that there are a few points (with a high error bar) that deviate quite far from the average. This has a clear explanation: as one can see in fig 4.3, some images do not show a clear standing wave pattern, as the amplitude peak is changing from one belly to the other. Those images are difficult to fit, so they do not yield an accurate value for the wave number, hence the large error bars. Because of the large error bars of these points, the relative standard deviation of the average of $k_{0}$ is still less than a percent. This leads us to believe we can accurately determine the wave number, and with that the sound speed.

Even though we appear to have a good method to determine the sound speed, we have not yet determined how exactly the sound speed depends on the density. We found from measurements that the $k$ (and so the peak spacing and the sound speed) does not depend on the density as eqn 4.9 predicts.


Figure 4.7: Wave number $k_{0}$ at different points in time for a series of measurements at $\omega_{r} /(2 \pi)=39.6 \mathrm{~Hz}$.

### 4.3.3 Wave number and condensate density

In this section we investigate how the wave number relates to the density in the axial direction. From eqn 4.14, one would expect the wave number to be proportional to $n^{-1 / 2}$. The power is a fit parameter from eqn 4.15. We proceed to fit several images from measurement series with radial trap frequency of 39.6 Hz . However, this is somewhat problematic, since most information about the power is located at the edges, and the edges are also the most unclear part of the images due to the low density. Fig 4.8 illustrates how the wavelength depends on the position in the condensate for a power of $1 / 2$ and 0.2 . Here, the position can be translated to the density with eqn 2.13 , while the wavelength equals $\lambda=2 \pi / k$. As one can see, the difference in power expresses itself as differences in wavelengths of more than a factor 2 towards the edges, while the center does not show a large difference. Fig 4.8 illustrates how clear the difference between the theoretical power and a power similar to our measurements is. Despite the fact that the edges are unclear, we found a power of $0.19 \pm 0.10$ from measurements, which is much lower than $1 / 2$. This value is obtained from the data of table 4.1 and discarding the values lower than 0 and higher than 0.8 , since these values are from images that do not contain a clear standing wave pattern, resulting in values that deviate much from all other values. Moreover, these few very high and very low values do not seem realistic when looking at the images. While the value of the power is not conclusive, we are certain that it is not the $1 / 2$ expected naively from theory.

In order to understand why our measurements disagree with theory, we compared the results with simulations (performed by S. Pratama). The simulations are done by solving the Gross-Pitaevskii equation (eqn 2.12) using the time-splitting spectral method for the time evolution [23]. In the simulations, the collective modes are also induced by modulating the radial trap frequency, but the trap frequencies used for the simulations are different from the trap frequencies used in the experiments. Fig 4.9 shows a typical frame of the simulation, with fit in order to extract the power. This particular data set is obtained by subtracting the non-excited condensate density from the excited condensate density. Fitting resulted in a power of $0.20 \pm 0.02$. While we do not fully understand all the details, we agree that it is very promising that the measurements agree with the simulation.


Figure 4.8: Normalized wavelength as a function of position for a Thomas-Fermi density profile, using a power of $1 / 2$ (blue) and a power of 0.2 (red).


Figure 4.9: Simulation of the standing wave pattern and a fit to find the power. This plot shows the total density minus the non-excited condensate density of a single frame, isolating the standing wave pattern (simulation by S. Pratama).

The question why both the simulation and the measurements disagree with the naive theory remains. The solution they both yield (a power of 0.2 ) does not seem trivial. Possible explanations may be that it is not a power law at all, but something completely different, like some sort of guided sound mode, or that there are more quantities that influence the wave number that we did not anticipate.

### 4.3.4 Long-term behavior

In this section we look at was how the pattern behaves over an extended period of time. We combined several consecutive series of measurements, in this case at a radial trap frequency of 39.6 Hz . Two series overlap for quite a bit, allowing us to see if they agree with each other and if they are consistent. Fig 4.10 shows how the standing wave amplitude behaves as a function of time after combining several measurement series. Note that the first ten points are before the pattern appeared, and the last ten points also did not have a clear pattern anymore, so these points were not included for research on the standing wave pattern.


Figure 4.10: Amplitude of the standing wave pattern as a function of time with $\omega_{r} /(2 \pi)=39.6 \mathrm{~Hz}$, now over an extended period of time.

The data is consistent with fig 4.5 , although $t=0$ is defined differently. In fact, the data of fig 4.5 is included in fig 4.10.

Note that we are driving the modes over the whole duration. We clearly see the amplitude changing over time. One question is if the amplitude really damps out or if it is a sort of "beating" behavior, in which case the pattern would return at a later point in time. We fitted the data with two different functions in order to determine the properties of the behavior. For the the standing wave pattern, we used a sine function, but for the amplitude of this sine function we tried two different functions to see which one works best:

$$
\begin{gather*}
A(t)= \begin{cases}e^{p_{1} t} & : t<t_{0} \\
a e^{-p_{2} t} & : t \geq t_{0}\end{cases}  \tag{4.16}\\
A(t)= \begin{cases}e^{p_{1} t} & : t<t_{0} \\
a\left(\sin \left[p_{2} t+p_{3}\right]+p_{4}\right) & : t \geq t_{0}\end{cases} \tag{4.17}
\end{gather*}
$$

Here $a$ is to make sure the function is continuous, while the different $p_{i}$ and $t_{0}$ are the fit parameters. The assumption here is that the amplitude grows exponentially up to a maximum (which is around $t=0.08 \mathrm{~s}$ in fig 4.10), after which it either starts damping out (eqn 4.16) or starts oscillating (eqn 4.17). The point in time this happens is defined as $t_{0}$. We tried to fit the data with both functions, but unfortunately they were almost exactly as good. However, eqn 4.16 (damping) seemed to be slightly better. This is also the function used in fig 4.10.

Even though we cannot rule either one out just from fitting this data set, we can draw some conclusions. The fact that we are still driving the oscillations suggests that it should be oscillatory behavior. On the other hand, we have a few series at even later times than in fig 4.10, and we still do not see the pattern returning. This suggests eqn 4.17 is wrong. While this suggest damping, we do not have many of these measurements, so there may be a third option: in the same way it takes some time for the standing wave pattern to appear, it is possible the standing wave pattern appears again at a later point. An explanation for this third option may be that the modes have a frequency that is slightly different from the radial trap frequency. This would result in damping at first, but the pattern would return at some later point when the


Figure 4.11: Width of the condensate (the quadrupole mode) as a function of time with $\omega_{r} /(2 \pi)=39.6 \mathrm{~Hz}$, now over an extended period of time.
phases match again. This option can be tested by doing measurements at even later times.

It is interesting to see that the quadrupole mode shows similar damping behavior, as seen in fig 4.11. This again suggests that the quadrupole mode and the standing wave pattern are coupled. Furthermore, the dipole mode does not show damping and the number of condensed particles remains approximately constant. The fact that the number of particles remains constant means that the damping is not caused by loss of particles.

## Chapter 5

## Conclusion

In conclusion, in chapter 3 we found that the particle loss processes inside traps are indeed exponential, while the temperature showed more interesting behavior. Namely, a MT with a high axial trap frequency causes a high heating rate, but with two different regions for the rate of temperature change. A decompressed trap hardly sees any change in temperature. The FORT causes exponential cooling, although we found highly distinct cooling rates between measurements: a higher starting temperature (just above $T_{c}$ ) caused much faster cooling than a lower starting temperature (just below $T_{c}$ ). This is likely due to the larger difference between the starting temperature and the asymptotic temperature of the trap.

As for the standing wave pattern, in chapter 4 we found several things. Firstly, we found that the dipole mode and the standing wave pattern oscillate with approximately the same frequency as the radial trap frequency, while the quadrupole mode oscillates twice as fast, which would be expected from theory [22]. We could not draw any definitive conclusions on the phase relation between the different modes yet. Secondly, we found that we can accurately measure the wave number $k_{0}$ and with that the sound speed $v_{0}$. Thirdly, we found that the spacing between the peaks is not proportional to $n^{1 / 2}$ as the theory naively suggests. Instead, both simulations and measurements suggest that it should have a power of 0.2 . However, it may not even be a power law at all. Lastly, we found that the standing wave pattern is damped over time, even though we cannot conclude if this really is damping or if it is some sort of oscillatory behavior.

We are still missing information on how the phases of the different modes relate to each other and to the radial trap frequency. We also have to understand why exactly the peak spacing we found does not agree with the theory. Lastly, we still do not completely understand the origin of the mode that so closely resembles a standing wave pattern. This understanding may come from solving the Gross-Pitaevskii equation together with the equations for superfluid motion and from collecting and analyzing more data.

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