Differences in estimation skills between children with and without mathematical problems: An eye-tracking study

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Preface

This master thesis was conducted in the context of the completion of the master Orthopedagogics at Utrecht University. I focused on differences in estimation skills between children with and without mathematical problems. This enabled me to gain knowledge about numerical representations and about strategies children use in solving number line tasks. It was nice to expand my test experience with tests on the estimation skills of both children with and without mathematical problems. Besides, it was instructive to work with an eye-tracker and to analyze the data thereof.

I would like to thank some people. First of all, my supervisor Jaccoline van ‘t Noordende for the support and feed-back during the writing process of this thesis. I also want to thank the school and the children for their collaboration, which made this research possible.

Elise Verkade

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Abstract

Differences in estimation skills between children with and without mathematical problems were examined, by using an eye-tracker. More precisely, the effect of mathematical problems on representations (logarithmic and linear) and strategies (percentage of gazes to the begin-, mid- and endpoint of the number line) was studied. **Method:** Twelve children with mathematical problems and fifteen children without mathematical problems (age 9-12) participated in this study. They performed two Number-to-Position tasks ranging from 0-100 and 0-1000. **Results:** Children without mathematical problems had a more linear and logarithmic representation than children with mathematical problems. Besides, children without mathematical problems used the begin- and endpoint strategy more than children with mathematical problems, while those children used the midpoint strategy more. In children with mathematical problems, representations and strategies were not related to each other. In children without mathematical problems they were only related in the 0-100 task. In the 0-1000 task, this relation was not present. **Discussion:** Results and recommendations for future research are discussed. This study provides a useful starting point for future studies on differences in estimation skills between children with and without mathematical problems, but more research is needed to clarify differences in estimation skills.

Keywords: mathematical problems, estimation skills, representations, strategy use, eye-tracker, children, elementary school

Differences in estimation skills between children with and without mathematical problems: An eye-tracking study

Mathematical problems are a severe disability for both individuals and governments (Butterworth, 2010); it affects 2 to 6% of the learning population (Kucian et al., 2011; Ruijssenaars, Van Luit & Van Lieshout, 2006). Therefore, there is growing interest in the causes of mathematical problems. To be able to help children who suffer from these problems, the factors that affect their mathematical ability need to be clarified. One of the factors that is expected to influence mathematical achievement, is estimation skills. Estimation skills are positively related to math achievement (Ashcraft & Moore, 2012; Booth & Siegler, 2006; Bull et al., 2011; Siegler & Booth, 2004). Although it has been shown that differences exist in estimation skills between children with and without mathematical problems between first through third grade (Booth & Siegler, 2006), less is known about those differences in fourth, fifth and sixth graders. The purpose of the present study is to examine differences in number line estimation skills between children with and without mathematical problems in this age group. An eye-tracker will be used to examine those differences.

Numerical estimation is a process of translation between alternative quantitative representations (Siegler & Booth, 2004; Bull et al., 2011). One of the ways of representing quantities, is the mental number line. The mental number line provides a conceptual structure in which number symbols are connected to non-verbal representations of quantity in an ordered, horizontally-oriented array with small numbers on the left and large numbers on the right (Booth & Siegler, 2008; Siegler et al., 2011). Numerical estimation can be measured with number line tasks (Hubbard, Piazza, Pinel & Dehaene, 2005; Schneider et al., 2008; Schneider, Grabner & Paetsch, 2009), also called Number to Position tasks (NP tasks). These tasks require a numerical-to-non-numerical translation, from numerals to positions on a line (Siegler & Booth, 2004; Sullivan, Juhasz, Slattery & Barth, 2011), through which children give a pure reflection of their sense of magnitudes of numbers (Fias, Lammertyn, Reynvoet, Dupont & Orban, 2003; Siegler Thompson & Schneider, 2011) and their ability to understand, approximate and manipulate numerical quantities (Dehaene, 2001; Sullivan et al., 2011).

Previous research has shown a substantial improvement in estimation accuracy during the period from kindergarten through third grade, which is demonstrated by decreased percentages of errors (Heine et al., 2010). Factors that may cause the improvement in estimation accuracy are the use of the right representation (Siegler & Booth, 2004) and the right strategy (Ashcraft & Moore, 2012).

Three theories about representations in number line estimation have been formulated: 1) the logarithmic-rules model; 2) the accumulator model; and 3) the multiple representations hypothesis, which is a combination of the logarithmic-rules model and the accumulator model. The logarithmic-ruler model assumes a logarithmic representation, which exaggerates the distance between the magnitudes of numbers at the low end of the range and minimizes the distance between magnitudes of numbers in the middle and upper ends of the range (Siegler & Opfer, 2003). The accumulator model assumes that quantities are equally spaced and linearly increasing magnitudes with scalar variability (Feigenson, Dehaene & Spelke, 2004; Siegler & Opfer, 2003). Linear representations should increase the likelihood that errors will be close misses (Laski & Siegler, 2007). The multiple representations hypothesis states that people know and use multiple numerical representations, depending on contextual variables. The range of situations in which children rely on each representation changes with age and numerical experience (Booth & Siegler 2006; Geary, Hoard, Nugent & Byrd-Craven, 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003). According to Heine and colleagues (2010), the phase in which children use both logarithmic and linear representations, is only a transitional phase, after which children proceed to the exclusive use of a linear representation. Between kindergarten and second grade, there is a shift from reliance on a logarithmic representation of numerical magnitudes to reliance on a linear representation for the 0-10 and 0-100 range (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007; Opfer, Thompson & Furlong, 2010). The same takes place between second and sixth grade for the 0-1000 range (Booth & Siegler, 2006; Opfer & Siegler, 2007; Siegler & Opfer, 2003). So the transition from logarithmic to linear representations occurs earlier for small numerical ranges than for larger ones (Siegler et al., 2011). This may be due to children’s experience with numbers. Experience with numbers leads to an improved sense of numerical magnitudes and number line estimation accuracy (Ebersbach, Luwel, Frick, Onghena & Verschaffel, 2008; Siegler & Booth, 2004; Whyte & Bull, 2008). Children gain experience with larger numbers at a later moment in their school careers, and therefore their representations in that number range proceed later to linear ones. In previous research, differences arise in first, second and third graders from the use of inappropriate representations (Kucian et al., 2011; Opfer & Thompson, 2008; Siegler & Opfer, 2003). The same applies to children with mathematical problems. They kept using an immature logarithmic representation (Geary et al., 2008; Siegler & Opfer, 2003), because their experience with numbers, especially the larger ones, is lacking (Geary, 2010). Based on these outcomes, children with mathematical problems are expected to have more difficulty with estimating larger numbers (Ashkenazi, Mark-Zigdon & Henik, 2009) than children without mathematical problems. The current study will add to existing research by comparing the representations of children with and without mathematical problems from fourth, fifth and sixth grade.

Besides representations of numerical magnitudes, the solution strategy affects estimation accuracy too (Ashcraft & Moore, 2012). There are three types of solution strategies: the starting point strategy, the endpoint strategy and the midpoint strategy. In the starting point strategy, children start at the beginning of the line and count up toward the target position. In the endpoint strategy, children start at one end of the line and count up or down until they reach the target position. In the midpoint strategy, children start at the middle of the line and count on from there (Newman & Berger, 1984; Petitto, 1990; Schneider et al., 2008). The midpoint strategy leads to a minimization of errors (Ashcraft & Moore, 2012; Newman & Berger, 1984). It appears that the starting point strategy and the endpoint strategy are especially used by first and second graders, while these strategies are increasingly replaced by the midpoint strategy since third grade (Ashcraft & Moore, 2012; Petitto, 1990; Schneider et al., 2008). The current study will compare the strategies from children with and without mathematical problems in fourth, fifth and sixth grade. Differences are expected to exist in the use of the midpoint strategy. As noted above, older children use all strategies, while younger children do not use the midpoint strategy. The children with mathematical problems in the current study are also expected to not make use of the midpoint strategy, because their level of mathematical achievement is expected to be approximately similar to that of younger children without mathematical problems. Mathematical achievement is positively related to estimations skills and strategy use (Ashcraft & Moore, 2012; Booth & Siegler, 2006; Bull et al., 2011; Siegler & Booth, 2004), and therefore children with mathematical problems are expected to make less use of the midpoint strategy. To compare the strategies of children with and without mathematical problems, an eye-tracker will be used to acquire eye movement data. This data helps to avoid the drawback of number line tasks, namely the difficulty to examine the processes underlying the solutions (Schneider et al., 2008). Eye movement measures can visualize how children orient themselves in problem situations, how they direct their attention to task-relevant features (Schneider et al., 2008) and which strategies they use (Green et al., 2007). In previous research, eye movement data reflects an increase in estimation competence and use of the midpoint strategy for first, second and third graders without mathematical problems (Schneider et al., 2008). In the current study, the eye movements will visualize the use of different strategies from children with and without mathematical problems from fourth, fifth and sixth grade.

The present study will also look at the relation between strategies and representations. Ashcraft and Moore (2012) were one of the first researchers looking at this relation. They found that continued reliance on logarithmic representations is related to the absence of the use of the midpoint strategy. Because of a continued reliance on logarithmic representations in children with mathematical problems (Geary et al., 2008; Siegler & Opfer, 2003), these children are expected to keep using the starting point and endpoint strategy (Ashcraft & Moore, 2012). Also, between first and fifth grade, there is a shift from the use of the endpoint strategy to increasing use of the midpoint strategy (Ashcraft & Moore, 2012). A similar shift occurs in representations. Between kindergarten and sixth grade, there is a transition from logarithmic to linear representations (Booth & Siegler, 2006). Because these shifts take place at almost the same time, they are expected to be related to each other. In both groups, the logarithmic representation is expected to be positively related to the begin- and endpoint strategy, while the linear representation is expected to be positively related to the midpoint strategy.

The aim of the present study is to look at differences between children with and without mathematical problems in estimation skills by using an eye-tracker. Much research has focused on age-related differences in representations and strategies, while practitioners may also benefit from knowledge about differences between children with and without mathematical problems, because estimation skills are positively related to mathematical achievement. The results will give a starting point from which future research can proceed in finding guidelines for practitioners to help children suffering from mathematical difficulties.

Method

Participants

Twenty-seven children from fourth, fifth and sixth grade participated in this study. The math problems group consisted of 12 children with a mean age of 10.69 (SD = 1.12), including 10 girls (M_{age} = 10.74; SD = 1.16) and 2 boys (M_{age} = 10.47; SD = 1.26). This were all children in the age of nine to twelve years old who came to the Ambulatorium of Utrecht University because of mathematical problems. The Ambulatorium is an academic centre, specialized in diagnosing learning disorders among others. The control group, with normal mathematical abilities, consisted of 15 children, who were randomly selected from three school classes (grade 4, 5 and 6) from one school. This group had a mean age of 11.17 (SD = 0.54) and included 5 girls (M_{age} = 11.07; SD = 0.60) and 10 boys (M_{age} = 11.22; SD = 0.53). Age differences between group and gender were not significant (F (1, 23) = 0.25; p = .62; η^2 = .01).

Before analyzing differences between groups in representations and strategies, differences within groups were tested. There only appeared to be significant gender differences in the logarithmic representation in the 0-100 task, F (1, 25) = 6.36; p = .02; η^2 = .20. This was a medium effect. There were meaningful differences between the boys (M_{log} = 0.85; SD = 0.03) and girls (M_{log} = 0.81; SD = 0.06) in their logarithmic fit scores in the 0-100 task. This should be taken into account when formulating conclusions.

Materials and Procedure

The research took place at the pedagogic lab of Utrecht University. To examine differences in strategies, two symbolic number line tasks were conducted at the computer with a Tobii eye tracker. Data from the eye-tracker provided insight in the way children approach number line tasks. During the experimental trials, no feedback was given.

All participating children completed two symbolic number line tasks, ranging from 0-100 and 0-1000. The children saw a horizontal line on the computer screen ranging from 0 at the left to 100 or 1000 at the right, after which the experimenter demonstrated the positions of the begin- and endpoint of the line (0 and 100/1000). Then, the children were asked to do this for the remaining numbers, after naming the numbers. The values 1-99 and 1-999 were divided in 33 equal groups. Thereafter, one value of each group was randomly selected to be used in the task. For the 0-100 range, the values were: 3, 5, 9, 10, 14, 18, 19, 24, 27, 28, 32, 34, 37, 41, 43, 46, 49, 53, 57, 60, 61, 64, 66, 72, 74, 78, 80, 83, 87, 89, 91, 96, 99; for the 0-1000 range, the values were: 4, 36, 68, 104, 135, 153, 201, 230, 261, 277, 308, 354, 385, 398, 422, 469, 510, 528, 542, 594, 613, 636, 684, 697, 723, 763, 804, 844, 862, 880, 919, 958, 996. The order of values within a task was random for each participant.

Data-analysis

The eye-movement data was collected by using a Tobii eye-tracker. First of all, the fixations located above and below the line were excluded. The size of the screen was 1280 pixels horizontally and 1024 pixels vertically. Only the fixations between Y = 307.2 and Y = 512 pixels were included. Then, for each participant, the percentage of the total time of gazing to the begin point (X = pixels 040-240), the midpoint (X = pixels 540-740) and the endpoint (X = pixels 1040-1240) of the line was calculated. This gave an impression of the amount of time watching at each point on the line during the tasks. It will be examined with a MANOVA whether significant differences exist between children with and without mathematical problems in the percentage of time gazing to the begin-, mid- and endpoint during number line tasks.

To analyze differences in representations between children with and without mathematical problems, individual linear and logarithmic fit scores were computed. The estimated magnitudes were compared to the target points with a curve estimation procedure, which computed linear and logarithmic fit scores for both tasks (0-100 and 0-1000) for each individual. It will be examined with a MANOVA whether significant differences exist between children with and without mathematical problems in their representations.

Finally, it will be examined whether strategies and representations are related to each other. A correlation analysis will be performed, which will show whether gazes to the begin-, mid- and endpoint are correlated to linear and logarithmic fit scores. Because of the small sample size, a significance level of .10 will be used in all analyses.

The assumptions of the multivariate analysis of variance (MANOVA), correlation analysis and Paired samples *t*-test were verified. The assumptions of the independent observations and interval dependent variables were fulfilled. Also assumption of the normality of difference scores was fulfilled. The Kolmogorov-Smirnov test of normality showed that the difference score of the linear and logarithmic fit score in the math problems groups was normally distributed in both the 0-100 (D (12) = .91; p = .38) and 0-1000 task (D (12) = .47; p = .98). The same applies to the control group in both the 0-100 (D (15) = .66; p = .77) and 0-1000 task (D (15) = .69; p = .74). Box' test showed that the assumption of equal variances (Box test: F (55, 1803) = 1.32; p = .06) was not fulfilled. Besides, the Kolmogorov-Smirnov showed that the linear fit score of the control group was not normally distributed in both the 0-100 (D (15) = 0.24; p = .02) and the 0-1000 task (D (15) = 0.21; p = .08). The logarithmic fit score of the control group was only not normally distributed in the 0-100 task (D (15) = 0.24; p = .02). All other variables were normally distributed in both groups. In interpreting the data, this must be taken into account.

Results

There appeared to be a multivariate effect of mathematical problems on the combination of the two fit scores and the three gaze points in the 0-100 and 0-1000 task, *Wilks' Lambda* = 0.34, F (10, 16) = 3.12, p = .02; η^2 = .66. This is a large effect. The univariate effects will be discussed separately below.

Representations

The descriptive statistics of the individual linear and logarithmic fit scores are presented in Table 1. First of all, differences within groups in linear and logarithmic fit scores were tested. The results suggest that the math problems group used the linear representation significantly more than the logarithmic representation in both the 0-100 (t (11) = 6.82; p < .01; r = .70) and 0-1000 task (t (11) = 6.34; p < .01; r = .68). These were large effects. The control group also used the linear representation significantly more than the logarithmic representation in both the 0-100 (t (14) = 10.85; p < .01; r = .81) and 0-1000 task (t (14) = 13.40; p < .01; r = .87). These effects were large. In the following, differences in logarithmic and linear fit scores between the groups are examined.

Number line task 0-100.

On average, the control group had higher linear and logarithmic fit scores than the math problems group. Only the difference in the linear fit score was significant. This was a large effect (see Table 1).

Number line task 0-1000.

The linear and logarithmic fit scores of the control group were higher than the fit scores of the math problems group. Differences in both fit scores were significant. These were large effects (Table 1).

Table 1

Descriptive Statistics and Results of Univariate Tests on Group Differences in Individual Linear and Logarithmic Fit Scores

		Math problems		Control				Univariate Effects		
		M	*SD*	*M*	*SD*	*F*	*df1*	*df2*	*p*	η^2
0-100	Linear	0.93	0.05	0.98	0.01	15.99	1	25	< .01	.39
	Logarithmic	0.82	0.06	0.84	0.04	1.08	1	25	.31	.04
0-1000	Linear	0.74	0.20	0.96	0.03	17.39	1	25	< .01	.41
	Logarithmic	0.62	0.20	0.78	0.05	9.14	1	25	< .01	.27

Note. $N_{\text{Math problems}}$ = 12; N_{Control} = 15.

Strategies

Before analyzing differences in strategies, the gazes of all children from the two groups in the 0-100 and 0-1000 task were displayed on the number line. Figure 1 and 2 show that the gazes of the children with mathematical problems were approximately evenly distributed over the number line, with an increase at the midpoint, while the gazes of the children without mathematical problems were mostly positioned at the begin-, mid- and endpoint. Altogether, the children with mathematical problems seem to have made less use of the begin- and endpoint. This may indicate that children with mathematical problems make more use of the midpoint strategy, while children without mathematical problems make use of all three strategies.

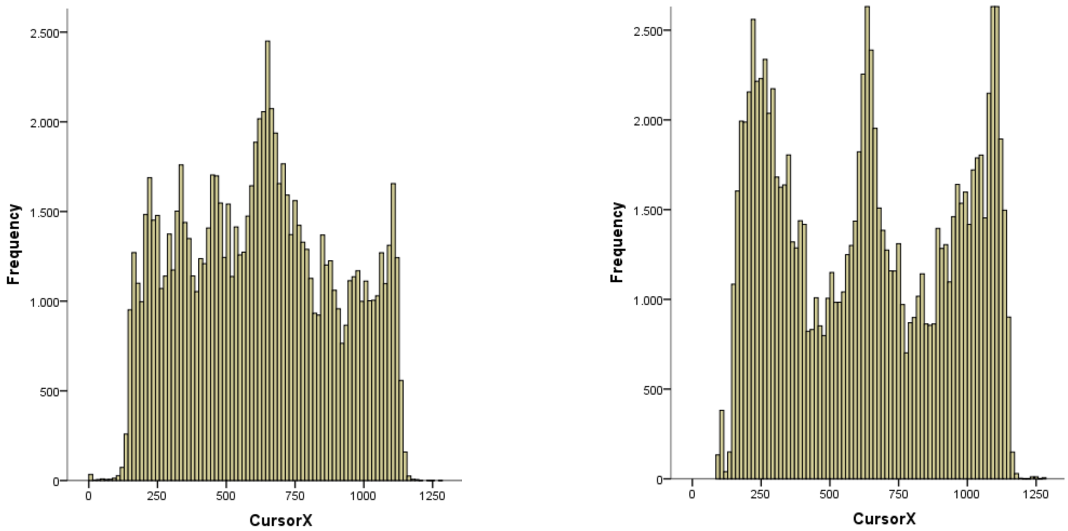

Figure 1. Distribution of gazes on the number line of the 0-100 task (left: children with mathematical problems; right: children without mathematical problems).

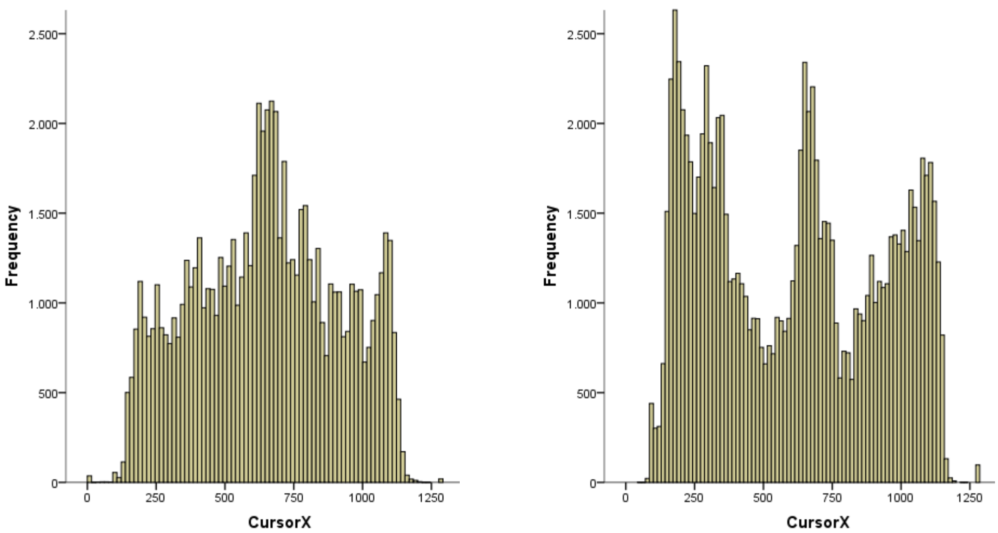

Figure 2. Distribution of gazes on the number line of the 0-1000 task (left: children with mathematical problems; right: children without mathematical problems).

To statistically analyze differences in strategies between children with and without mathematical problems, the percentage of time gazing to the three points on the number line was used. The descriptive statistics of the percentages of time gazing to the three points on the number line are presented in Table 2.

Number line task 0-100.

On average, the control group gazed more to the begin- and endpoint, and the math problems group more to the midpoint. Only the difference in gazes to the endpoint was significant. This was a large effect. However, the other differences were also relevant. The difference in gazes to the begin point was a medium effect and the difference in gazes to the midpoint was a small to medium effect (see Table 2).

Number line task 0-1000.

Like in the 0-100 number line task, the control group gazed more to the begin- and endpoint and the math problems group more to the midpoint. These differences in gazes to the begin-, mid- and endpoint were all statistically significant with large effects (see Table 2).

Table 2

Descriptive Statistics and Results of Univariate Tests on Group Differences in Percentages of Time Gazing to Begin- Mid- and Endpoint of the Number Line

		Math Problems		Control		Univariate Effects				
		M	*SD*	*M*	*SD*	*F*	*df1*	*df2*	*p*	η^2
0-100	Begin	10.24	4.98	12.70	5.46	1.46	1	25	.24	.06
	Mid	24.08	7.50	20.97	5.99	1.43	1	25	.24	.05
	End	9.46	3.66	14.78	3.96	12.88	1	25	< .01	.34
0-1000	Begin	7.83	5.18	16.07	6.30	13.30	1	25	< .01	.35
	Mid	27.60	5.65	20.66	6.66	8.25	1	25	< .01	.25
	End	9.26	4.08	12.43	3.00	5.42	1	25	.03	.18

Note. $N_{\text{Math problems}} = 12$; $N_{\text{Control}} = 15$

Relation representations and strategies

Table 3 shows the correlations between representations and strategies. It appeared that representations and strategies did almost not correlate significantly, but there were relevant correlations. Those will be discussed in the following.

Table 3

Correlations between Gaze Points and Representations per Group

			Math Problems		Control	
			r_s	p	r_s	p
0-100	Linear representation	Begin point	.11	.74	- .44	.10
		Midpoint	- .15	.64	.33	.23
		Endpoint	.34	.28	- .24	.38
	Logarithmic representation	Begin point	- .33	.30	- .12	.68
		Midpoint	- .28	.37	- .04	.90
		Endpoint	- .04	.91	.15	.59
0-1000	Linear representation	Begin point	.43	.16	.13	.66
		Midpoint	- .48	.12	- .02	.94
		Endpoint	.00	.99	.23	.41
	Logarithmic representation	Begin point	.41	.19	.49	.06
		Midpoint	- .49	.11	- .21	.46
		Endpoint	- .20	.54	- .01	.98

Number line task 0-100.

The linear representation was positively related to the midpoint and negatively related to the begin- and endpoint in the control group. These were small and medium effects. The opposite applies to the math problems group. In that group, the linear representation was positively related to the begin- and endpoint and negatively related to the midpoint. These were also small and medium effects (see Table 3).

The logarithmic representation was positively related to the endpoint and negatively related to the begin point in the control group. These were small effects. In the math problems group, the logarithmic representation was negatively related to the begin- and midpoint. These were medium effects (see Table 3).

Number line task 0-1000.

The linear representation was positively related to the gazes to the begin- and endpoint in the control group. These were small effects. In the math problems group, the linear representation was positively related to the begin point and negatively related to the midpoint. These were medium to large effects (see Table 3).

The logarithmic representation was positively related to the begin point and negatively related to the midpoint in the control group. These were large and small to medium effects. In the math problems group, the logarithmic representation was positively related to the begin point and negatively related to the mid- and endpoint. These were small to medium and medium to large effects (see Table 3).

Discussion

This study examined differences between children with and without mathematical problems in their number line estimation skills. More specifically, the effect of mathematical problems on representations and strategies was examined, this latter by using an eye-tracker. This knowledge will provide a useful starting point for future studies on differences in estimation skills between children with and without mathematical problems. Knowledge coming from studies on this topic will provide information to practitioners about the extra support children with mathematical problems need.

When comparing the representations of the children with and without mathematical problems, it is remarkable that the estimations of both groups fitted better to the linear than to the logarithmic model. This does not correspondent with previous research, which stated that children with mathematical problems keep using an immature logarithmic representation (Geary et al., 2008). The deviation may have arisen from differences in age between the participants in this study and previous studies. Geary and colleagues (2008) used first and second graders, while the present study used fourth, fifth and sixth graders. This may indicate that children with mathematical problems use the logarithmic representation longer, but move to the linear representation when they get more experience with numbers. However, the estimations of the children without mathematical problems fitted significantly better to both models than those of the children with mathematical problems in the 0-1000 task, which is in contrast with the expectations. Children with mathematical problems were expected to keep using an immature logarithmic representation (Geary et al., 2008; Siegler & Opfer, 2003), while children without mathematical problems were expected to rely on a linear representation. Perhaps, children with mathematical problems use other representations than a logarithmic or linear one. Another explanation might be that children with mathematical problems do not use a fixed representation in estimating at all. In the 0-100 task, only the difference in the linear fit score was significant. The children with and without mathematical problems used the logarithmic model about the same. This may also indicate that all children use multiple representations, depending on the situation. This corresponds with the multiple representations hypothesis, which states that people know and use multiple numerical representations depending on contextual variables (Siegler & Booth, 2004; Siegler & Opfer, 2003). The shift from reliance on the logarithmic to reliance on the linear representation in the 0-100 task between kindergarten and second grade (Berteletti et al., 2010; Geary et al., 2007; Opfer et al., 2010) is not reflected in the results. Based on this shift, the children from fourth, fifth and sixth grade were expected to rely on the linear representation only in the 0-100 task. This does not appear to be the case. The current study revealed that children with mathematical problems make less use of the logarithmic and linear representation than children without mathematical problems, especially when estimating larger numbers.

In addition, there appear to be differences in strategy use. In both tasks, the children without mathematical problems used the endpoint strategy more than the children with mathematical problems. The children with mathematical problems, on the other hand, used the midpoint strategy more than the children without mathematical problems in both tasks. In the 0-1000 task, these differences were significant. So, the children with mathematical problems used the midpoint strategy and the children without mathematical problems the endpoint strategy more often. These results are in contrast with the expectations. The children with mathematical problems were expected to not make use of the midpoint strategy, because their level of mathematical achievement is worse than that of children without mathematical problems. Previous research indicated that mathematical achievement is positively related to estimation skills and strategy use (Ashcraft & Moore, 2012; Booth & Siegler, 2006; Bull et al., 2011; Siegler & Booth, 2004). The deviation may have arisen because of more difficulty with adapting strategies to the task. Low mathematical achievement leads to less use of different strategies. As Figure 1 and 2 showed, the gazes of the children with mathematical problems are approximately evenly distributed over the number line, with an increase at the midpoint, while the gazes of the children without mathematical problems were mostly positioned at the begin-, mid- and endpoint. Probably, the children without mathematical problems adapt their strategy to the task, while children with mathematical problems have difficulty doing this. This is consistent with previous research, which stated that there is a shift from using one to using three strategies (Ashcraft & Moore, 2012). It can be concluded that children with mathematical problems make less use of the endpoint strategy, while children without mathematical problems use all three strategies, but the endpoint strategy the most.

Finally, the relation between representations and strategies was examined. This revealed that children without mathematical problems who had a more linear representation, gazed more to the midpoint and less to the begin- and endpoint in the 0-100 task and more to the begin- and endpoint in the 0-1000 task. The results from the 0-100 task are consistent with previous research, which stated that the use of the midpoint strategy and the use of a linear representation are based on growing number knowledge (Ashcraft & Moore, 2012). So, the children without mathematical problems probably had enough knowledge of the numbers ranging 0 to 100, through which they were able to use both the linear representation and the midpoint strategy. A lack of knowledge about the numbers ranging 0 to 1000, may have led to the absence of a relation between the linear representation and the midpoint strategy. This finding is consistent with previous research, which concluded that the transition from logarithmic to linear representations occurs earlier for smaller numerical ranges than for larger ones (Siegler et al., 2011) and that the starting point strategy and endpoint strategy are increasingly replaced by the midpoint strategy since third grade (Schneider et al., 2008). The shift from logarithmic to linear representation for the 0-1000 range, takes places between second and sixth grade (Siegler & Opfer, 2003), which is the age group used in this study. The participants may be trying to adapt their representations en strategies to the task, which they do well in the 0-100 task but not yet in the 0-1000 task. The results show that the math problems group had difficulties with this adaptation in both tasks. Children with mathematical problems who had a more linear representation, gazed more to the begin- and endpoint and less to the midpoint in the 0-100 task and more to the begin point and less to the midpoint in the 0-1000 task. This corresponds with the expectation that children with mathematical problems have less number knowledge than children without mathematical problems (Geary, 2010). It also points out that children with mathematical problems who had a more linear representation, still used other strategies than just the midpoint strategy.

The results from children who had a more logarithmic representation were about the same in both groups. The control group gazed more to the endpoint and less to the begin point in the 0-100 task and the math problems group gazed less to the begin- and midpoint. These results are partly consistent with previous research. Ashcraft and Moore (2012) found that the continued reliance on logarithmic representations is related to the absence of the midpoint strategy. This also emerges from the present study. The children who had a more logarithmic representation, gazed less to the midpoint in both groups and both tasks. This may be explained by the fact that the distance between numbers in this representation exaggerates at the low end of the range and minimizes at the middle and upper ends of the range (Siegler & Opfer, 2003). In the 0-100 task, the children with and without mathematical problems used the begin point less, while they used it more in the 0-1000 task. Perhaps, the experience with numbers also plays a role here. When children know more about the value of the target, they can make less use of the begin point, while they use the begin point more when the numbers are higher and the children know less about their value. The results point to a negative relation between the logarithmic representation and the gazes to the midpoint, which may be explainable by the shift from a begin- and endpoint strategy to a midpoint strategy and the shift from a logarithmic to a linear representation, which take place at almost the same time (Ashcraft & Moore, 2012; Booth & Siegler, 2006). According to those studies, children use the logarithmic representation and the starting point and endpoint strategy before turning to the use of the linear representation and the midpoint strategy. This study replicates these findings for the 0-100 task only. The children without mathematical problems who had a more linear representation, gazed more to the midpoint and the children with mathematical problems more to the begin- and endpoint. In the 0-1000 task, this relation was not present. It can be concluded that all children with a more logarithmic representation, used the midpoint strategy less. Children without mathematical problems who had a more linear representation, used the midpoint strategy in the 0-100 task and the endpoint strategy in the 0-1000 task.

The findings of this study provide information about differences in estimation skills between children with and without mathematical problems. However, there are some limitations to this study that should be addressed in future research.

First of all, the number of participants was limited. Only 29 children participated in this study. The group of children with mathematical problems consisted of almost all girls (10 girls and 2 boys), while the group of children without mathematical problems consisted of more boys (5 girls and 10 boys). Future research should include more children to be able to draw more reliable conclusions about differences between children with and without mathematical problems. Despite the fact that there only appear to be significant gender differences in the logarithmic representation, there also should be a better balance between boys and girls in future research.

Secondly, future research might use the amount of time instead of the percentage of time gazing to the begin-, mid- and endpoint of the line. By using the percentage of time, information about the time children need to solve the tasks is lost. By using the amount of time, differences between children with and without mathematical problems in the solution time can be considered. For practitioners, it can be useful to have this information, to be able to adapt the assignments better to the time in which children work independent.

Thirdly, there has to be attention for the place in which the target value is shown. In the present study, the target value was in the middle. This may have ensured that the children with mathematical problems gazed more to the midpoint than expected. They may have used this point as an indication for the midpoint, through which they used it more than they would when this point was not indicated this clear. In future research, this can be prevented by placing the target value at another place every time, so the child can not focus on one place on the screen for the target values.

Fourthly, there has to be attention for the fact that the numbers of the begin- and endpoint were displayed on the number line in the current study. This increases the likelihood that children gaze at these points before placing the number on the number line. In future research, this numbers can be better removed, so that the gazes of the children are based more on their own representations and strategies instead of on the stimuli on the screen.

It can be concluded that children without mathematical problems use the linear and logarithmic representation more often than children with mathematical problems. Besides, children without mathematical problems gaze more to the begin- and endpoint, while children with mathematical problems gaze more to the midpoint. In children with mathematical problems, representations and strategies are not related to each other. In children without mathematical problems they are only related in the 0-100 task. In the 0-1000 task, this relation is not present. For practitioners, it is important to give extra support to children with mathematical problems, because they seem less able to adapt their strategies and representations to the task. The findings add to the current body of research, in which it has been proposed that shifts exist in representations and strategies (Ashcraft & Moore, 2012; Booth & Siegler, 2006; Petitto, 1990; Schneider et al., 2008).

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