Relevance of the Yablo paradox

Whether the Yablo paradox is self-referential

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But one must not think ill of the paradox, for the paradox is the passion of thought, and the thinker without the paradox is like the lover without passion: a mediocre fellow. But the ultimate potentiation of every passion is always to will its own downfall, and so it is also the ultimate passion of the understanding to will the collision, although in one way or another the collision must become its downfall. This, then, is the ultimate paradox of thought: to want to discover something that thought itself cannot think.[1]

- Søren Kierkegaard

[1] Søren Kierkegaard, *Philosophical Fragments*, translated by David F. Swenson, translation revised by Howard V. Hong, (New Jersey: Princeton University Press, 1936).

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Introduction

Let L be the sentence:

'L is not true'

At first sight sentence L, usually referred to as the Liar sentence, does not look problematic; saying of something that it is not true is a completely innocent thing to do. It seems to be true to assume that 'if L, then L is true' and 'if L is true, then L'. If however we look more closely at L, we see that these assumptions lead to a contradiction. The informal argument to a contradiction proceeds as follows: if sentence L is not true, then it is not true that L is not true. So L is true, which contradicts the assumption that we started with: sentence L is not true. Therefore the sentence must be true. Hence it is true that L is not true. So the sentence is not true. Now we have a contradiction with our conclusion that L is true. Thus: L is true if and only if L is not true. So, after looking closely, it appears that sentence L is problematic after all. We call such a sentence, that is based on apparently true assumptions but leads to a contradiction, a logical paradox.

Many philosophers blame paradoxical sentences for being circular or, more specifically, for being self-referential. Challenging the idea that the root cause of logical paradoxes is circularity, Stephen Yablo introduced a logical paradox consisting of a denumerable list of sentences each of which says that the sentences occurring after it in the list are not true. This list of sentences, usually referred to as Yablo's paradox, might also not look problematic at first sight, but after a quick analysis you will see how this list leads to a contradiction too.[2] Yablo claims that his paradox is, unlike the Liar paradox, not self-referential and therefore that self-reference is not necessary for (liar-like) paradoxes.[3]

One can categorise Yablo's paradox as a liar-like paradox for it implies an infinite list of sentences similar to the Liar sentence. The sentences of both paradoxes refer to the untruth of some sentence. This resemblance between the two might be reason to believe that the Yablo paradox could be helpful in finding a solution to liar-like paradoxes and, more generally, to all paradoxes concerning theories of truth. In order to establish the significance of the Yablo paradox in solving logical paradoxes, we need to be sure that the Yablo paradox is indeed not self-referential. Opposing the belief that circularity is an essential feature of logical paradoxes, Yablo tried to demonstrate that circularity should not have the focus when working on (a) solution(s) to logical paradoxes. Yablo intended to construct a paradox that is different from the traditional liar-like paradoxes. The question is whether Yablo did succeed in avoiding circularity in his paradox. Since Yablo's discovery, first described in *Truth and Reflection*[4] and later more

[2] I will show this in chapter two.

[3] Stephen Yablo, "Paradox without Self-Reference", *Analysis* 53.4 (1993), pp. 251-252.

[4] Stephen Yablo, "Truth and Reflection", *Journal of Philosophical Logic* 14 (1985), pp. 297-349.

explicitly in *Paradox without Self-Reference*, numerous papers for and against the claim that Yablo's paradox involves circularity have appeared. As one of the first commenters on Yablo's article, Graham Priest argues that Yablo did not succeed in avoiding circularity.[5] Priest's argument relies on the following three assumptions:

i. Fixing the reference of the Yablo list necessarily needs description;

ii. The description of the Yablo list necessarily makes use of self-reference;

iii. A referent that can only be referred to by a circular description must itself be circular.[6]

These three assumptions have been under attack as well as defended by, amongst others, James Hardy,[7] Roy Sorensen,[8] JC Beall,[9] Otávio Bueno and Mark Colyvan,[10] Jeffrey Ketland[11] and Roy T. Cook.[12] An important subject in this discussion is the (dis)analogy between the Liar paradox and the Yablo paradox. For the Yablo paradox to be relevant, it must be similar to the Liar paradox in most ways (because if it would be too different, it would not be an example of a liar-like paradox and therefore not be helpful in finding a solution to liar-like paradoxes). For example we need to establish whether the Yablo paradox is, similar to the Liar paradox, genuinely inconsistent. If the Yablo paradox appears to be another type of paradox, for example one that does not involve a negation-inconsistency but rather something weaker,[13] the Yablo paradox might not be so helpful at all. In that case, solutions to the Yablo paradox would not necessarily work for all liar-like paradoxes and vice versa. If on the other hand the Yablo paradox appears to be, just like the Liar paradox, circular, it will lose much of its interest (because then it will not challenge the belief that circularity is essential to liar-like paradoxes). Hence to be relevant to the search for solutions to logical paradoxes, the Yablo paradox must be *not self-referential* (unlike the Liar paradox) and *genuinely inconsistent* (like the Liar paradox). The purpose of this paper is to give a meaningful analysis to whether Yablo succeeded in providing a new insight to possible solutions to logical paradoxes. To do this, I will give an answer to the following question: 'is the Yablo paradox *self-referential*?'

[5] Graham Priest, "Yablo's paradox", *Analysis* 57.4 (1997), pp. 236-242.
[6] The referent is the object to which the description is referring to.
[7] James Hardy, "Is Yablo's paradox Liar-like?", *Analysis* 55.3 (1995), pp. 197-198.
[8] Roy A. Sorensen, "Yablo's Paradox and Kindred Infinite Liars", *Mind* 107.425 (1998), pp. 137-156.
[9] JC Beall, "Is Yablo's paradox non-circular?", *Analysis* 61.3 (2001), pp. 176-187.
[10] Otávio Bueno and Mark Colyvan, "Yablo's paradox and referring to infinite objects", *Australasian Journal of Philosophy* 81.3 (2003), pp. 402-412.
[11] Jeffrey Ketland, "Bueno and Colyvan on Yablo's paradox", *Analysis* 64.2 (2004), pp. 165-172.
[12] Roy T. Cook, "There *Are* Non-circular Paradoxes (But Yablo's Isn't One of Them!)", *The Monist* 89.1 (2006), pp. 118-149.
[13] I will discuss this type of paradox in more detail in chapter 3.2.

To be able to give a good analysis I will look closely to the Yablo paradox and some influential attributions to the discussions about the paradox. Before I look at the discussion regarding the Yablo paradox, I will start with briefly discussing the characteristics of the Liar paradox in chapter 1. In the second chapter I will introduce the Yablo paradox and demonstrate how to derive to a contradiction. To be able to conclude whether the Yablo paradox is self-referential or not, the third chapter will be dedicated to Priest's three assumptions and different objections to these assumptions. Finally I will conclude in chapter four whether or not the Yablo paradox will help us find (a) solution(s) to logical paradoxes.

1 The Liar paradox

There is little discussion about the circularity and inconsistency of the Liar paradox:

'L is not true'

Therefore I dedicate this first chapter to the demonstration of self-reference and genuine inconsistency in the Liar paradox. First I will discuss the position of the Liar paradox within the categories of logical paradoxes. Then I will show that self-reference of the Liar paradox is evident, followed by an introduction of an alternative version of the Liar paradox with indirect self-reference. In the fourth paragraph the inconsistency of the Liar paradox will be analysed. In the final paragraph of this chapter I will name some possible solutions to the Liar paradox.

1.1 Semantic paradox

There are many ways to define a paradox. In this work I will follow the definition given by Thomas Bolander: "a *paradox* is a seemingly sound piece of reasoning based on apparently true assumptions that leads to a contradiction".[14] Logicians spent years on trying to solve such paradoxes. Some are hoping to find a universal solution that is applicable to all paradoxes, while others focus on one subset of paradoxes at the time. There is a large set of paradoxes that is characterised by its use of self-reference. Sticking to Bolander's definitions, "*self-reference* is used to denote a statement that refers to itself or its own referent".[15] Paradoxes of self-reference can be categorised as semantic, set-theoretic or epistemic. The Liar paradox is a genuine semantic paradox, because it concerns theories of truth.

1.2 Self-reference in the Liar paradox

Yablo claims that "paradoxes like the Liar are possible in the complete absence of self-reference",[16] so I assume that Yablo thinks that other liar-like paradoxes are circular. Here I will demonstrate that Yablo is correct in claiming that the Liar paradox is circular.

Some people might use the demonstrative 'this' in the Liar sentence:

'This sentence is not true'

Writing the sentence like this immediately exposes self-reference. The fact that demonstratives are context-depending, and therefore cause variable truth-values, makes me more careful in drawing conclusions from them. To draw a

[14] Thomas Bolander, "Self-Reference", in *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, (Fall 2012 Edition) 1 (b).
[15] Bolander, "Self-Reference", 1 (a).
[16] Yablo, "Paradox without Self-Reference", p. 251.

more reliable conclusion, I prefer to formalise the Liar in the language of first order logic L, supplemented with the truth-predicate $T(x)$ for 'it is true that'. We call this language L_T. Alfred Tarski introduced the so called 'T-schema' in order to define what it takes to be a theory of truth:

$$\text{for all sentences } \phi,\ \ulcorner\phi\urcorner \text{ is true if and only if } \phi$$

where $\ulcorner\phi\urcorner$ is a name for the sentence ϕ in L_T.[17] Note that we can only use the T-schema for sentences and not for predicates. We can use predicate T(x) for the formalization of the Liar sentence:

$$\neg T(\ulcorner \text{L} \urcorner)$$

where $\ulcorner \text{L} \urcorner$ is the name for the Liar sentence $\neg T(\ulcorner \text{L} \urcorner)$. The fact that the sentence $\neg T(\ulcorner \text{L} \urcorner)$ includes $\ulcorner \text{L} \urcorner$, which is the name for that specific sentence, demonstrates that the Liar sentence is self-referential.

1.3 Indirect self-reference

Consider the dialogue between Aristotle and Plato:

Plato says: "Aristotle's claim is true"
Aristotle says: "Plato's claim is not true"

Suppose Plato's claim, P, is true. Then Aristotle's claim, A, is true. Since Aristotle speaks the truth, P is not true. This is in contradiction with our assumption that P is true. Therefore, by reductio ad absurdum, P is not true. This means that A is not true, hence it is not true that P is not true: P is true. We now have that P is true and that P is not true. A contradiction has arisen in the dialogue that is concerning notions of truth. Thus the dialogue is a genuine semantic paradox.

This dialogue-paradox looks like the Liar paradox, since Aristotle's claim is similar to the troubling Liar sentence. When we construct both paradoxes in L_T the analogy becomes even more clear. The classical Liar paradox can be represented with the Liar biconditionals:

$$\neg T(\ulcorner \text{L} \urcorner) \iff \text{L}$$

The Liar-dialogue paradox can then be formalised as follows:

$$T(\ulcorner \text{A} \urcorner) \iff \text{P}$$
$$\neg T(\ulcorner \text{P} \urcorner) \iff \text{A}$$

We must look at the Liar-dialogue, consisting of P and A, as a loop: the dialogue is only paradoxical when we take what Plato says and what Aristotle says together. This is where self-reference makes its entrance. There is no P

[17] Alfred Tarski, "The Concept of Truth in Formalized Languages", *Logic, Semantics and Metamathematics*, translated by J.H. Woodger, Indianapolis: Hackett (1983), pp. 152-278.

in $T(\ulcorner \mathrm{A} \urcorner)$ and there is no A in $\neg T(\ulcorner \mathrm{P} \urcorner)$, so P and A individually are not self-referring. But P refers to A and A refers to P, so circular reasoning comes up when we combine P and A. The Liar-dialogue is an example of a semantic paradox that does not use explicit self-reference but does end up, like the standard Liar, with circular reasoning. A part of the dialogue is in fact referring to another part of the dialogue, which is referring to the first part. Both sentences of the Liar-dialogue are indirectly self-referring, making the Liar-dialogue paradox genuinely circular. This shows that circularity does not only appear in sentences that refer directly to themselves.

1.4 Inconsistency of the Liar paradox

In the introduction of this paper I already demonstrated in words how to derive to a contradiction with the Liar sentence. By giving a more technical proof, I will now demonstrate the inconsistency of the Liar paradox in L_T.

Take the Liar biconditionals:

$$\neg T(\lceil \mathrm{L} \rceil) \iff \mathrm{L}$$

and the T-schema:

$$T(\lceil \phi \rceil) \iff \phi$$

Suppose L is true:

$$T(\lceil \mathrm{L} \rceil)$$

By T-schema, we have:

$$\mathrm{L}$$

Thus we have:

$$\neg T(\lceil \mathrm{L} \rceil)$$

By reduction ad absurdum, our assumption that L is true is false. So L is not true. Hence:

$$\neg T(\lceil \mathrm{L} \rceil)$$

But this exactly what the Liar sentence says, so:

$$\mathrm{L}$$

By T-schema, we now have:

$$T(\lceil \mathrm{L} \rceil)$$

This is in contradiction with:

$$\neg T(\lceil \mathrm{L} \rceil)$$

So the Liar sentence is inconsistent.

No extra rules are needed to derive a contradiction.[18] In this paper I define a sentence with this sort of inconsistency, where no extra rules are added to L_T, as 'genuinely inconsistent'.

1.5 Solutions to the Liar paradox

When Bertrand Russell discovered a self-referential paradox,[19] related to the class of set-theoretic paradoxes, he imposed an absolute ban on all self-reference to avoid all self-referential paradoxes and possibly all logical paradoxes. Banning self-reference will definitely block the Liar paradox. Unfortunately this restraint "work[s] like a cannon against a fly",[20] since this restraint also blocks all self-referential sentences that are not paradoxical, e.g. 'this sentence is in English'. Quite comprehensibly many philosophers are not satisfied with this solution.

Other solutions to the Liar paradox include, amongst many others: non-classical truth theories, Tarski's hierarchy of truth predicates[21] and Burge's hierarchies of contexts.[22]

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[18] If we take the standard definition for $(A \iff B)$, being '$(A \implies B)$ and $(B \implies A)$', to be part of L_T.

[19] The Russel paradox goes as follows: consider the set of all sets that are not members of themselves. This set is a member of itself if and only if it is not a member of itself, hence paradoxical.

[20] Yablo, "Paradox without Self-Reference". p.251.

[21] Alfred Tarski, "The Semantic Conception of Truth", *Philosophical and Phenomenological Research* 4.3 (1944), pp. 341-376.

[22] Tyler Burge, "Semantical Paradox", *Journal of Philosophy* 76 (1979), pp. 169-198.

2 The Yablo paradox

Now that we have seen how the Liar paradox works, we can look closely at the Yablo paradox to be able to compare the circularity and inconsistency of the two paradoxes in chapter three. In this chapter I will first show how Yablo presents his paradox and how the informal argument of the Yablo sequence leads to a contradiction. Then I will show how the paradox does not look self-referential at first sight.

2.1 The informal argument to contradiction

Imagine an infinite sequence of sentences $S_1, S_2, S_3, ...$, each to the effect that every subsequent sentence is untrue:[23]

$$
\begin{gathered}
S_1\text{:for all } k > 1, S_k \text{ is untrue} \\
S_2\text{:for all } k > 2, S_k \text{ is untrue} \\
S_3\text{:for all } k > 3, S_k \text{ is untrue} \\
\vdots
\end{gathered}
$$

The argument to a contradiction proceeds as follows: suppose that some S_n is true, then for all $k > n$, S_k is untrue. This means that (a) S_{n+1} is untrue and (b) for all $k > n+1$, S_k is untrue. Because of (b) what S_{n+1} says, 'for all $k > n+1$, S_k is untrue', is actually true. So S_{n+1} appears to be the case. This is contrary to (a). So we must conclude that there is no S_n that is true, and thus that all S_n are untrue. But then S_n is true again, since all $k > n$ are untrue. Now we see that S_n is true if and only if S_n is false.

2.2 No self-reference at first sight

Since the Yablo list is using semantic notions of truth it is to be classified as a semantic paradox. Even though semantic paradoxes belong to a subclass of self-referential paradoxes, this does not mean that the Yablo paradox is a self-referential paradox by definition. There can also be semantic paradoxes of a non-self-referential kind. Notice that we end up with circular reasoning (when S_n is true, S_n is false, and when S_n is false, S_n is true). The question is whether the argument necessarily uses circularity and whether the paradox itself is self-referential.

The most important ingredient of the Yablo paradox is the Yablo sentence 'all subsequent sentences are false'. A quick analysis tells us that each sentence in the list only refers to sentences that occur below it in the list, so there is no circularity in the pattern of reference involved. But since there are however a lot more things to say about the sequence, it would not be wise to stop analysing

[23] Yablo, "Paradox without Self-Reference", p. 251.

the paradox already at this point.

Will self-reference appear when we formalise the Yablo paradox? Not at first sight. The Yablo sentences can be reformulated as biconditionals, using n for all natural numbers. Then the Yablo biconditionals are all instances of:

$$T(\ulcorner S_n \urcorner) \iff \forall k > n : \neg T(S_k)$$

There is no actual object that appears on both sides of the biconditional, but the fact that the variable n appears on both sides of the biconditional should make us suspicious. For now, formalising the Yablo sentences does not expose any circularity in the sentence. This is however not the only way to look at the Yablo sentences, as we will see in the next chapter.

3 Is Yablo's paradox self-referential?

Stephen Yablo introduced his paradox to show that self-reference is not necessary to liar-like paradoxes. Liar-like paradoxes are semantic paradoxes that state the untruth of some sentence(s). Yablo did not specify what he meant by 'liar-like paradoxes', so I am aware that this definition of the expression 'liar-like' is only an assumption.

Not everyone agrees that Yablo's paradox is not self-referential. An ongoing debate on this claim followed Yablo's article. In this chapter I will elaborately discuss the most influential articles on this discussion. In the first paragraph I will explain how Priest argues that Yablo's paradox is self-referential. In the next three paragraphs I will discuss objections to Priest's argument and try to counter them. In the final section I will conclude whether Yablo's paradox is self-referential or not.

3.1 Priest's arguments for self-reference

Graham Priest wrote an article in 1997 where he objects to Yablo's claim that his paradox is not self-referential. Priest tries to prove that self-referential circularity is involved in Yablo's paradox, by rewriting the Yablo sentences. Priest starts by giving the formal argument to contradiction.[24]

> For any n:
> $$\begin{aligned} T(S_n) &\implies \forall k > n : \neg T(S_k) \ (*) \\ &\implies \neg T(S_{n+1}) \end{aligned}$$
> But:
> $$\begin{aligned} T(S_n) &\implies \forall k > n : \neg T(S_k) \ (*) \\ &\implies \forall k > n+1 : \neg T(S_k) \\ &\implies T(S_{n+1}) \ (**) \end{aligned}$$
> Hence, $T(S_n)$ entails a contradiction, so $\neg T(S_n)$. But n was arbitrary. Thus for all $k, \neg T(S_k)$, by Universal Generalisation. In particular, then, for all $k > 0 : \neg T(S_k)$, i.e., S_0 and so $T(S_0)$. Contradiction (since we have already established $\neg T(S_0)$).[25]

Priest then points out that we cannot justify the sentence marked (*) with the T-schema. T-schema applies only to sentences, not to formulas with free variables. This argument involves free variables (note that we are applying the rule of universal generalisation). Priest says that it is necessary to generalise the T-schema to formulas containing free variables. This involves the notion of satisfaction. Satisfaction is the two-place relation $Sat(x, y)$ between numbers and predicates. Instead of writing 'if it is true that S_n, then all subsequent sentences of S_n are untrue', we write 'if n satisfies $\dot{s}$, then all subsequent sentences of S_n

[24] I choose to maintain my notation, while Priest uses a slightly different notation.
[25] Priest, "Yablo's paradox", p. 237.

are untrue'.
Here, ṡ is the name for the predicate:

$$\forall k > x : \neg T(S_k)$$

When we replace truth by satisfaction in every line of the argument, the argument goes as follows: For any n, suppose:

$$Sat(n,\dot{s})$$

Then, by the definition of ṡ,

$$\forall k > n : \neg Sat(k,\dot{s})$$

So:

$$\neg Sat(n+1,\dot{s})$$

But we said before that $Sat(n,\dot{s})$ and for all $k > n : Sat(k,\dot{s})$, thus also:

$$\forall k > n+1 : \neg Sat(k,\dot{s})$$

Hence:

$$Sat(n+1,\dot{s})$$

We now have $\neg Sat(n+1,\dot{s})$ and $Sat(n+1,\dot{s})$, so $Sat(n,\dot{s})$ entails a contradiction. Thus, by reductio ad absurdum:

$$\neg Sat(n,\dot{s})$$

By Universal Generation, then:

$$\forall k : \neg Sat(k,\dot{s})$$

So:

$$\forall k > 0 : \neg Sat(k,\dot{s})$$

Thus, by the definition of ṡ:

$$Sat(0,\dot{s})$$

We now have a contradiction, for we have for all $k : \neg Sat(k,\dot{s})$ and $Sat(0,\dot{s})$. So replacing truth by satisfaction does not change anything for the argument to leading to a genuine contradiction.

After replacing truth by satisfaction, the predicate ṡ becomes

$$\forall k > n : \neg Sat(k,\dot{s})$$

Predicate ṡ is equivalent to 'no number greater than n satisfies ṡ'. One could rephrase ṡ as 'no number greater than n satisfies this predicate', using the demonstrative 'this', which could indicate that the predicate is self-referential. But since I do not completely trust demonstratives, I will show in a more formal way that the predicate ṡ is self-referential.

A fixed point of a predicate is a point that is mapped to itself by the function. So "if we have a mapping $f : X \to X$ and we *stipulate* that, for some $r \in X, r = f(r)$, then this r is a fixed point of the function f".[26] The fact that $\dot{s} = \forall k > x : \neg Sat(k, \dot{s})$ shows that ṡ is a fixed point for the satisfaction predicate $Sat(x, y)$. Remember that ṡ is the name of the predicate that uses satisfaction to state for every n what S_n (any Yablo sentence) says. Predicate ṡ's being a fixed point indicates self-reference.

In order to prove that the Yablo sentences are fixed points for the Yablo paradox, we need to show that the fixed points exist. Priest established the existence of the fixed point with the help of a diagonalisation argument.[27] While Bueno and Colyvan complain that "Priest's argument seems to presuppose the existence of the list, in order to establish that to derive a contradiction from the latter",[28] Ketland argues that "on Priest's analysis of the paradox, one *first* directly constructs the Yablo list, using standard diagonalisation techniques, *and then* proves the inconsistency" of the Yablo paradox. Indeed we can be sure that the Yablo sequence exists because we can prove diagonalisation which implies the existence of each sentence in the list.[29]

Rewriting the sentence with satisfaction does seem to expose self-reference. As Priest argues that this is the only correct description of the Yablo sentence, circularity is evident to him. In the next sections I will discuss the three assumptions Priest makes to come to this conclusion:

i. Fixing the reference of the Yablo list necessarily needs description;

ii. The description of the Yablo list necessarily makes use of self-reference;

iii. A referent that can only be referred to by a circular description must itself be circular.

[26] Ketland, "Bueno and Colyvan on Yablo's paradox", p. 169.

[27] The argument goes: if $\alpha(x, y_1, \ldots, y_m)$ is any formula, there is a number n such that n is the code of the formula $\alpha(\underline{n}, y_1, \ldots, y_m)$. Here, $\underline{n}$ is the numeral of n and n is the fixed point. See Priest, "Yablo's paradox", p. 238.

[28] Otávio Bueno and Mark Colyvan, "Paradox without satisfaction", *Analysis* 63.2 (2003), p. 156.

[29] For the exact argument see Priest, "Yablo's paradox", p. 238.

3.2 Fixing the reference of the Yablo list necessarily needs description (assumption i.)

J.C. Beall clarifies this first assumption by saying that there are exactly two ways of fixing the reference of any term t: by demonstration or by description. Both Beall and Priest believe that fixing the reference of the Yablo list necessarily needs description. In this paragraph I will discuss the most important attempts to prove that we can also fix the reference of the Yablo list by demonstration.

As one of the first reactions to Priest, Sorensen wrote an article arguing that Yablo's paradox is not self-referential. Sorensen claims that "if I see a queue of students, I can specify a sequence just by pointing at the queue - even if the queue is infinitely long."[30] Before I will elaborately argue why I disagree with this claim, which I will do later in this chapter, I can already say that my conclusion comes down to this: there are no infinitely long queues in our world, so we cannot possibly be pointing at one.

In 2001, Beall wrote an article in response to Sorensen. In his article Beall claims that Sorensen's defence fails to address Priest's basic point, therefore giving us no reason to believe that Yablo's paradox is not self-referential. Let $\delta(t)$ be the referent of t. If we fix the reference of t by demonstration "we need to see $\delta(t)$ or otherwise stand in close (spatial) proximity to $\delta(t)$."[31] If we fix the reference of t by description we do not necessarily have to see $\delta(t)$. Term t simply denotes whatever satisfies the description. Like me, Beall believes that we cannot point at an infinite sequence, because we are finite beings who cannot do that. We can only look at a part of the infinite sequence and then point at it. But then, we will never know of which whole it is part of. Also we cannot write down an infinite sequence by using vague descriptions like 'etcetera', 'and so on' or '. . . ' (I will say more about this argument later in this chapter). Since we cannot use demonstration, says Beall, we must use description.

In a reaction to Beall, Otávio Bueno and Mark Colyvan argue that there is no reason to believe that Yablo's paradox is circular. They claim that descriptions are not the only way to refer to Yablo's sequence. Their arguments are somewhat similar to Sorensen's, so I will only go through the undiscussed arguments.

As we have seen above, Beall argues that we cannot refer to Yablo's paradox via demonstrations because any finite segment that we have access to underdetermines the complete infinite list. Bueno and Colyvan object to this that "if this were correct, we would not be able to refer, via demonstration, to *any* infinite sequences".[32] They argue that the standard description of the natural numbers involves both description and demonstration:

[30] Sorensen, "Yablo's Paradox and Kindred Infinite Liars", p. 145.
[31] Beall, "Is Yablo's paradox non-circular?", p. 179.
[32] Bueno and Colyvan, "Yablo's paradox and referring to infinite objects", p. 405.

(1) 0 is a natural number

(2) For every x, if x is a natural number, then so is $x + 1$

Although (2) is a descriptive clause, (1) is a demonstrative clause. As always in the inductive definition of a class, the base clause needs to be fulfilled by an object that is assigned as an element of this class.

Even though Bueno and Colyvan provide us with a good argument that fixing the reference of the infinite list of natural numbers necessarily needs demonstration, this does not change anything for our conclusion that the Yablo paradox necessarily needs description to fix the reference. The crucial point is that fixing the reference of the Yablo list necessarily needs description (and that the description necessarily involves self-reference). Proving that fixing the reference would *also* need demonstration does not change anything about the fact that it necessarily needs description. Therefore, this argument of Bueno and Colyvan did not achieve to prove that the Yablo paradox is not self-referential.[33]

Basically, I agree with Priest, and Beall that no-one can point at an infinite queue while Sorensen and Bueno and Colyvan believe that some beings can. For Sorensen and Bueno and Colyvan it is enough that we know by demonstration how an infinite sequence most plausibly continues. I will now give my reasons why I believe that this is not enough.

Suppose we try to demonstrate the set of natural numbers to a computer by pointing at the first ten elements:

$$< 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots >$$

A human being would know by experience how the sequence continues, but a computer would not be able to understand. People understand how the sequence possibly continues, but you can never be sure. We could also be talking about the sequence that continues with:

$$< 10, 20, 30, 40, 50, \ldots >$$

or with:

$$< 10, 12, 14, 16, 18, 20 \ldots >$$

Basically, we could be talking about any sequence that begins with:

$$< 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots >$$

People use their experience to conclude that this sequence is referring to the infinite sequence of natural numbers. But since it is an infinite sequence, you can never be sure about the sequel. If we use demonstration to fix the reference of a sequence, we need to see the sequence in full in order to be completely sure

[33] Bueno and Colyvan do speak about this gap in their argument, but in my comprehension they did not manage to provide a good answer to this (p. 408).

which sequence we are referring to. Therefore we are unable to see an infinite sequence and we cannot use demonstration to fix the reference of the Yablo sequence.

Saul Kripke provides us with a well-known argument to support my sceptical observation. He does this in an article where he analyses Ludwig Wittgenstein's contribution to rules and private language.Saul Kripke, *Wittgenstein on Rules and Private Language*, Cambridge MA: Harvard University Press (1982). Wittgenstein states the rule-following paradox in his *Philosophical Investigations*: "This was our paradox: no course of action could be determined by a rule, because any course of action can be made out to accord with the rule".[34] Kripke outlines a situation that leads to this conclusion. Suppose that you have never added numbers greater than 50. Say that you are asked to add 68 to 57. We would expect you to apply the addition function as you have before and calculate that the correct answer is '125'. We could also ask a sceptic with the same experience to add 68 to 57. The sceptic believes that there is no fact about his past usage of the addition function that determines '125' as the right answer. He says that nothing justifies him in giving the answer '125' rather than another. Remember that he has never added numbers greater than 50 before. It would be consistent with his previous use of 'plus' that he actually meant it to mean the 'quus' function, defined as:

$$\text{if } x, y < 57 \text{ then } x \text{ quus } y = x + y$$
$$\text{if } x, y \geq 57, \text{ then } x \text{ quus } y = 5$$

The answers he gave when adding numbers up to 50 do not indicate whether he used the quus function or the plus function. There is no fact about the sceptic that determines that he ought to answer '125' rather than '5'. The same sceptic could be invited to complete the sequence

$$< 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots >$$

For him, the fact that the sequence uses the successor formula to the start of the sequence gives the sceptic no reason to believe that the sequence ought to follow the successor rule for the rest of the sequence. It would be just as correct to have it continue in any other way. This means that we cannot point an infinite sequence by demonstration, which was exactly what I wanted to demonstrate.

Objecting this conclusion, Sorensen argues that an infinite being, such as God, can point at an infinite sequence and that this proves that demonstration can fix the reference of infinite sequences. Sorensen writes that an infinite being could enumerate a denumerable list of sentences. "For example. God could write the first sentence during the first minute, the second after [the] following 30 seconds, the first in the following 15 seconds, and so on. By writing faster and faster, God could finish the sequence in two minutes."[35] God could write the

[34] Ludwig Wittgenstein as cited in "Wittgenstein on rules and private language",
[35] Sorensen, "Yablo's Paradox and Kindred Infinite Liars", p. 145.

Yablo sequence in the same way and then point at it. To Sorensen, this means that we know that the Yablo sequence is paradoxical for the infinite being and therefore paradoxical *simpliciter*. "Our use of a self-referential specification is merely a useful heuristic."[36] To people who believe in the existence of a God, this would be a strong argument. To people, like myself, who do not believe that a God exists, this argument is less convincing. I have troubles imagining an infinite being that writes sentences. What is an infinite being anyway? I find it remarkable that God is always used as an example of an infinite being. What are other examples of infinite beings? As long as non-believers are not provided with other examples of infinite beings, they will not be convinced of Sorensen's argument that an infinite being's ability to enumerate a denumerable list of sentences proves that the reference of the Yablo sequence can be fixed by demonstration.

One could also argue against Priest's second assumption as follows: take a finitely long line and point at it. Since any finitely long line consists of infinitely many *points*,[37] this way we can point at an infinite sequence of *points*.[38] Pointing at infinitely many *points* is however something else than pointing at an infinitely many Yablo sentences. We can point at a finitely long line, but we cannot point at every singular *point* of which the line exists. The *points* on a line are not discretely but densely ordered. Therefore we cannot point at, for example, the second *point* and the tenth *point*. On the opposite, we can point at every singular of the Yablo sentences. We cannot point at all of the *points* at the same time, but we can point at, for example, the second and the tenth sentence. The argument that an imaginary being with infinitely many arms could then point at the whole list at once has already been tackled above: there exists no creature with infinitely any arms that could point at all the sentences in the Yablo list. This should make the disanalogy of the Yablo sequence and a finitely long line obvious. One could then argue that in order to make the finitely long line analogous with the Yablo list we could name all the *points* that are on this line and then construct a list with these names. This way we can point at (the representation of) one of the *points* of the line. But now we have the same problem as we encountered before: we cannot point at an infinite list. Even though we can point at the finitely long line, we cannot point at the infinite representation of the *points* on this line.[39] So to conclude, pointing at a

[36] Sorensen, "Yablo's Paradox and Kindred Infinite Liars", p. 145.

[37] In order not to get confused by the use of the verb 'pointing at' and the noun 'point', I write the noun in italics.

[38] Thanks to Michael De for challenging me with this objection.

[39] In private communication, Michael De suggested to "identify some discretely ordered subset of the line, (supposing the points are constituted by the closed interval [0, 1], then e.g. 0.1, 0.2, 0.3, . . . , 0.010, 0.011, . . . , 0.00100, 0.00101, . . .) and then correspond to each member of the subset a Yablo sentence in the list. Then we could point to the Yablo list by pointing at the subset of the line, or perhaps just by pointing at the line as a whole while indicating the subset to be picked out amongst all the points." If we could identify a discretely ordered subset of the line, this would be a convincing argument. However we cannot do that and the fact that a line consists of *infinitely* many *points* proves this. We can zoom in on any part of the line *infinitely* many times and thus we will never be able to separate two distinct

finitely long line is irrelevant to the argument that we cannot point at the Yablo sequence.

Also, one might argue that we can 'see' an infinite sequence with a mental eye. According to this argument, it would be sufficient to fix the reference of the Yablo paradox by demonstration in our minds. Like Beall, who first came up with this objection,[40] I do not have knockdown arguments against this argument. More must be said about this in order for me to be convinced that we can see with a mental eye. To me, an infinite sequence within one's mind (actually everything that is in one's mind) is an ambiguous thing to speak about, while we are looking for an unambiguous way of talking about a specific infinite sequence. For now, this argument leads to even more uncertainty.

There is no way for us humans to fix the reference of an infinite sequence by demonstration, nor are there other beings who can fix the reference by demonstration. Therefore I conclude that demonstration cannot fix the reference of the Yablo sequence. Since we know of only two ways to fix the reference, we must use description to fix the reference. If, of course, anyone can think of another way of fixing the reference than description or demonstration, this would be another alternative for demonstration. Until then, I conclude that the Yablo sequence necessarily needs description to fix the reference.

3.3 The description of the Yablo list necessarily makes use of self-reference (assumption ii.)

In this section I will discuss Bueno and Colyvan's objection to the claim that the description of the Yablo list necessarily makes use of self-reference. Recall Priest's argument for satisfaction in paragraph 3.1. Priest argues that we are not allowed to use T-schema for justification of the Yablo formula, because there are free variables in it. Priest then demonstrates that satisfaction exposes a fixed point, which is a sign that the paradox is self-referential. Sorensen does admit that "if a finite being wishes to describe which sequence is my demonstratum, then his description must be recursive and so self-referential in a sense."[41] With the word 'demonstratum' Sorensen means 'the object we refer to'. Thus Sorensen accepts the fact that the description of Yablo's infinite sequence must be self-referential.

In *Paradox without Satisfaction* Bueno and Colyvan think otherwise. They show how to rewrite the Yablo sentences to avoid the use of free variables in order for us to use T-schema correctly. If that works, they believe, this would mean that we would not need satisfaction and hence that the Yablo paradox is not circular. In this paragraph I will first discuss Bueno and Colyvan's article on

points. To conclude: we cannot identify a discretely ordered subset of the line so neither can we match them up with the Yablo sentences.

[40] Beall, "Is Yablo's paradox non-circular?", p. 185-186.

[41] Sorensen, "Yablo's Paradox and Kindred Infinite Liars", p. 145.

this and then describe how their specification of the sequence leads to a weaker kind of inconsistency than the Liar paradox.

Bueno and Colyvan reformulate the Yablo sentences without a satisfaction relation, because "if a contradiction can be established from the Yablo list *without* invoking such a relation, there's no fixed point, and so Priest's argument is blocked."[42] Bueno and Colyvan succeed in avoiding free variables by using only specific numbers. In this way they derive to a contradiction from the truth or untruth of a particular sentence and then they prove that the whole sequence is paradoxical. This way, the use of T-schema is legitimate. The specific sentence they used is arbitrary, but this arbitrariness is not used to derive to the contradiction. Bueno and Colyvan's argument to contradiction does, contrarily to Priest, not use universal generalisation. Their argument goes as follows:[43]

> Consider 'S_1' in the Yablo list. Suppose 'S_1' is true [...].
> $$\begin{aligned} T(S_1) &\implies \forall k > 1 : \neg T(S_k) \\ &\implies \neg T(S_2) \end{aligned}$$
> But,
> $$\begin{aligned} T(S_1) &\implies \forall k > 1 : \neg T(S_k) \\ &\implies \forall k > 2 : \neg T(S_k) \\ &\implies T(S_2) \end{aligned}$$
> So, given that '$T(S1)$' entails a contradiction, $\neg T(S_1)$. This means that there is at least one true sentence in the Yablo list. Let the first such sentence be 'S_i'. (Note that 'i' is not a variable, but an unknown, particular natural number.) Now consider 'S_i'.
> $$\begin{aligned} T(S_i) &\implies \forall k > i, \neg T(S_k) \\ &\implies \forall k > i+1, \neg T(S_k) \\ &\implies T(S_{i+1}) \end{aligned}$$
> Thus, a contradiction can be derived from the truth or untruth of a particular sentence, 'S_1', in the Yablo list.[44]

This way, Bueno and Colyvan show how to derive to a contradiction without the use of satisfaction and a fixed point. The question is whether their contradiction implies the same sort of inconsistency as the contradiction that we obtain when we do use satisfaction.

One year later, Ketland wrote an article objecting to Bueno and Colyvan's method. In this article Ketland claims that this method does not lead to genuine inconsistency but to omega-inconsistency. This type of inconsistency occurs in paradoxes where $A(t)$ for every term t of the language, while there is also an x for which $\neg A(x)$. This means that in Bueno and Colyvan's method the

[42] Bueno and Colyvan, "Paradox without satisfaction", p. 153.
[43] I choose to maintain my notation, while Bueno and Colyvan use a slightly different notation.
[44] Bueno and Colyvan, "Paradox without satisfaction", p. 155.

T-schema proves S_0, S_1, S_2, and so on (that is, for every natural number n, the T-schema proves that S_n holds), but that there is some natural number n such that S_n fails. Omega-inconsistency is weaker than 'normal' inconsistency, because it may not be able to prove for any specific value of n that S_n fails.

Ketland was not the first one to call the Yablo paradox omega-inconsistent. Two years after Yablo published his paradox, James Hardy argued that the Yablo sentences are only omega-inconsistent while the traditional Liar sentence leads to 'negation-inconsistency'.[45] "If there were a negation-inconsistency, then, by compactness, that inconsistency would be confinable to a finite set of sentences," says Hardy.[46] I will now explain that this claim is true.

The compactness theorem states that a set of first-order sentences has a model if and only if every finite subset of it has a model. To be able to apply this theorem to the Yablo paradox, we would have to check the following two claims:

(a) The set of Yablo sentences is a set of first order sentences

(b) Every finite subset of the set of Yablo sentences has a model

We can translate the Yablo sentences to L_T:

$$\forall n : S_n \iff \forall k > n : \neg T(\ulcorner S_k \urcorner)$$

Thus we can verify claim (a).

We now have to show that no finite subset of the Yablo sentences is inconsistent. Take a random finite subset of the Yablo sequence:

$$S_n, S_{n+1}, S_{n+2}, \ldots, S_m$$

Since there are no $k > m$, S_m is vacuously true. A predicate A is vacuously true if A states something about all x while there are no x. Now we have for $S_n, S_{n+1}, S_{n+2}, \ldots, S_{m-1}$ that there is a $S_{k>n}$ that is true, making $S_n, S_{n+1}, S_{n+2}, \ldots, S_{m-1}$ untrue. This subset appears not to be inconsistent. Every finite subset has a last element S_m, so this accounts for every finite subset of the Yablo sequence. So, claim (b) is also verified.

Summing up the conclusions of (a) and (b) and adding them to the compactness theorem, we can conclude that the set of Yablo sentences has a model and hence that the Yablo sequence is consistent (of this kind).

We do get an omega-inconsistency in Yablo's paradox. As Hardy puts it:

> Suppose that each S_n is false. The falsity of S_1 implies that there is some n such that S_n is true, but it does not require that any particular S_n be true. There is no inconsistency between the falsity

[45] 'Negation-inconsistency' is a genuine inconsistency.

[46] Hardy, "Is Yablo's paradox Liar-like?", p. 198.

> of S_1 and the falsity of any other finite set of S_n. The inconsistency on which Yablo's paradox feeds is between 'for some $n > 1$, S_n is true', which is implied by the supposed falsity of S_1, and 'for all $n > 1$, S_n is untrue' which seems to be a consequence of the fact that each S_n is false. But 'for all $n > 1$, S_n is untrue' does not follow from the falsity of the various S_n without the further assumption of omega-completeness. So Yablo's paradox is based on an omega-inconsistency, whereas the traditional Liar is based on a negation-inconsistency.[47]

This is a very clear representation of the point Ketland makes in his article.[48] We see that the Yablo list is inconsistent of a weaker kind than the Liar sentence is. While the Liar paradox is genuinely inconsistent (or as Hardy calls it 'negation-inconsistency'), the Yablo paradox is omega-inconsistent.

We saw that we can define the Yablo predicate (the generalisation of all the sentences in the Yablo list) in two ways: with the truth-predicate and with satisfaction. While the latter leads only to omega-inconsistency, the former leads to negation-inconsistency. Is there another way to define the Yablo predicate in terms of truth, without ending up with only omega-inconsistency? Ketland argues that there is, but that this specification will then involve self-reference. This is because the notions of truth and satisfaction are interdefinable, says Ketland.[49] We have already seen that Priest defines truth in terms of satisfaction, exposing self-reference. In his paper, Ketland defines satisfaction in terms of truth: "A number x *satisfies* a 1-place predicate y if and only if the result of substituting the *numeral* for x for any free variable in y is *true*."[50] The Yablo formula S_n can then be expressed as follows: for any number $k > n$, the result of substituting the numeral for k for any free variable in S_n is not true. This could be an alternative way to express the Yablo predicate, that is different from the original formulation of the Yablo predicate that is also expressed in terms of truth. However when we do this, argues Ketland, the Yablo formula S_n is still a fixed point of a predicate, namely the predicate 'for any $y > x, z(num(y))$ is not true'. So by completely avoiding free variables, as Bueno and Colyvan did, we get omega-inconsistency and by defining satisfaction in terms of truth, we get self-reference.

[47] Hardy, "Is Yablo's paradox Liar-like?", p. 198.
[48] For more details on this argument, see Ketland, "Bueno and Colyvan on Yablo's paradox", pp. 165-172.
[49] As long as we have a system of naming for objects, which we have.
[50] Ketland, "Bueno and Colyvan on Yablo's Paradox", p. 168.

Sorensen argues that the disanalogy between the inconsistency of the Liar and this specification of the Yablo can be weakened by the construction of a standard Liar that is obviously self-referential but only omega-inconsistent:

1. There is a falsehood on this list.
2. $2 = 2$

...

n. $n = n$

In order "to get the contradiction that the first statement is true if and only if it is not true, we would need to show that the successors of the first statement are all true. Thus this infinite Liar requires an omega-rule just as much as Yablo's paradox".[51] Hardy's point is not touched by this example, since Hardy never claims that every version of the Liar is negation-inconsistent. Sorensen's example is just another paradox that can be classified as an omega-paradox.

Roy T. Cook provides a new perspective on the discussion about the Yablo paradox. Amongst some other claims does Cook claim that any predicate $\phi(x)$ is a weak predicate fixed point of some binary predicate in the language.

> Given a particular theory Th in a language L:
> Φ is a *weak sentential fixed point* of Th iff there is a unary relation $\Sigma(x)$ in L such that "$\Phi f_{\Sigma}(\Phi)$" is a theorem of Th.[52]

What Cook seems to overlook in his paper, is that Priest (and with him Beall) argues that the *only* way to fix the reference of the Yablo paradox is by this relation (satisfaction) that involves fixed point. Instead, Cook writes that there is *a* relation in L.

To conclude, Ketland proved in a reply to Bueno and Colyvan the same as Hardy did almost ten years earlier: that by using T-schema the best we can get is omega-inconsistency. To derive to a genuine contradiction we need a description that implies self-reference. We can avoid self-reference in the Yablo paradox by not using satisfaction, but then we can only get omega-inconsistency. Therefore I conclude that it is true that the description of the Yablo paradox necessarily leads to self-reference.

3.4 A referent that can only be referred to by a circular description must itself be circular (assumption iii.)

We have come to the third and last assumption that Priest has made. This is probably the most difficult assumption to defend. As we have seen in the previous paragraph, Priest proved that the description of the Yablo paradox is self-referential. There are philosophers who agree with Priest on this, while it is

[51] Sorensen, "Yablo's Paradox and Kindred Infinite Liars", p. 143.

[52] Roy T. Cook, "There *Are* Non-circular Paradoxes (But Yablo's Isn't One of Them!)", p. 128.

not enough for them to accept the fact that the Yablo paradox *itself* is also self-referential. The sequence alone does not give us reason to call it self-referential, since no sentence in the sequence ever refers to itself or to any sentence above it. In this section I will address the discussion whether properties of the descriptions are transferred to the referent.

Before I will discuss what various authors have written about this, it is important to have clearly in mind what component of the Yablo paradox exactly everyone claims to be self-referential. We can distinguish the Yablo sentences, the Yablo predicate and the Yablo list/sequence. They all relate closely to the Yablo paradox, but we must not forget the subtle differences between them. The Yablo paradox is the contradiction that follows from combining infinitely many Yablo sentences and the truth-biconditionals. A Yablo sentence is a sentence on the list, say the fifth, that says: 'for all $k > 5$, S_k is untrue (where '5' can be replaces by any n.). All of the sentences together form the Yablo list/sequence. The Yablo predicate is the generalisation of the Yablo list/sequence and can be seen as a description of the Yablo paradox. Priest proved that this predicate is a fixed point for the satisfaction predicate and thus self-referential (in similar way that the Liar sentence is a fixed point for the truth predicate). The question now is whether this means that the Yablo paradox is also circular.

To start with, Beall tries to explain why the fact that the Yablo sequence is circular, follows from the fact that any description of the sequence is circular:

> From here, however, it is a small step to the circularity of the sequence itself. [...] The situation, however, is this: that the satisfaction conditions of our available reference-fixing descriptions require a circular satisfier – a sequence that involves circularity, self-reference, a fixed point. Given all this, it follows that the reference of 'Yablo's paradox' is circular. [...] The upshot of this is that, unless we find some other way of fixing the reference of 'Yablo's paradox', we are stuck fixing it on a circular sequence – a sequence containing fixed points, self-reference, etc.[53]

Beall seems to claim that since we can only describe the Yablo sequence with the help of fixed points, the sequence *contains* a fixed point and is therefore self-referential. I find this explanation not very convincing, because it does not give reasons to believe this. Sorensen and Bueno and Colyvan try to counter this claim by giving counterexamples.

Sorensen lets us consider the description 'any sentence with fewer words than this very sentence'. This description is circular because it is referring to itself. This circular description does indeed denote non-circular expressions, e.g. 'the cat is on the mat'.[54] Hence Sorensen correctly claims that 'a self-referential description does not guarantee that the referent is self-referential'. The problem

[53]Beall, "Is Yablo's paradox non-circular?", p. 180.
[54]Sorensen, "Yablo's Paradox and Kindred Infinite Liars", p. 148.

with Sorensen's counterexample is that it would have to prove that his description is the *only* way to describe the non-circular object. In the case of Sorensen, we can easily prove that there are also non-circular descriptions to the sentence 'the cat is on the mat'. For example we can describe 'the cat is on the mat' with the non-self-referential sentence 'write 'cat' for 'dog' in the sentence 'the dog is on the mat' '. Sorensen does not give a counterexample where the description of a non-circular sentence necessarily implies circularity, so he fails to counter Priest.

Bueno and Colyvan give it another try. They provide a counterexample of a non-circular object, the last upper bound s of a subset σ of real numbers, whose description is circular: "the least upper bound s of a subset σ of real numbers is the smallest number that is larger than every member of σ."[55] They say that there are non-algebraic real numbers to which we can only refer to by the definition of the least upper bound (and not by demonstration). Bueno and Colyvan's counterexample is relevant, contrarily to Sorensen's, because they claim that the *only* way to denote these non-algebraic real numbers is via a circular description.

But their counterexample does not quite feel the same as the Yablo paradox or the Liar paradox. I will try to explain why we cannot conclude anything about the Yablo paradox by analysing the least upper bound. To see whether Bueno and Colyvan's counterexample hits the target, we must compare the Yablo sequence and the least upper bound. Recall that we can only denote both objects by a circular description. We can see the least upper bound as a sequence that consists of infinitely many numbers behind the comma.[56] These numbers (call them *numbers) are however unequal to the Yablo sentences. Each *number is part of the specification of the least upper bound we are considering, but each *number says nothing about other elements of the least upper bound, while each Yablo sentence expresses something about the subsequent sentences. This is an important difference, because reference is after all the main subject of debate in this discussion. To sum up: because the elements of the least upper bound have a different content than the elements of the Yablo sequence (respectively *numbers and Yablo sentences), the least upper bound and the Yablo sequence are disanalogous. Furthermore, because the objects we are comparing are disanalogous, we cannot conclude anything about the one (the Yablo sequence) by looking at the other (the least upper bound). Therefore, Bueno and Colyvan's counterexample does not give us reason to believe that the Yablo paradox is not self-referential.

So no-one succeeded in giving a good reason why circularity in the only description means that the referent is also circular. Until someone will come up with a good counterexample that is comparable to the Yablo paradox, I conclude for the time being that indeed a referent that can only be referred to by a circular

[55] Bueno and Colyvan, "Yablo's paradox and referring to infinite objects", p. 409.

[56] The relevant least upper bounds have infinitely many numbers behind the comma.

description must itself be circular.

3.5 Conclusion on self-reference)

Now we have closely looked at the three assumptions that Priest made when objecting to Yablo, we can decide whether the Yablo paradox is self-referential or not.

Firstly, Priest claims that we can only speak about Yablo's paradox via a description. Opponents of this claim must either give a convincing example of an infinite being or find a third method for fixing the reference (besides demonstration and description). Until anyone comes up with a good argument objecting to Priest's claim, we must conclude that description is the only way to fix the reference of the Yablo sequence.

Secondly, Priest has shown how a description of the Yablo paradox will always express self-reference. Bueno and Colyvan tried to avoid self-reference by using T-schema for the argument to contradiction. This way they avoid the fixed point, but since this will only lead to omega-inconsistency they changed the Yablo paradox into weaker kind of paradox than the Liar paradox is. Thus I conclude that Priest was right in claiming that a description of the Yablo paradox always expresses self-reference. Together with the conclusion to the first assumption, I claim that fixing the reference necessarily implies self-reference.

Thirdly, Priest claims that self-reference in the description implies self-reference in the object. I cannot say that there exists no proper counterexample. I can only say that so far, no one has presented a good counterexample. We need a counterexample of a non-circular object, that is similar in most ways to the Yablo sequence, whose description is always circular, in the same way as the Yablo paradox is. Since no one has presented such a counterexample, Priest's argument (that self-reference in the description does imply self-reference in the object) is most plausible at the moment. Summing up the conclusions to the discussions on Priest's three assumptions, I conclude that the Yablo paradox is self-referential.

Conclusion

The ongoing debate about Yablo's paradox gives us a lot to think about. Yablo originally introduced his paradox to challenge the belief that self-reference is essential to liar-like paradoxes. After looking closely at the specification of the paradox, I concluded that Yablo did not succeed in challenging this claim. His paradox is, just like the Liar paradox, self-referential. This means that the Yablo paradox is less helpful to finding solutions to logical paradoxes than Yablo hoped. His paradox is just another self-referential semantic paradox. Hereby I do not say that Yablo's paradox is not helpful at al. The paradox is simply another version of the Liar paradox, proving once again that these paradoxes are not an easy target to solve.

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