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Multiple stellar generations in globular clusters

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1 Introduction

Globular clusters are spherical shaped objects which consist of a dense distribution of thousands of stars and which orbit the centers of galaxies. Unlike the stars in the disk of a galaxy, globular clusters are found in the halos around galaxies. The Milky Way hosts approximately 200 globular clusters, from which ω Centauri is the brigthest and largest. It is commonly believed that most globular clusters consist of a single generation of stars, which are mostly very old and have low metallicity (these stars are called population II stars). Therefore some clusters possibly have even higher ages than galaxies themself. The common method of measuring the age of a globular cluster is by fitting isochrones to the colour-magnitude diagram (CMD) of the cluster. A colour-magnitude diagram is a diagram which plots the colour (for example $V - I$) versus the magnitude in a certain filter (for example V) for all observed stars in a cluster. See for example figure 1, which is a CMD of the globular cluster NGC 1705-1.

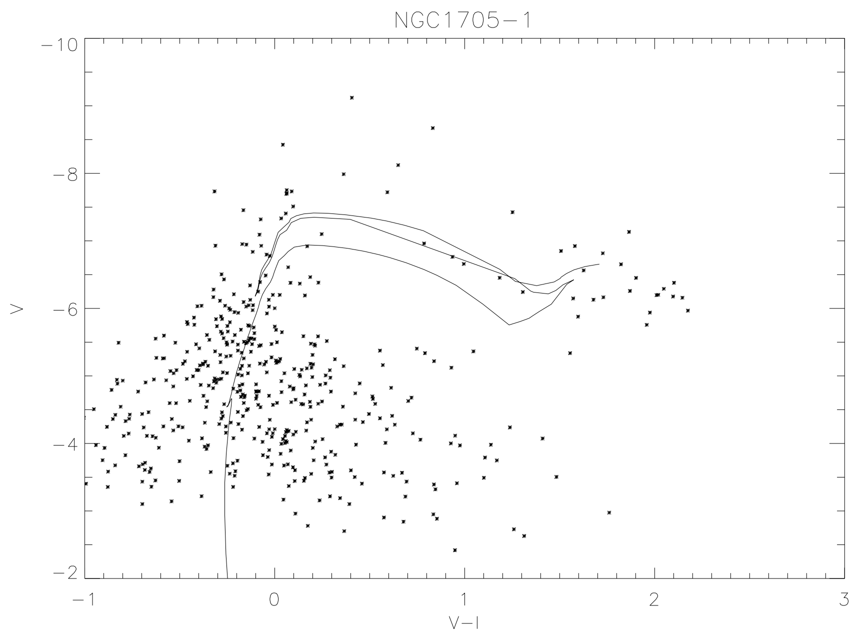

Figure 1: CMD for NGC 1705-1 with fitted isochrone for log t/yr = 7.1 and Z=0.008.

Isochrones in a CMD are based on isochrone tables which provide stellar properties like absolute magnitudes and abundancies for different masses with the same age (hence 'isochrone', which means 'same age' in Greek). One of the mostly used isochrone tables are calculated by the Padua Observatory (Marigo et al. 2008), these are also used in this paper. Isochrones can be used to illustrate some major features from the theory of stellar evolution. The main sequence can be seen as a vertical branch near $V - I = 0$ and also the turnoff point can be seen, where stars start the nuclear fusion of helium and heavier elements and become giants or supergiants. Another prediction from the isochrone CMDs is that clusters

of stars will become fainter as they age, this is because of extinguishing massive stars.
An important input-parameter for isochrones is Z, which represents the metallicity of the stars. Traditionally in astronomy X is used for the proportion by mass of hydrogen in a star at age zero, Y for the proportion of helium and Z for the proportion of heavier elements than helium (which are called metals in astronomy, hence metallicity). The metallicity of 0.019 is that of the sun. Older stars have a lower metallicity, because metals are formed in the cores of stars and then spread in the universe by stellar winds and (mostly) supernovae, so the proportion of metals in the gas clouds from which stars are formed generally increases with the age of the universe. It should be noted that this actually is only correct for the average metallicity, because there are many possible causes which could increase or decrease the local metallicity, for example galactic winds which blow the newly formed metals away or metal-poor gas that falls into galaxies or clusters.

It is easy to fit an isochrone to the CMD data and therefore estimate a clusters age, an example again is figure 1. There are however some complications in this method. The first one is that stellar models need a metallicity as an input to calculate isochrones, so it is necessary to make an accurate observation of the metallicity. A second complication is that some isochrone fits are quite poor, due to the following characteristics of observed CMDs, which cannot be explained by observational errors: observed stars in the "Blue Hertzsprung gap" of the CMD, where according to stellar evolution models none should appear, a larger spread in luminosities of supergiant stars and supergiants which are redder than predicted (Larsen et al. 2011). These complications have led to the suggestion that not all the stars in a globular cluster are formed at the same time, so that there possibly are multiple generations of stars in globular clusters. Multiple generations of stars in globular clusters would explain the complications in the colour-magnitude diagrams, because they could lead to multiple main sequences, a split sub-giant branch and multiple red giant branches (Milone et al. 2010). The first time that multiple generations of stars in a cluster were detected was in 1997 in ω Centauri. ω Centauri actually is a rather extreme case, because it is believed that the cluster is a remnant dwarf galaxy which collided with our Galaxy and therefore it isn't a good representation of common globular clusters. So far, multiple generations of stars have been detected in globular clusters by the following methods (Milone et al. 2010):

Light-elements correlations
Spectroscopic analysis can study the abundance of light elements like sodium and oxygen. In several clusters different ratios between these two elements can observed, where stars from a second generation have higher Na and lower O abundances. The two populations with anti-correlated light-elements result in different red giant branches in the colour to magnitude diagram. What has happened is that massive stars from the old population have enriched the interstellar gas in wich the young population is born. Multiple populations have been detected in NGC 6121 with this method (Marino et al. 2008).

Multiple main sequences
In accurate CMDs multiple main sequences can be detected. This has been done for NGC

2808 (Piotto et al. 2007), where three different main sequences can be observed. Multiple main sequences correspond to multiple generations with different helium contents and an age spread between them.

Spread or split sub-giant branches

The sub-giant branch is an important feature of the CMD and accurate photometry can show signs of multiple generations in a cluster in the sub-giant branches. Research on the clusters NGC 1851 and NGC 6656, where actually split sub-giant branches are observed, has shown that these spread or split sub-giant branches can be explained by multiple populations in a cluster with slightly different metallicities and ages.

Multiple red giant branches

Model isochrones like the one in figure 1 show that a population of stars which consists of one generation has a very well defined red giant branch in the CMD, where the red giants are observed. In several CMDs multiple red giant branches can easily be observed, which can be explained by multiple populations of stars in the cluster.

Of course the previous methods are very correlated and most of the time a CMD of a globular cluster which consists of multiple generations shows several characteristics at the same time. Almost all the methods have in common that most information comes from the colour to magnitude diagram of a large sample of stars in a cluster. This brings however one important difficulty, namely that only clusters can be studied for which (partially) resolved photometry of individual stars is available. Even using the Hubble Space Telescope (HST) it is very hard to do this in galaxies beyond the Local Group. This is a pity because by increasing the search volume it is possible to get data from several young, massive globular clusters which provide better statistics than most of the (relatively) small globular clusters which can be observed already (Larsen et al. 2011). Larger telescopes in space, like the James Webb Space Telescope (to be launched in 2018), would solve this problem, but a less expensive way would be to study photometry of clusters as a whole: integrated photometry.

The goal of this paper is to see whether it is possible to find clues for multiple generations of stars in globular clusters by using integrated photometry. To do this, two different models of clusters with multiple generations of stars are studied. The first model calculates the integrated colours of a cluster with two different generations of stars with a typical age spread according to previous studies. It then looks how those colours evolve in time, so a colour versus cluster-age diagram is made. The model shows how a possible young generation of stars influences this diagram. The second model simulates a cluster which consists of two different populations of stars. By assumption the youngest population is centered more than the old population. Then the integrated colours for stars in several concentric rings with different distances to the center of the cluster are calculated and plotted in a colour versus distance to the center diagram. In this diagram the influence of the young population in the center can be studied. Finally both models are compared with data mainly from the Hubble Space Telescope.

The relevance of research on star clusters in astronomy is quite significant. Long it has been thought that globular star clusters are simple stellar populations as all stars were thought to have the same age. Because they are simple stellar populations, star clusters are very suitable to test stellar evolution models, mainly by comparing observed CMD's with modeled ones. Finding multiple generations of stars in clusters changes the view on clusters and this could lead to more constraints on stellar evolution models. Therefore a better understanding of globular clusters leads to a better understanding of stellar evolution and this is, of course, one of the main objectives of whole astrophysics.

2 Model 1: Colour evolution

By definition a model is a simplification of reality, but still we are able to learn major properties of the system. It is however important to note all the assumptions which are made in the model and to justify them. Because star clusters are huge objects seen from even larger distances we can assume the stars in the clusters having all the same distance, also because the variations in their distances probably is much smaller than the systematic errors in the measurement of the distance of the cluster.

This model doesn't take the number of stars in a cluster into account, therefore the integrated magnitudes will just give a qualitative description and not a quantitative one. This simplification however isn't very troubling, because it is not necessary to know the number of stars in a cluster for certain observations. The reason for this is that the observed parameter used ($B - V$) is not a function of the total number of stars in a cluster. The magnitudes in B and V do depend on the number of stars, but $B - V$ doesn't, this can be seen by the following derivation:

$$L_\lambda = 4\pi r^2 N F_\lambda \tag{1}$$

Where L_λ is the luminosity in a certain colour, r is the distance to the cluster (with the assumption that all stars have the same distance), N is the number of stars in a cluster and F is the mean flux of a cluster.

$$m_\lambda \sim \log L_\lambda \tag{2}$$

With m_λ the absolute magnitude in a certain color (in this paper mostly absolute magnitude will be meant with magnitude, unless mentioned otherwise). Note there is a $\sim$ sign there in stead of a $=$ sign. This is because magnitudes mostly are given relative to a reference magnitude. Now B is by definition equal to m_{λ_B} and so on, so we can write $B - V$ (and $V - I$ likewise) as:

$$B - V = \log(4\pi r^2 N F_B) - \log(4\pi r^2 N F_V) \tag{3}$$

Which equals:

$$B - V = \log \frac{F_B}{F_V} \tag{4}$$

Now it is easy to see that the colours $B - V$, $V - I$ etc. do not depend on the number of stars (and not on the distance either, if we assume no extinction, which could be distance-dependent).

Figures 2 and 4 actually plot seperate U, B, V and I magnitudes as a function of the age of the modelled cluster, but this diagram is not used quantitatively and the behavior of the plots doesn't depend on the number of stars, it only shifts it upward or downward. It is however possible to calculate the number of stars in a cluster with use of the measured total mass in combination with an initial mass function, or by using a mass-luminosity relation.

Another assumption made is that there is no interstellar extinction of the light and that there is no interstellar dust between the star clusters and earth.

These are the major simplifications and assumptions in the modeling of the star clusters, other assumptions which are made specific for one calculation will be mentioned when necessary.

2.1 Single generation model

The main goal of the single generation star cluster model is to create diagrams which show the behavior of the integrated magnitude by varying the clusters age for constant metallicity. While most star clusters aren't fully resolved or even totally unresolved the integrated magnitude is still a property which can be observed by our telescopes, therefore it is important to compute integrated magnitudes with the data provided by the Padua isochrones. To compute the integrated magnitudes there are several steps to be taken:

1. convert the magnitudes to luminosities
2. use a normalized stellar initial mass function (stellar IMF) to make sure the masses have correct weights
3. use a summation to compute the integrated luminosity

2.1.1 Converting magnitudes to luminosities

The Padua data provides absolute magnitudes in the standard photometric system for stars with varying initial masses of a certain age. It is however not convenient to use magnitudes in calculations, because of their logarithmic nature. Therefore the following formula is used to convert magnitudes to luminosities:

$$m_\lambda = -2.5 \log \frac{L_\lambda}{L_0} \tag{5}$$

With m_λ the magnitude in a certain wavelength and $\frac{L_\lambda}{L_0}$ the luminosity in the certain wavelength relative to the absolute magnitude of the Sun. Normally the magnitude of Vega is used as reference, but because the Padua data contains absolute magnitudes in stead of apparent magnitudes, the magnitude of the Sun observed at distance of 10 pc must be used. Rewritten to $\frac{L}{L_0}$ as a function of the magnitude:

$$\frac{L_{star}}{L_0} = 10^{-m_\lambda/2.5} \tag{6}$$

2.1.2 Stellar Initial Mass Function

It is important to use an IMF to make sure that the number of stars with different masses are weighted correctly. An IMF describes how the mass of stars is distributed at the time of formation. The function is related to the fundamental process of star formation. Salpeter was the first one to define a simple powerlaw as initial mass function and it counts as the most basic IMF and therefore is most commonly used in astronomy. Salpeters function is not directly derived from theory, but was determined empirically by the study of stars in the Sun's neighborhood in 1955. The function can be used to determine the total number of stars between masses M_{min} and M_{max} with the following formula:

$$N = \int_{M_{min}}^{M_{max}} \xi(M) \ \mathrm{d}M \tag{7}$$

Where $\xi(M) = \xi_0 M^{-2.35}$ is the Salpeter IMF, with ξ_0 is a constant for the local stellar density. The function is not defined for very low mass stars, as the number of stars goes to infinity when the mass reaches low values. Observations showed that the function is valid for masses >0.5 $M_\odot$ (Kroupa, 2002).
It is easy to see that with a Salpeter IMF most of the stars are low mass stars, which contain most of the total mass, but that most of the luminosity comes from high mass stars. This is because of the well known mass-luminosity relation:

$$L \sim M^{3.5} \tag{8}$$

Salpeters IMF looks like $\xi(M) \sim M^{-2.35}$, so to be able to normalize it, the function will be defined as $\xi(M) = kM^{-2.35}$. With k a constant. The reason to normalize the Salpeter function is that the weight of the total mass equals one. This way it is easy to calculate k by saying:

$$k = \frac{1}{\int_{M_{min}}^{M_{max}} M^{-1.35} \ \mathrm{d}M} \tag{9}$$

2.1.3 Computation of integrated luminosity

The formula used to compute these integrated luminosities is:

$$L_\lambda = \int_{M_{min}}^{M_{max}} \xi(M) \ L_\lambda(M) \ \mathrm{d}M \tag{10}$$

(Larsen et al. 2011)

$L_\lambda(M)$ is the luminosity with a certain wavelenght λ for mass M, these are provided as magnitudes by the Padua isochrone data and were converted in the first step.
In this case there is no luminosity-formula which relates L_λ to M with infinitesimal steps, so the integral changes to a summation and de dM to ΔM.
The summation looks like this

$$\sum_{M_{min}}^{M_{max}} \xi(M) \ L_\lambda(M) \ \Delta M \tag{11}$$

With the Padua data and a ΔM which is small for the masses in the post main sequence phases (where the evolution is rapid, so $\frac{dL}{dM}$ is large), a program calculates integrated luminosities relative to the luminosity of the Sun at distance 10 pc. As a final step the luminosities are converted again to magnitudes, which makes comparing with observations and other models easier. Because the magnitudes have been converted to luminosities and now are being converted back to magnitudes, the reference terms discussed earlier drop out and therefore don't influence the calculations.

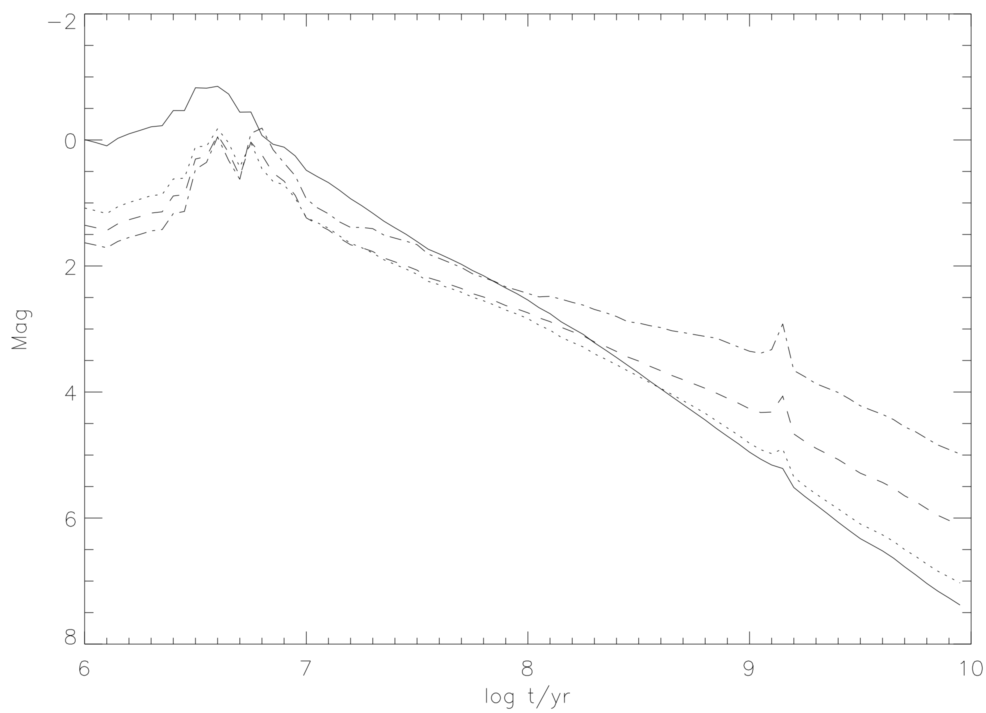

Figure 2: Time evolution of magnitudes of modelled cluster in different bands. U-band: normal line, B-band: dotted line, V-band: Striped line and I-band: dotted-striped line.

2.1.4 Single generation model diagrams

The three steps mentioned in the previous section have to be taken for every colour-band (in this case U, B, V and I) and for varying stellar ages to create the desired data-set. To do this a computerprogram was written in IDL (called INTMAG.pro), which is included in the appendix. With the created data we can qualitatively plot the evolution of magnitudes of a cluster in time and quantitatively plot how colors ($U - B$, $B - V$, $V - I$ and $U - I$) change in time. The created plots are shown and discussed in this section. They all have a metallicity of 0.008, which is typical for massive star clusters in the galaxies around which the observed clusters orbit (Larsen et al. 2011).

The prediction from the isochrone CMDs that star clusters become fainter as they age, can easily be seen in figure 2. Until a cluster has an age of approximately log t/yr = 7, the integrated magnitudes increase in all bands. This is due to the evolution of the stars on the main sequence. At the age of ten million years the most massive stars have reached the turn-over point in the Hertzsprung-Russell diagram and start the fusion of helium and heavier elements. They turn into bright red (super)giants, but won't have long to live. The integrated magnitude decreases as massive stars extinguish at the end of their lifes. So the effect of the extra brigthness a red supergiant has over when it was on the main sequence cancels out as more and more stars extinguish. The diagram plots the evolution of

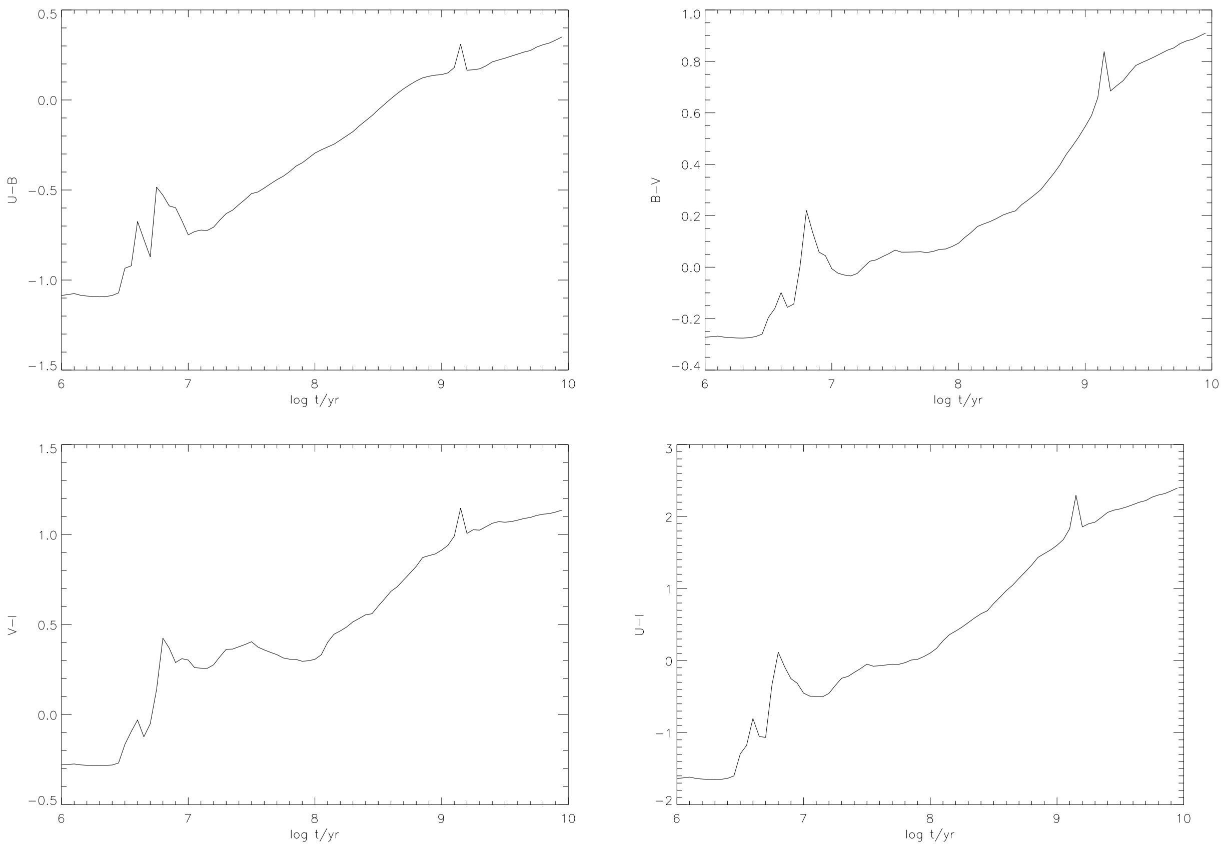

Figure 3: Top left: $U - B$ versus log t/yr. Top right: $B - V$ versus log t/yr. Bottom left: $V - I$ versus log t/yr. Bottom right:$U - I$ versus log t/yr.

magnitudes in different bands and it can be seen that the magnitude in the U and V band (blue stars) drops more and faster than the magnitude in for example the I band (which is the band where red giants emit most of their light). This has the same explanation as above: blue stars evolve into red stars.

Figure 3 shows how integrated colours change over the age of a single generation star cluster. These plots show actually the same observations already made from figure 2: all integrated colours become redder. They all have a local peak around the age of ten million years (log t/yr =7), which is the point where the first main sequence stars reach the turn-over point. The reason why this peak is so clearly visible is that approximately the heaviest few percent of the stars become red (super)giants very rapidly. It can be seen from the mass-luminosity relation (equation 8) that a relative large amount of the total luminosity of star clusters can be attributed to the heaviest stars. Therefore their evolution will have quite a significant impact on the integrated luminosity and therefore integrated colour of star clusters. This statements can also be seen more quantitatively by the following example. By making a colour versus mass diagram of the Padua data for different ages, it can be seen that there are no red giants at an age of one million years. For an age of log t/yr = 6.8, stars with masses higher than 29.41 M_{sun} are red giants (this can be seen in the colour versus mass diagram, because they are way redder than stars with a lower mass). Because according to the Padua isochrones stars with an age of log t/yr = 6.8 can maximally have a mass of 32.1495 M_{sun}, the percentage of red giants in a population can be calculated by using the Salpeter IMF. Once the Salpeter function is normalized to 1 and therefore ξ_0 is known, the fraction of red giants can be calculated with the following formula:

$$f_{rg} = \xi_0 \int_{29.4113}^{32.1495} M^{-2.35} \; \mathrm{d}M \tag{12}$$

From this it follows that f_{rg} at an age of log t/yr = 6.8 is equal to 0.009 %. The contribution of the red giants to the integrated luminosity can be calculated by changing the values for M_{min} and M_{max} in equation 11. It follows then that this small fraction of red giants contributes to almost 94 % of the total luminosity in the B-band.

The slope of all the diagrams in figure 3 is slightly declining before those local peaks, which can be explained as the evolution from stars on the main sequence (they become a bit more a blue). After the first 'reddening peak', which just has been discussed, the modelled star cluster becomes redder with a more constant slope, this slope however differs between the colours.

2.2 Multiple generation model

Once the model for a single generation star cluster is made, this will be a good starting point for the multiple generation model. A multiple generation model consist of a superposition of single generation models with different ages. It is assumed that the initial conditions for the two different generations are the same, i.e. same metallicity (Z = 0.008) and same initial mass function (Salpeter). This leaves the possibility to experiment with different metallicities, IMFs or different boundaries for the IMF later. A multiple generation star cluster model contains two extra parameters in comparison with the single generation model, namely the time at which the second generation is born and the fraction of the total mass which is in the second generation. The IDL-program which was written to calculate the integrated magnitudes for a star cluster with two generations (called MULTIGEN.pro) took the following steps:

1. use the data from single generation model to find integrated magnitudes for the two generations
2. transform these magnitudes to luminosities
3. adress correct weight to luminosities from different generations
4. add luminosities and change to magnitudes again

2.2.1 Finding correct integrated magnitudes for the generations

The program MULTIGEN.pro uses the integrated magnitude data which was created by INTMAG.pro. This data consists of a table which gives the integrated magnitudes in U, B, V and I for ages ranging from one million to 10 billion years. The major programming challenge for this model is to address correct integrated magnitudes to the ages of the second generation. Because the ages in the INTMAG-data are in logarithmic units, the first step is to transfer these into linear units, such that the age of the second generation can be calculated by:

$$t_{secgen} = t_{firstgen} - t_{burst} \tag{13}$$

where $t_{firstgen}$ is in linear units and t_{burst} is the age of the first generation when the second generation is born.

Now these ages from the second generation must be transfered back into logarithmic units. This results in an array which at low ages differs significantly from the array of the ages of the first generation, but almost looks the same for larges ages. Then the IDL procedure INTERPOL can be used to find the corresponding magnitudes in the four different filters, based on the INTMAG-data.

One limitation on the model here is that the steps in the first generation age-array are sometimes not small enough to make a distinction between the age of the first generation and second generation, while they actually are present. The INTERPOL procedure slightly corrects for this error, but because it is based on linear interpolation and the function of luminosity versus time is very probably not linear, it isn't a hundred percent right.

This procedure however gives some problems because it generates negative ages for the second generation when t_{burst} is larger than one million years. To solve this, luminosities for those ages will be set to zero by using an if-statement. Physically this is also very sensible, because the second generation hasn't been born yet. Finally two arrays will be created which contain the magnitudes in the four filters with corresponding cluster ages. These magnitudes are transfered to luminosities in the second step. The second step has already been done in a very similar way in creating the single generation model, so that step won't be discussed further here.

2.2.2 Addressing correct weight to luminosities

The third step involves the input parameter n, which is defined as the fraction of the total mass which is in the second generation. It is easy to see that the fraction of the total mass which is in the first generation equals $1 - n$. To find the corresponding luminosities for the first and second generation stars, the following formulae are used in MULTIGEN.pro:

$$L_{firstgen} = (1 - n)L_t \tag{14}$$

$$L_{secgen} = nL_t \tag{15}$$

Here L_t equals the cluster's luminosity at corresponding ages of the generations.

Now the weights of the luminosities are corrected for both generations, the luminosities can

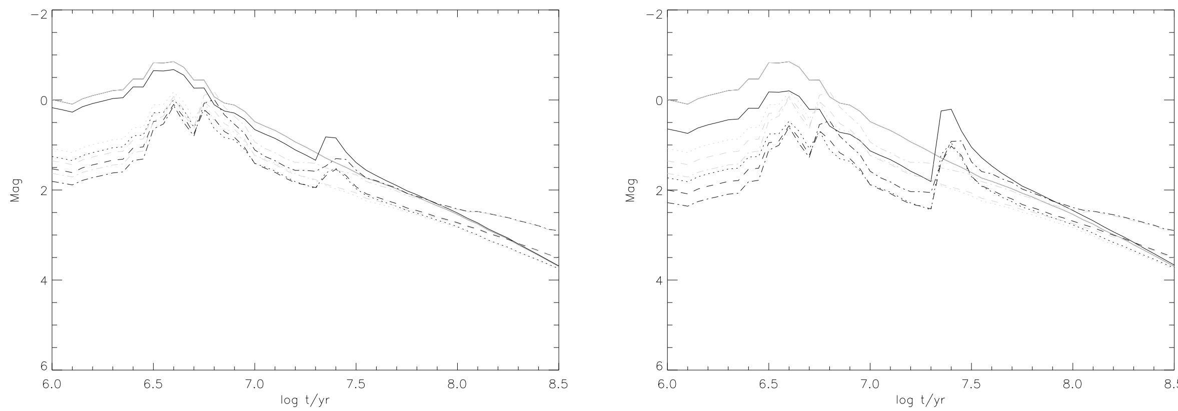

Figure 4: Time evolution of magnitudes of modelled multiple generation cluster in different bands. U-band: normal line, B-band: dotted line, V-band: Striped line and I-band: dotted-striped line. The results for the single generation model are printed in a lighter colour. Left: n = 0.15. Rigth: n = 0.45

be added up to receive the total luminosity from a star cluster which contains stars from two different generations. The final step to make the model-data ready for comparison with the single generation cluster is to change luminosities back to magnitudes. How to do this has been explained earlier.

In the above section it has been explained how to make a model for a star cluster which contains stars from two different generations. Of course it could be done for more generations in the same way, but for the moment two generations are satisfactory.

2.2.3 Multiple generation diagrams

To see how a second generation of stars in a cluster affects proporties as integrated magnitude and colors, diagrams of magnitudes versus age and colours versus age will be produced, as has been done for the single generation model before. Because there are two different input-parameters (tburst and n) a literature study was needed to find which one of them to vary and which one of them to set as a constant.
It seems that significant age spreads in clusters are between 10 - 30 Myr (Larsen et al. 2011) and 10 - 16 and 30 - 100 Myr (Vinko et al. 2009). There is also evidence that there are stars with ages of a few million years in the cluster NGC 1850 (which has an age of $\sim$ 50 Myr)(Fischer et al. 1993). Because of this information, tburst was set to 20 million years for the following diagrams. Parameter n will vary between 0.15 and 0.45, accordingly with previous studies (Larsen et al. 2011).
Just as has been done for the single generation model, the first diagram to study is the diagram which plots the integrated magnitudes versus the age. This has been done for varying n and here the results for $n = 0.15$ and $n = 0.45$ will be shown in figure 4.

In general the same observations as in figure 2 can be made from figure 4, there are however some important differences. The most important one is by far the extra peak around log t/yr = 7.4, which is the time were the second generation is born. Before this peak the curve of the multiple generations model is actually the same as the one in the single generation model, but then shifted a bit downward (this is because of formula 9). Because the parameter n is higher for the diagram below, this one has shifted downwards the most. The peak which occurs around log t/yr = 7.3 corresponds with the time difference of twenty million years between the birth of the first generation and the second. It is easy to see that the higher the fraction of the total mass which is in the second generation, the higher the peak is. The diagrams for $n = 0.25$ and $n = 0.35$ are therefore somewhere inbetween the presented diagrams and have no extra remarkable features. The peak is visible untill the cluster has an age of approximately one hundred million years, where the curves of the single generation model and the multiple generation model are almost indistinguishable.

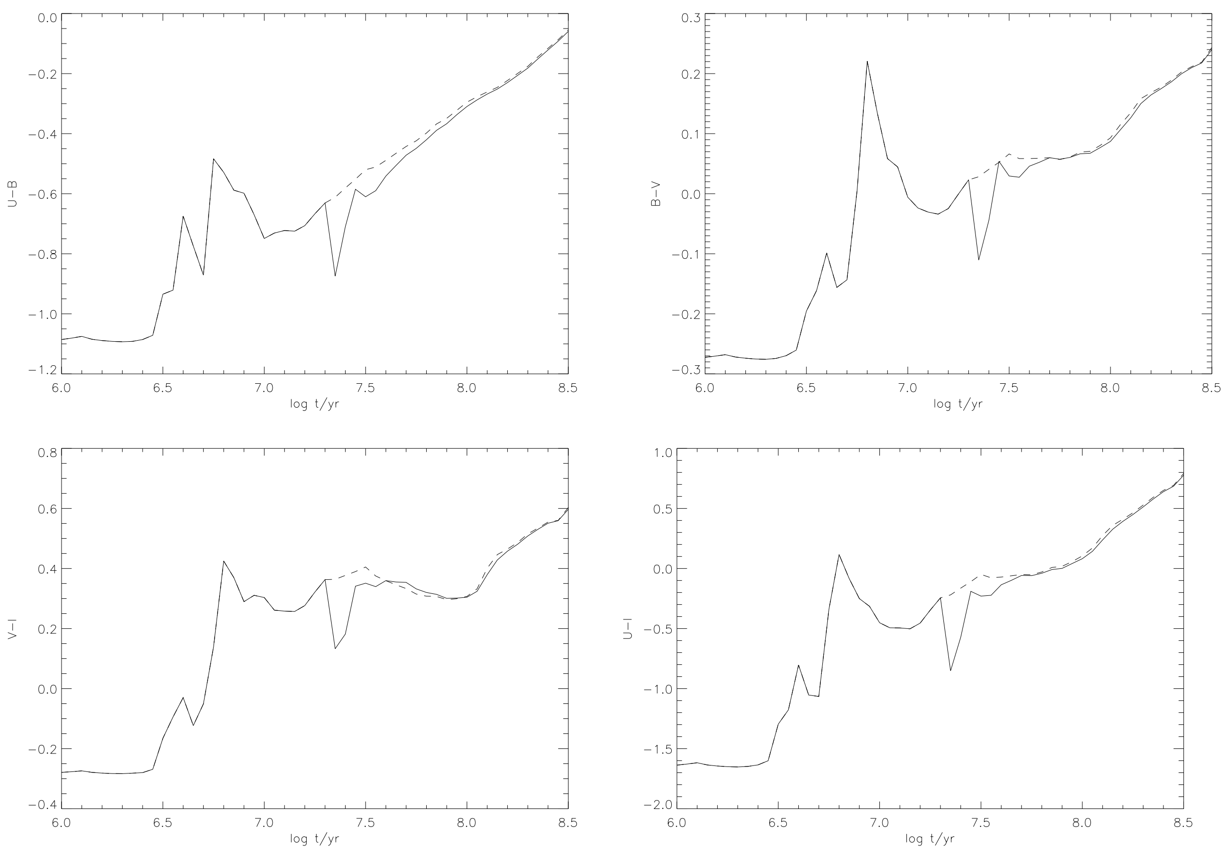

Figure 5: Top left: $U - B$ versus log t/yr. Top right: $B - V$ versus log t/yr. Bottom left: $V - I$ versus log t/yr. Bottom right:$U - I$ versus log t/yr. The dashed lines are the curves for the single generation model.

In figure 5 four different colour versus age diagrams are shown. Just as before tburst has been set to 20 million years. The weight parameter n is fixed on 0.25. The main features don't differ very much from figure 3, which shows the same diagrams for the single generation model. The curves from those plots are plotted in the multiple generation diagrams as

dashed lines, so it is easy to see the differences.
As in figure 3 the most significant feature is the reddening peak around log t/yr = 6.8. As explained earlier, this corresponds to the age where the most massive (and therefore most luminous) stars become red (super)giants. The most important and significant difference due to the second generation of stars is the downward pointing (bluening) peak around log t/yr = 7.4. This corresponds to the time at which the second generation is born and therefore it can be explained easily: in an instant there are relatively more main sequence stars in the cluster. Main sequence stars in the second generations have blue colours, especially compared to the stars of the first generation which are in a later evolutionary phase (mostly) as red giants. For a higher fraction of mass in the second generation the peak is even more significant. The presence of the downward point peak might be a succesful way to find evidence for multiple generations in star clusters, for example when the measured color is much more blue than predicted by the single generation model. However there could be other reasons for this, for example extinction or the presence of luminous blue supergiants. More about how extinction could influence the diagrams is discussed in the extinction section later in this paper.

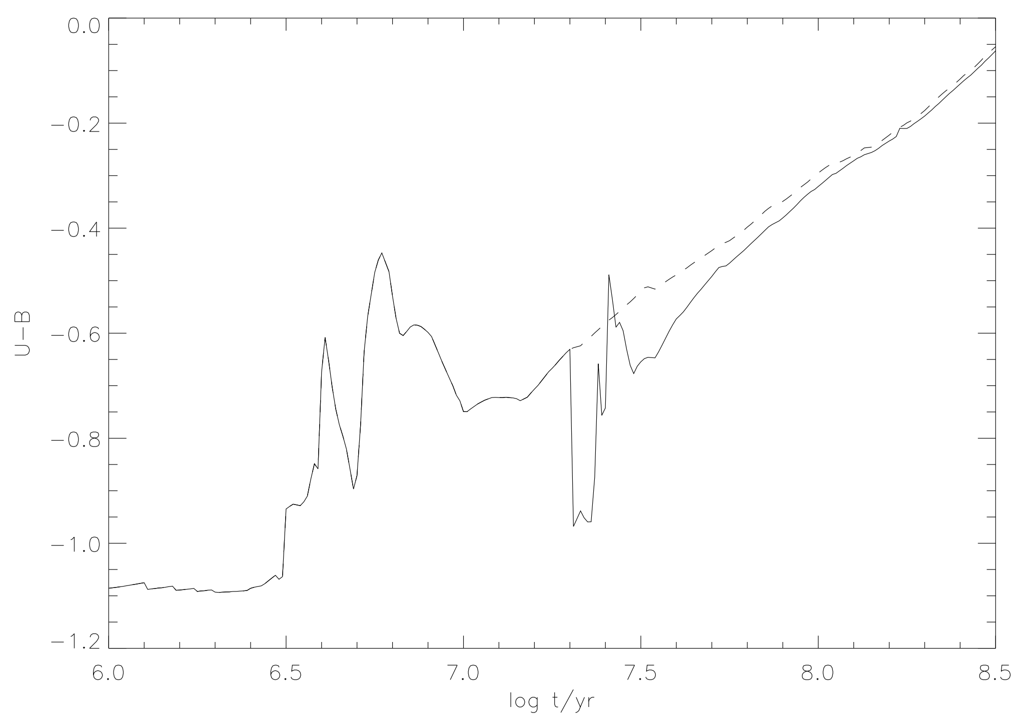

Figure 6: $U - B$ versus log t/yr. This is the same diagram is the top left from figure 5, but constructed from data with a higher resolution. The dashed line is the curve from the single generation model.

A second reddening peak should be expected after the first appearance of the stars of the second generation, but this peak is not clearly visible in the diagrams. For this there are two complementary explanations. The first is a physical one, namely that as time passes by even more stars from the first generation become red giants, so at the time were the expected second reddening peak should be, already relatively more stars have a red colour, so the peak will be less significant. The second reason is a computational one. The evolution of stars near the turn-over point (where they become red giants) goes very rapidly. The data for the multiple generation model was created by the interpolation of data which had a resolution of log t/yr = 0.05, so between log t/yr = 7.4 and log t/yr = 7.45 there are more than 3 million years. This could be solved if our data had a higher resolution, for example log t/yr = 0.01. This howevever brings other difficulties in programming, because at least double precision numbers are needed for those calculations. It has been done once and the results are shown in figure 6. Notice that the second reddening peak is a bit more visible now, but the other main characteristics of the diagram are the same.

Some careful conclusions can be made of the model shown in this section. The single generation model of the change of the integrated colours in time shows that star clusters become redder as they age. First there is a reddening peak-period, where the heaviest and therefore most luminous stars evolve into red (super)giants. The reddening peak-period is relatively very short, because the life-time of the most luminous red supergiants isn't very long. After this peak, other stars in the cluster, mostly main-sequence stars, gradually evolve in red giants and the cluster gradually becomes redder. The model which included a second generation of stars shows that in the colour-age diagram a blueening peak occurs. This peak is located at the point in time where the second generation is born. It however vanishes quite fast, because the stars in the first generation continue getting redder. It could carefully be concluded that a cluster with a blueer colour than predicted by a single generation model contains a second (or third) generation, but the arguments for this conclusion aren't very strong. The blueening peak mostly has an even smaller period than the reddening peak mentioned in the single generation model and therefore is hard to measure, because it is not possible to observe clusters over a large enough timescale (which should be millions of years). The biggest problem is that there isn't a method to estimate the age of a cluster independent on the colour of the cluster. The age estimates are based on CMD-fitting, which is based on stellar model isochrones (like Padua's), as is our integrated colour-model. Proving a second generation with a integrated-colour model is therefore some kind of circle argumentation and it is actually more like a test whether the model is consistent.

3 Model 2: Colour gradient

Another method to find possible multiple generations in star clusters is to look for colour gradients in a colour versus distance diagram, where the distance is the distance to the center of the cluster on the projection on the sky. Therefore $r^2 = x^2 + y^2$, where x and y are coordinates on an image of a cluster. This model is kind of in between using photometry of resolved individual stars and integrated photometry of clusters, because integrated photometry of the light in concentric rings is used here. Spherical symmetry is assumed and also the most assumptions which were made in model 1 hold here. One difference however is that the number of stars N in the cluster actually is included in the model here. The cluster-age will be an input parameter in this model. It is expected that in a cluster where all the stars have the same age and where the masses are randomly distributed when compared to the distance, that the colour versus distance diagram is approximately flat. From model 1 it can be seen that a possible second generation of stars in the clusters core will result in a bluer colour for small distances and thus in a colour gradient. The stars of a second generation will be called the second population in this model, to avoid confusion with model 1. So instead of generations this model deals with populations, but essentially the same is meant.

3.1 Single population model

The model basically generates N stars with distances corresponding to an intensity formula and with a Salpeter mass distribution. Again the Padua data is used to find corresponding magnitudes to the simulated stars. Once a table is created which contains N number of stars with generated distances, masses and magnitudes, the generated cluster will be sliced into twenty disks with width 0.05 R_{max}. Where R_{max} is the maximum distance a star can have from the center of the cluster. Then the model will calculate the integrated magnitudes in the disks and finally a colour versus distance diagram can be produced by distracting two magnitudes to receive a colour. The steps mentioned here will be described to more extent in the following subsections.

3.1.1 Distance-Intensity formula

The distance of stars to the center of a cluster is distributed according to a certain intensity formula, because there are more stars near the clusters core than farther away from the center. By fitting clusters in the Large Magellanic Cloud Elson et al. (1987) found the following intensity formula:

$$I(r) = I_0(1 + (\frac{r}{r_c})^2)^{-\eta} \tag{16}$$

Where I_0 is a normalizing constant, r is the distance to the center, r_c is the core radius and η is a parameter to determine how fast the envelope falls off at large distances, which must be larger than 1.
The normalizing constant I_0 can easily be calculated by a computer. Once a set of radial sampling points is created, corresponding intensities are calculated by equation 16 and then

devided by the sum of all intensities. This normalizes the sample distribution. It must be noted that as r is two dimensional, $I(r)$ also is. It should actually be seen as a profile of the number of stars per unit area as a function of the distance. This will be of importance when calculating the integrated magnitudes in the disks.

3.1.2 Generating stars

To generate a star cluster with good statistics it is necessary to use a large number of stars, N, otherwise outliers with a high mass (and therefore high luminosity) will dominate the integrated colours too much. A program in IDL, called SIMPOP.pro, generates a table with N rows and with colomns containing a star identification number, distance to the center, mass, age and U, B, V and I magnitudes. Distances are generated independent of the masses, so the masses are randomly distributed compared to the distance.
To generate the distances to the center first a set of radial sampling points is created. This is an array of N values between zero and one, multiplied by the R_{max}. Then corresponding intensities are calculated and normalized, which results in a discrete probability-distribution for each value of r. To let the computer generate stars according to this probability-distribution the probabilities have been binned in the following way: the total bin consists of N bin-parts, corresponding to discrete probabilities. The first bin-part includes the values of the probability that a star cluster has the smallest distance, the second bin-part has the value of the first bin plus the value of the probability that a star cluster has a one step larger distance, etc. This can easily be illustrated by the example of a dice. The probabilities for all six values are $\frac{1}{6}$, so the total bin consists of 6 bin-parts. The first one has values from 0 to $\frac{1}{6}$, the second one from $\frac{1}{6}$ to $\frac{2}{6}$ and so on. When the bin is created, the computer generates a random number between zero and one, then looks in which bin-part the number belongs and finally assigns the distance corresponding to the bin-part to the simulated star. It does this for all N stars. To test the method a histogram of generated r values can be plotted, as in figure 7. It can be seen that the method works perfectly.

Secondly, SIMPOP.pro generates the masses of the stars in the cluster. To do this the function POWERLAWDIS was used, which uses a Salpeter IMF. It is important to assign the correct upper limit for the mass for the age which has been used as imput parameter, because otherwise stars with a heavier mass than allowed according to the Padua stellar evolution models can be created. The final step is to assign the magnitudes to the generated stars. Once the masses of the stars are calculated, the INTERPOL procedure in IDL can read out the correct U, B, V and I magnitudes according to the Padua tables. As all these steps have been done, a star cluster with 100,000 stars is generated.

3.1.3 Integrated disk-magnitudes & colours

To be able to find colour gradients in the cluster, integrated colours have to be calculated for different distances to the center. This has been done by slicing the cluster into twenty disks with width 0.05 R_{max}. The reason to use twenty disks is because the disks are small enough

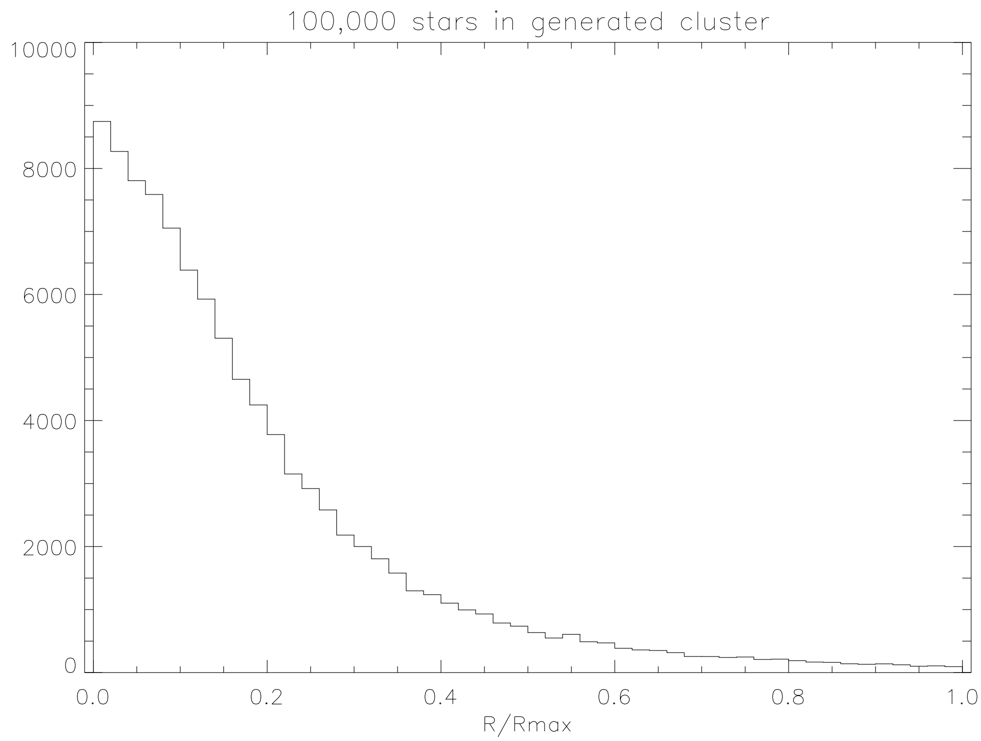

Figure 7: Histogram of generated distances to the center of the cluster for a cluster with 100,000 stars.

to detect possible colour-gradients and large enough to contain a statistical large enough number of stars (at least in the clusters core). A program (DISKS.pro) was written to select which stars in the generated cluster belong in each disk. Then for each disk the integrated disk-magnitude is calculated by first converting magnitudes to luminosities, secondly adding up all luminosities in the disk and finally converting the total luminosity to an integrated magnitude. As mentioned before, the intensity formula for the number of stars as a function of the distance is two dimensional. Star clusters however are three dimensional, so therefore the total number of stars in a disk is proportional with $2\pi r \cdot I(r)\mathrm{d}R$. This however is not very troubling when looking at relative intensities, which is done in this model.

3.1.4 Single population model diagrams

Once SIMPOP.pro and DISKS.pro have been run, the data from the generated cluster has been transformed to an array which contains integrated magnitudes for disks with different distances to the clusters center. Finally, a colour versus distance to the center diagram can be made by plotting the difference between integrated disk-magnitudes (for example $B-V$) versus the disks. This has been done for a generated cluster with 500,000 stars from a single population and the result can be seen in figure 8. The input-parameters are the age of the population, r_c and η. The age has been set to log t/yr = 7.5 (for reasons which will become clear when discussing the multiple population model) and the structural parameters r_c and

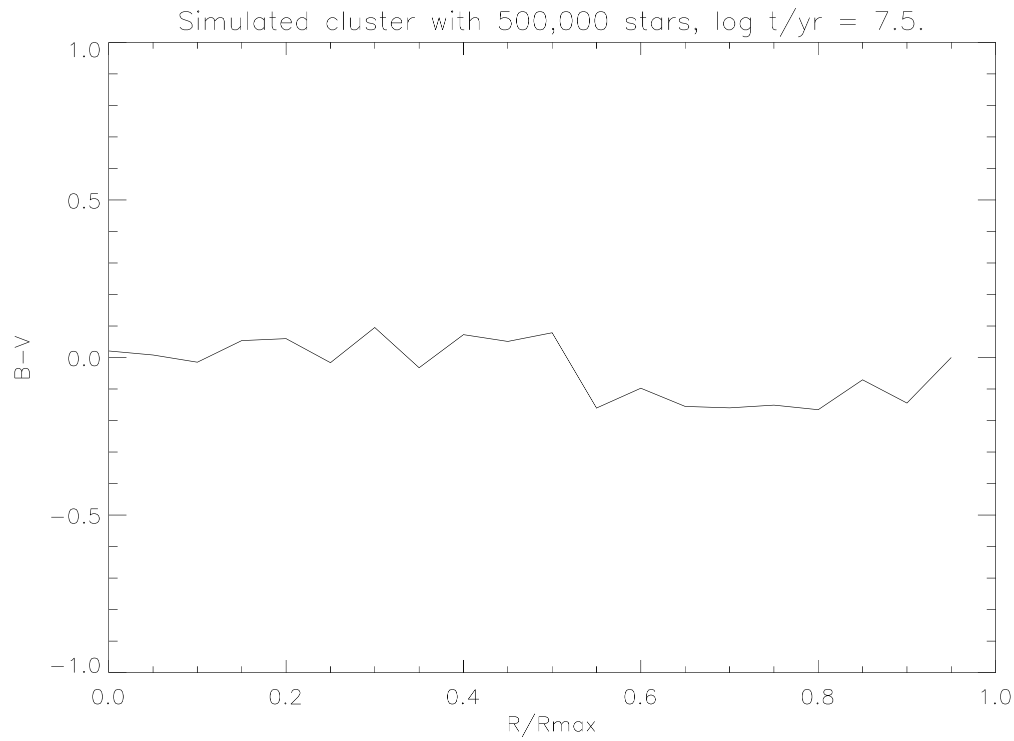

Figure 8: $B - V$ versus disks in a single population cluster with age log t/yr = 7.5, Z = 0.008.

η to be 0.25 and 1.55 respectively (the exact values are not very important, only the relative values when compared to the multiple population model matter for the scope of this paper).

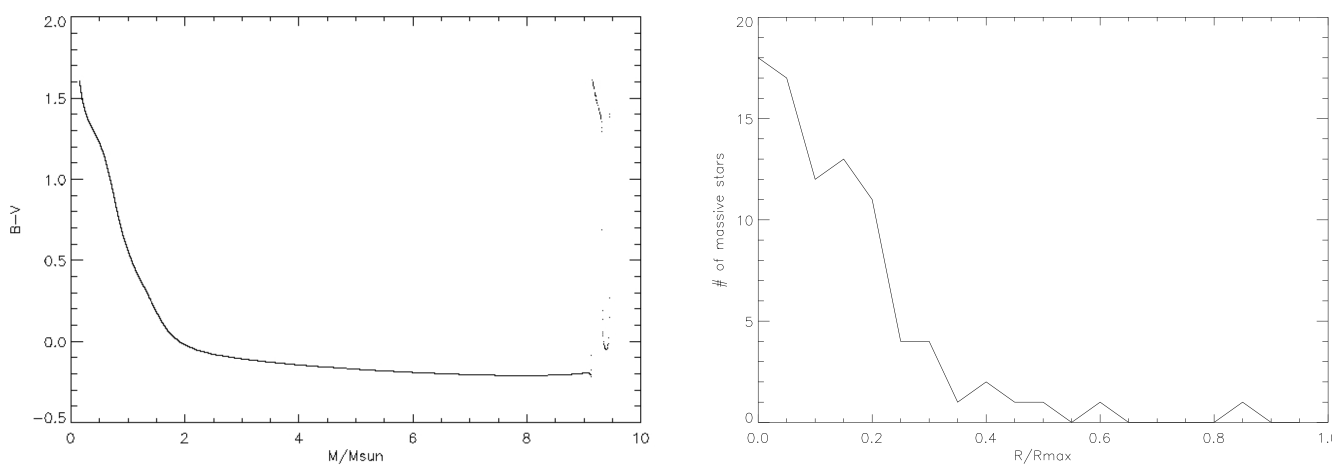

Figure 9: Left: $B - V$ versus mass in the generated cluster with 500,000 stars. Right: Number of very massive stars ($M > 9.12M_{sun}$) in the generated cluster versus the distance to the center.

As can be seen in figure 8, the colours almost fit a straight line. This is as expected, because there is no correlation between the generated mass and distance to the clusters center and therefore no correlation between a stars colour and its position. The $B - V$ colour is numerically between 0 and 0.1, which matches the prediction by model 1 for a cluster with age log t/yr = 7.5, quite well (see top right part of figure 3). There are some small fluctuations and they are due to random errors. These fluctuations will significantly be larger for generated clusters with a smaller number of stars and should vanish when even more stars are generated. Generating 500,000 stars however already took the computer between two and three days and as the computation time seems to depend exponentially on the number of generated stars, the errors are taken for granted. Another error in the diagram is the blueer colour of the cluster for $R/R_{max} > 0.5$. This is due to a smaller number of very massive red giants in the outer regions of the cluster and this is again due to random sampling. The right part of figure 9 shows the number of very massive stars in each disk. It can be seen that there are none in the outer regions of the cluster. To distinguish massive stars from very massive stars, a colour versus mass plot has been used, like the one in the left part of figure 9. It can easily been seen that the very massive stars, which colours differ significantly from other massive stars, have a mass larger than 9.12 M/M_{sun}. Because they are so massive they are also very luminous and therefore have great infuence on the integrated colours.

3.2 Multiple population model

The interesting part of the second model is how multiple populations influence the $B - V$ versus distance diagram. When there is a significant age spread between two populations, it can be seen from model 1 that the young population mostly will have a blueer colour than the old population (this is only not the case when the young population is exactly on its reddening peak once most of the stars turn into red giants, see figure 3). If then the assumption is made that the two populations have different spatial distributions, colour gradients should appear in the colour to distance diagram.

The single population model can easily be extended to a multiple population model by generating star clusters of different ages. In stead of generating one cluster with N stars, one cluster with $n_{first} \cdot N$ and one cluster with $(1 - n_{first}) \cdot N$ are generated. n_{first} is the fraction of stars in a cluster which belong to the first, oldest population. Of course the generated clusters have different ages and different spatial distributions. In this model the only way the spatial distribution of the second population varies from the first population is by a smaller value of r_c. This means that the second population is concentrated in the center of the cluster.
Once the two clusters have been generated, the program DISKS2.pro is used to read out which stars from both populations belong to the disks. Then it calculates the total luminosity in each disks by adding the luminosities from stars from both populations and finally converts the values to magnitudes.

3.2.1 Multiple population model diagrams

The parameters used to generate the stars in the oldest population are the same as for the single population model. n_{first} is set to 0.8, such that about twenty percent of the stars in the cluster are from a second population. This has been chosen in correspondence with the values used for model 1. For the young population only the age has been changed to be log t/yr = 6.6 and r_c to 0.05. Figure 3 has been used to determine this age, because it can be seen from the $B-V$ diagram that when the old population has an age of log t/yr = 7.5, there should be a significant difference in colour between the two populations for an age gap of around 20 - 30 Myr. The difference in colour will be more visible when the age-gap between the populations is larger. It however should be noted that the young population can actually be redder than the old population when it has an age where the reddening peak occurs. Of course these input parameters only have an illustrative purpose for this part of the paper and should be adjusted when more data is known.

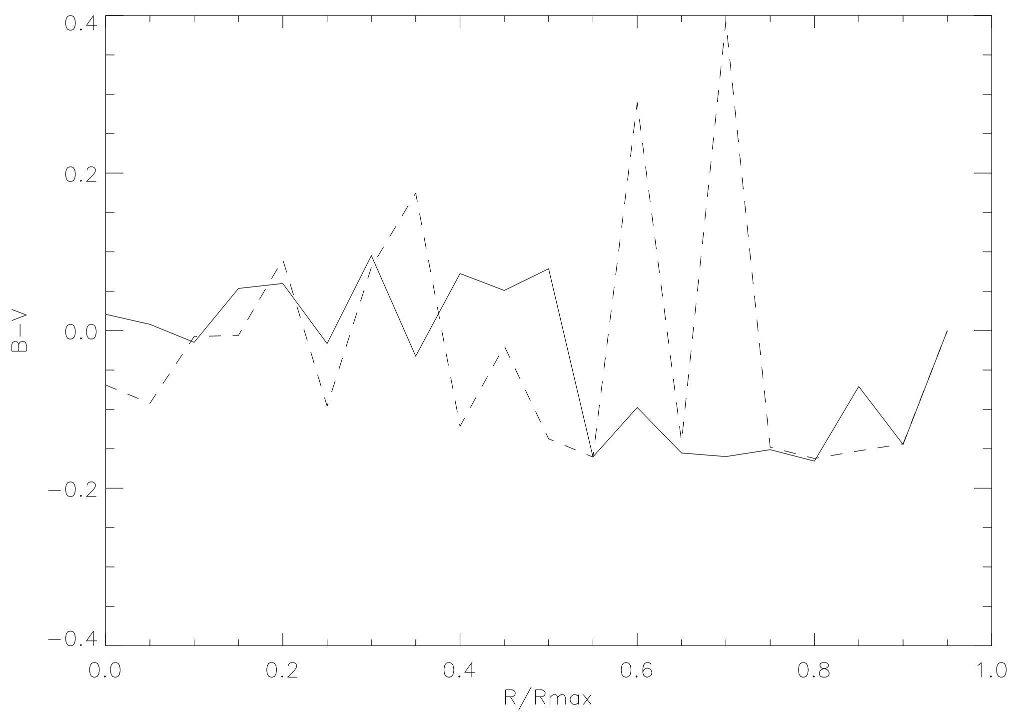

Figure 10: $B-V$ versus distance to the center of the cluster. The normal line is the generated cluster with one population (figure 8). The dashed line is the generated cluster with two populations.

Once all programs have been run a $B-V$ versus distance plot can be made. The direct result is shown in figure 10. At first sight, the diagram seems rather messy. Especially the colours of the cluster with two populations show various peaks and don't fit a straight line at all. All these errors however can be simply explained by random-sampling. To prove this, it is a good starting point to see how the young population influences the colours of the total cluster.

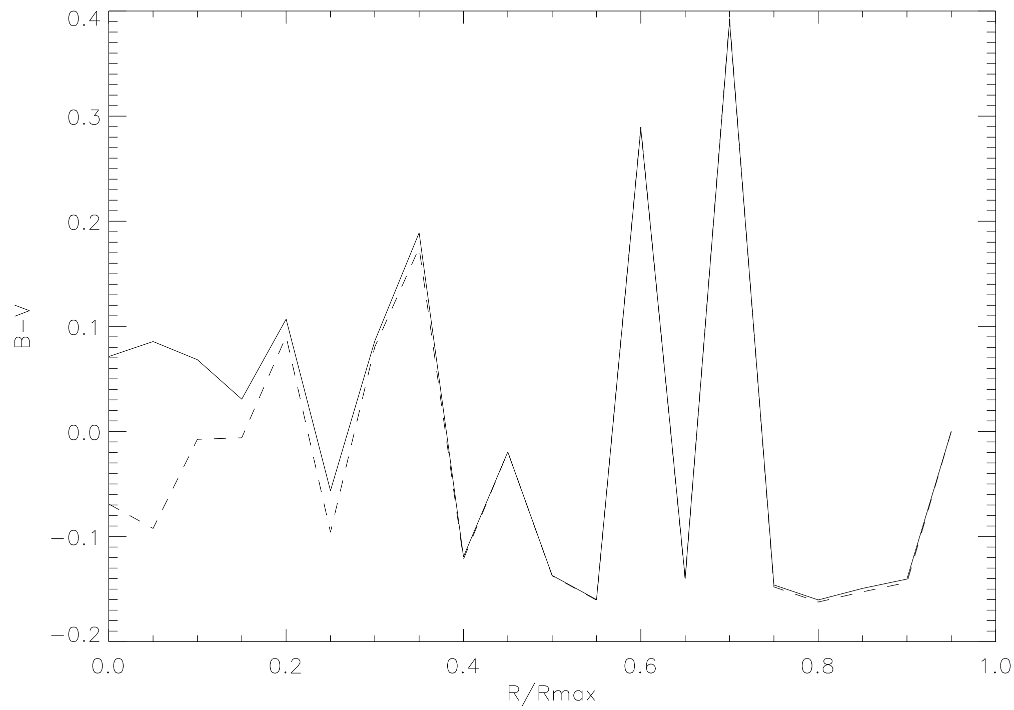

Figure 11: $B - V$ versus distance to the center of the cluster. The dashed line is the generated cluster with two populations. The normal line is the generated cluster with 2 populations, but only the light from the oldest population is counted.

From figure 11 it can be seen that the peaks and errors in the two population cluster are caused by the integrated colours of 400,000 stars in the oldest population. The only difference due to the presence of the young population is the blueer colour in the center. So now it seems that the old population causes peaks and fluctuations in the colours due to random-sampling errors. In the discussion of the single population model it already was suggested that they are caused by very massive stars, which have very different colours from main sequence stars and which dominate the integrated colours of the cluster because of their high luminosity. There are three ways to solve the random-sampling problem.

The first way would be to ignore the very massive stars, but this would devalue the model because information is thrown away. The very massive stars can be detected in colour versus mass diagrams, like the left diagram of figure 9, and then it is easy to run DISKS.pro with data which excludes the very massive stars. This has been done for both the single population model and the multiple population model and the results can be seen in figure 12. It can be seen that the very massive stars are indeed the causes of the errors and fluctuations and that without them, the colours for the single generation cluster on the left diagram fit a straight line very well. The redder part at the outer edge of the cluster is

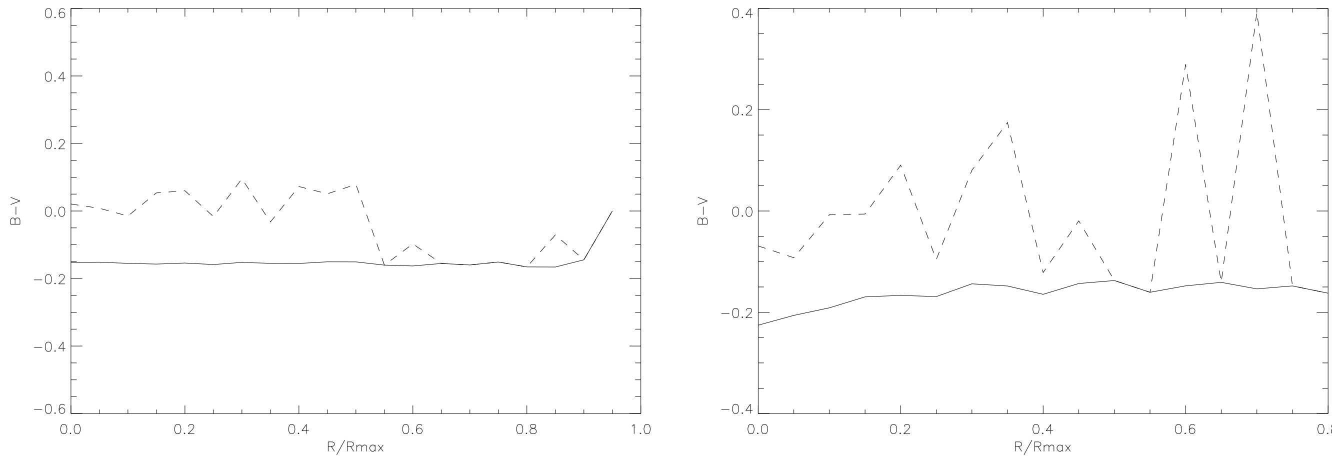

Figure 12: Left: $B-V$ versus distance for the single population cluster without very massive stars. Right: $B-V$ versus distance for the multiple population cluster without very massive stars. Dashed lines represent the $B-V$ versus distance diagram for clusters with very massive stars.

due to a programming error and should not be considered (that is why the right diagram x-ranges are adjusted). From the right diagram it can be seen that it fits a straight line further away from the center and that the colours in the clusters core are redder than the colours from the single population model. These two diagrams can be added together to see colour gradients due to multiple populations in clusters which contain only main sequence stars. This result is shown in figure 13. From this figure it can cleary be seen that when very massive stars are ignored, a younger population in the center of a cluster results in a gradient in the integrated colours.

A second way to solve the random-sampling problem is just to increase the number of stars in the generated cluster, or to generate a high number of clusters with the same numbers of stars and other parameters and then take averages. Both processes however take long computation time, since generating one cluster with 500,000 stars already took between two and three days. There is however one improvement which can be made fairly easily already and that is to use the data from the single population cluster as the old population in the multiple population cluster. By doing this, 500,000 stars are in the old population in stead of 400,000 stars, while there still are 100,000 stars in the young population. The result will be that the stars in the young population will have less influence on the integrated magnitudes and colours than before. The result of this improvement is shown in figure 14. This diagram shows actually the same results as figure 13, namely that a young population in the center of a cluster will result in a colour gradient. Figure 14 however also includes very massive stars like blue and red supergiants and therefore is a better model.

The third way to handle the problem of the very massive stars is an accomodation between the other two ways, namely raising the lower-mass limit when generating the masses of the stars in the cluster. This leads to more massive stars in a cluster and therefore better statistics, for the price of losing information of low-mass stars. For illustrative purposes the

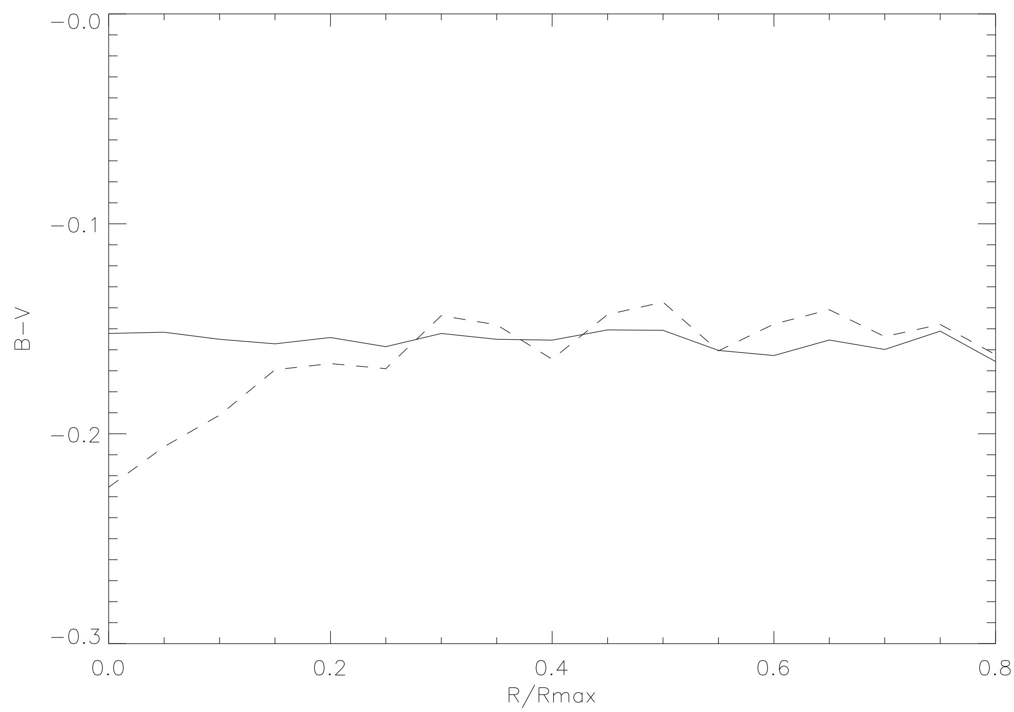

Figure 13: $B - V$ versus distance to the center. The normal line shows the colours of a cluster with one population of stars, the dashed line shows the colours of a cluster with two populations of stars, where the younger one is concentrated in the clusters core. Note that very massive stars are ignored.

lower-mass limit for the following diagrams has been set to 1.5 M_{sun}, which is ten times larger than the lower-mass limit used before. The absence of stars with low mass doesn't influence the integrated colours very much, because it can be seen from the mass-luminosity relation that they aren't very luminous compared to more massive stars. A resulting colour versus distance diagram by using this method is shown in figure 15. The two generated clusters from figure 15 consist of 200,000 stars, from which 20 percent is in the young generation in the cluster with two populations. It can be seen that integrated colours now fit a straight line very well, this is because the statistics for the distribution of very massive stars is improved. Also the feature from the young population in the center, namely the blueer integrated colour, can easily be seen here. Note that when this diagram is compared to figure 13, where the very massive stars were neglected, it can be seen that the clusters which contain many very massive stars are somewhat redder than the clusters which contain no massive stars.

To finish the section on this second model, there are some general conclusions which can be made. First of all it might be possible to see a colour gradient in a colour to distance diagram due to the presence of a young population in the center of a cluster, but it is very unlikely. The greatest problem in simulating star clusters is a stochastic one. The initial mass function allows that there is a tiny fraction of stars in a population which has a very high mass and therefore very high luminosity. These 'outliers' have much influence on the integrated colours

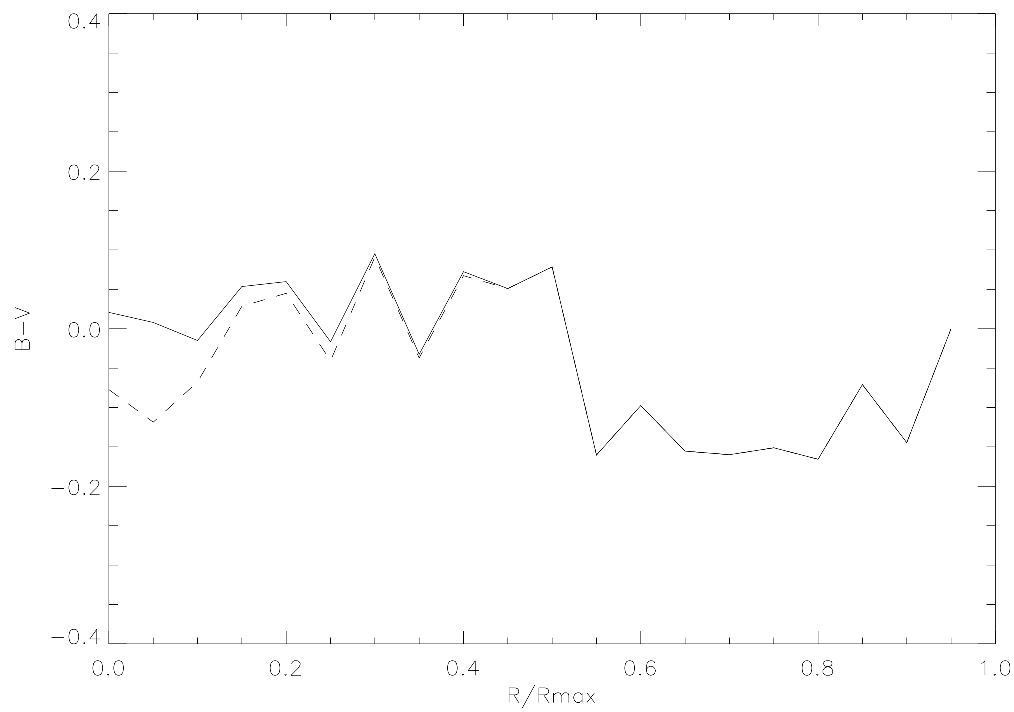

Figure 14: $B - V$ versus distance to the center. The normal line shows the colours of a cluster with one population of stars, the dashed line shows the colours of a cluster with two populations of stars, where the younger one is concentrated in the clusters core.

and therefore a large number of stars must be simulated in a cluster to prevent that they have too much effect. This however is not per definition a computer problem, it could be a more fundamental problem too. This model shows that a cluster should host approximately 500,000 stars to have a sample with good statistics, so the clusters must be very massive. Only the few largest clusters which can be observed can get even close this number of stars, like ω Centauri. Most clusters have a much lower number of stars and therefore it is possible, even likely, that those clusters won't have nice, flat and smooth colour to distance diagrams, because it is very likely that there are very massive outliers which influence the integrated colours greatly. Another limitation to this model is that it assumes a young population which is considerably more concentrated in the center of a cluster than the old population, this is however far from necessary, because there can be other radial distributions which are less visible in the colour to distance diagram.

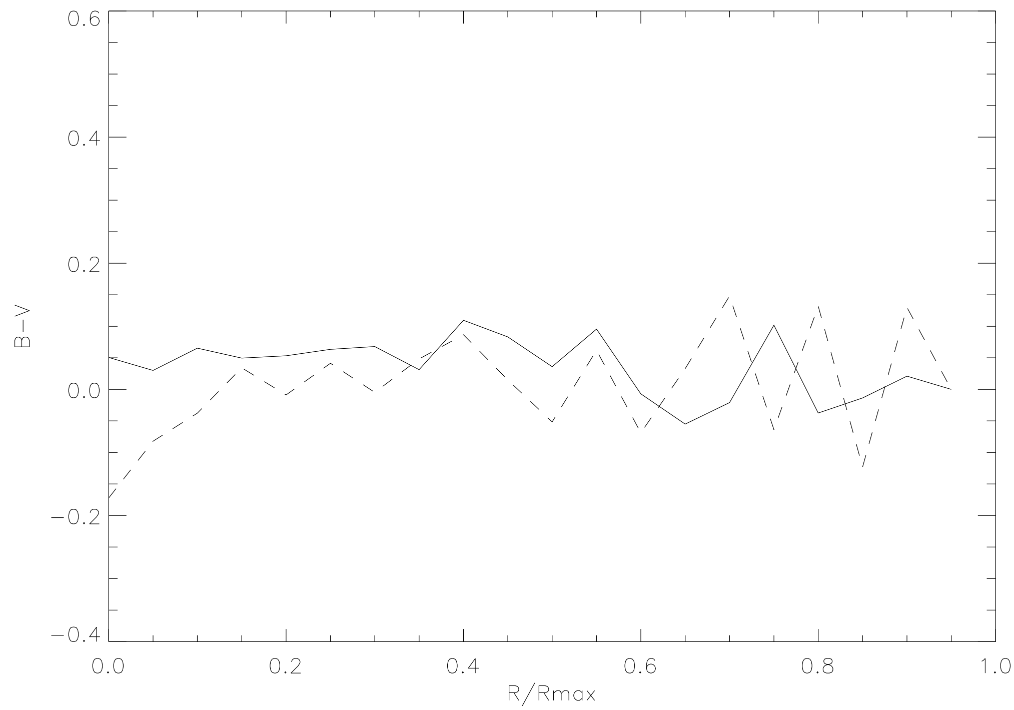

Figure 15: $B - V$ versus distance to the center. The normal line shows the colours of a cluster with one population of stars, the dashed line shows the colours of a cluster with two populations of stars, where the younger one is concentrated in the clusters core. This time the lower-mass limit is ten times larger than in previous diagrams.

4 Extinction and Colour-Colour diagrams

One of the most important problems in observational astrophysics is that the observed light from stars, galaxies and other objects is not exactly the same as the emitted light. This is due to several reasons as the expansion of the universe, gravitational lensing, the object's movement, but most important: extinction. Extinction is a combination of scattering and absorption of electromagnetic waves (Larsen, 2011) because of dust in the interstellar medium between the observer and the source. This dust can consist of matter or gas, or a combination of the two. A difference can be made between scattering due to very small dust (where the size is much smaller than the wavelength of the electromagnetic waves, this is called Rayleigh scattering) or due to large dust grains. For the latter the scattering is independent of the wavelength, for the first it has a strong wavelenght dependence ($I \sim \lambda^{-4}$). This occurs for example in our atmosphere. The problem of extinction is that it is not trivial to measure the correct amount of interstellar dust between the observer and the source and therefore makes it neccessary to make additional observations to measure the correct extinction in the different wavelengths.
To give extinction a mathematical description, an optical depth A_λ is introduced. The observed absolute magnitude in a certain wavelenght can then be given by:

$$m_\lambda = m_{0,\lambda} + A_\lambda \tag{17}$$

Now it is possible to define the colour excess in for example the colour $B - V$ by:

$$A_B - A_V = E(B - V) \tag{18}$$

Then the emitted $B - V$ colour can be calculated by:

$$(B - V)_0 = (B - V)_{obs} - E(B - V) \tag{19}$$

This reduces the extinction problem to finding the colour excess in the desired band.

There are several ways to determine the colour excess, for example by fitting CMD's and using colour-colour diagrams. These two will be described here shortly. The stars in a cluster which is at least for a significant part resolved can be plotted as points on a colour-magnitude diagram. When it is assumed that all stars in the cluster are from the same age and have the same colour excess, an isochrone can be fitted to the data. One fitting parameter is the extinction A_B (from which the colour excess can be derived from formula 16 and an extinction curve. Reddening extinction shifts an isochrone to the right in a CMD and it is relatively easy to determine the extinction.
The colour-colour diagram is a plot of (mostly) $U - B$ versus $B - V$. Using cluster models a curve can be derived for the case of zero extinction, like figure 16. Data-points which are not corrected for reddening extinction do not lie on the model-curve and a reddening vector can be drawn between the data and the model (Gordon & Clayton, 1998).

When comparing the single generation model with the second generation model, it is interesting to see whether the colour-colour diagrams are different. This could be important

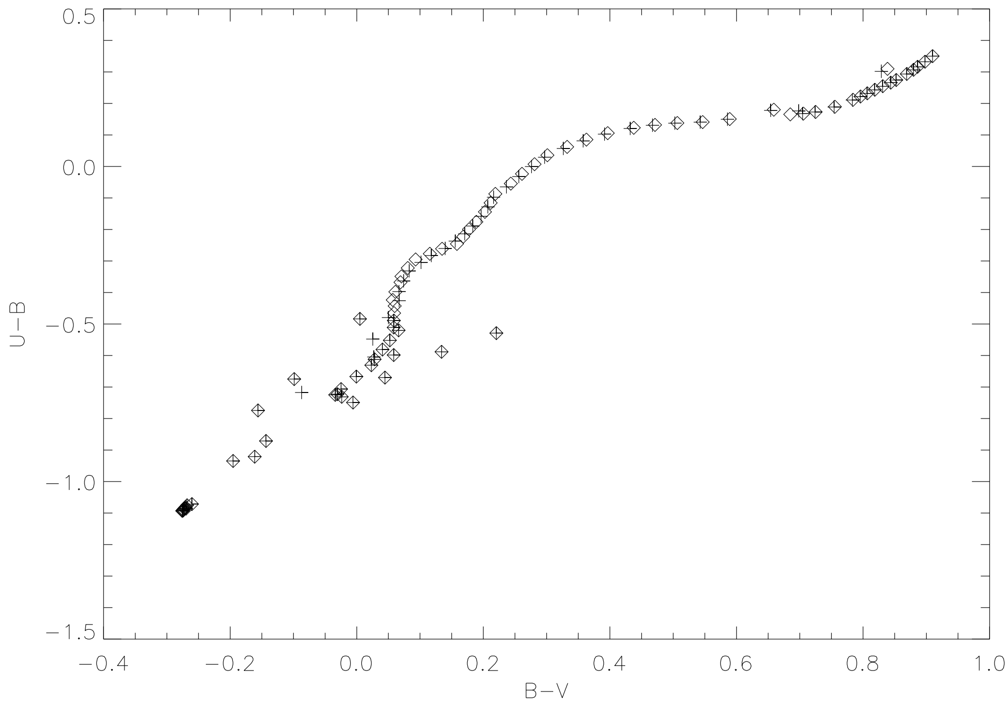

Figure 16: $U - B$ versus $B - V$ in a single generation cluster and a multiple generation cluster. $\diamond$ symbols are the single generation points, $+$ symbols the ones from the multiple generation.

because when it is possible to find a second generation in a colour-colour diagram, it is not needed to use a model to calculate the ages for clusters, as is needed for the colour versus age diagrams. Figure 16 plots both colour-colour models in one diagram and because almost all diamonds are 'filled' with plus signs, the important conclusion is that multiple generations cannot be seen in colour-colour diagrams.

5 Data comparison with model 1

There are two different approaches which can be done to compare the predictions of the first model with actual observations. The first one is to find photometric data for clusters for which multiple generations are already suspected and see how those datapoints lie in the $B - V$ versus age diagrams. The second approach uses catalogues which contain many star clusters (50+) in certain galaxies. When a large sample of clusters in one galaxy is used, it is likely that metallicities, extinction and other assumed or unknown parameters are the same for all the objects. The catalogues contain photometric data and age estimates (mostly based on CMD fitting). These data is also plotted on a $B - V$ versus age diagram. Using the fact that a second generation gives a bluening peak, this could be used to trace clusters with multiple generations.

Data for four clusters with possible multiple generations (NGC 1313-F3-1, NGC 5236-F1-1, NGC 5236-F1-3 (Larsen,1999) and NGC 1569-B (Anders et al. 2004)) has been used to compare with the predictions of the two generation model for the $B-V$ versus age diagram. As can be seen in figure 17, some datapoints quite nicely match the predicted blueening peak, but the data point for NGC 1313-F3-1 doesn't really match the data.

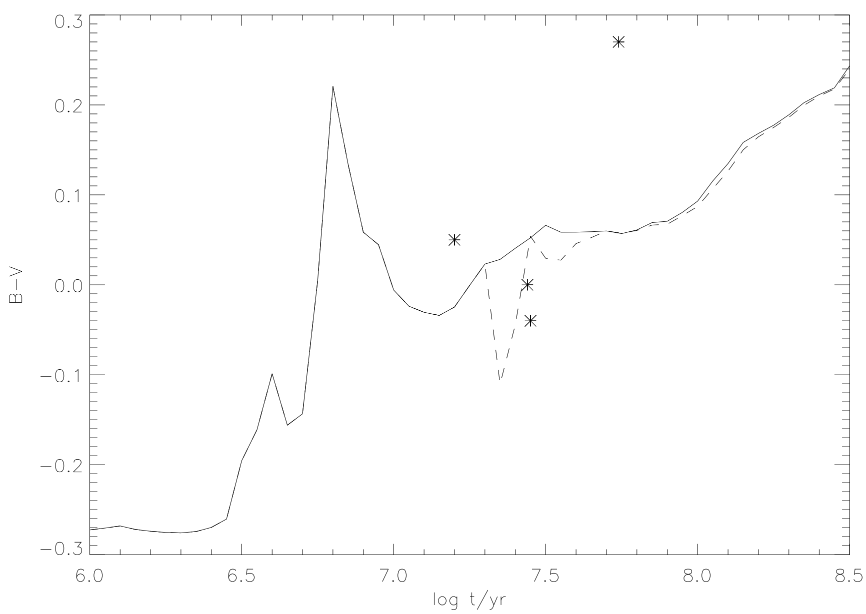

Figure 17: $B - V$ versus age. The dotted line represents the changes due to a second generation. As metallicity, Z is set to be 0.008 in the model.

For the second approach, the VizieR service was used to find a number of catalogues with photometric data and age estimates for a large enough sample of star clusters. Three suitable catalogues were found, namely:

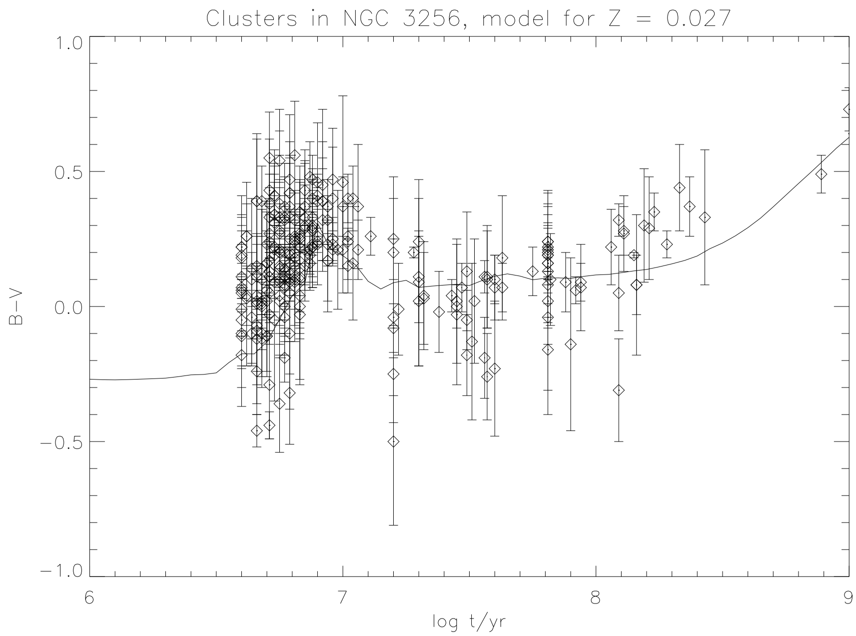

Figure 18: $B - V$ versus age for the single generation model. The data points are from the Goddard catalogue.

1. Goddard et al., Star clusters in NGC 3256, 2010.
   The Goddard catalogue contains 276 clusters in the galaxy NGC 3256. It contains photometric data and ages derived from the CMD. The metallicity used to estimate the cluster ages varies from one to two times the metallicity of the sun, so the metallicity for the model is set to 1.5 times the sun's metallicity, hence Z = 0.027.

2. Mora et al., UBVI photometry of NGC 45 star clusters, 2007.
   The Mora catalogue contains 47 clusters in NGC 45. Again it contains photometric data and age estimates, where a metallicity of Z = 0.008 is used.

3. Silva-Villa et al., Photometry of star clusters in 5 nearby galaxies, 2011.
   Finally the Silva-Villa catalogue contains 2999 clusters in the galaxies NGC1313, NGC45, NGC 4395, NGC 5236 and NGC 7793. Ages are estimated with a LMC-like metallicity (Z = 0.008).

The results of this comparison is plotted in figures 18, 19 and 20. Only data for one of the five galaxies in the Silva-Villa catalogue is shown, because the diagrams for the other galaxies are very similar.
The data from the Goddard catalogue seems to match the predicted colours very good and there are some clusters which appear blueer than the model, which could lead to the conclusion that the cluster contains multiple generations of stars (however most of the time the error bars include the predicted colour).

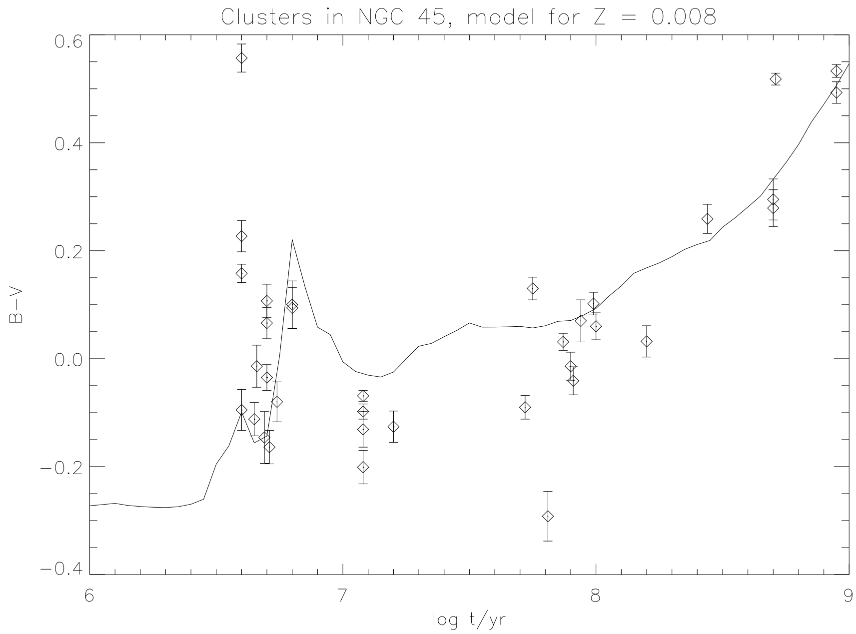

Figure 19: $B-V$ versus age for the single generation model. The data points are from the Mora catalogue.

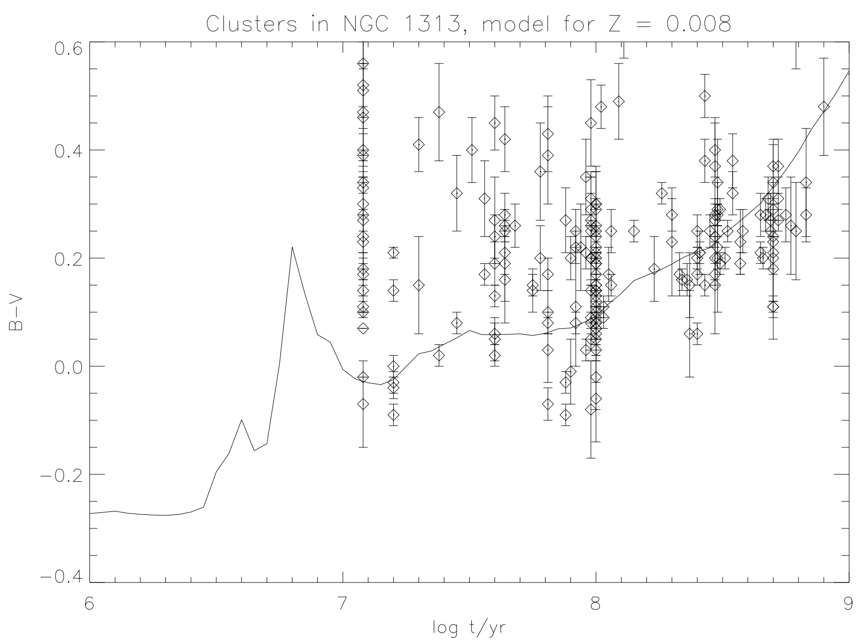

Figure 20: B-V versus age for the single generation model. The data points are from the galaxy NGC 1313 in the Silva-Villa catalogue.

The data from the Mora catalogue has smaller error bars but also less clusters, so there is less data to compare with. Most of the clusters have colours which aren't really different from the model, but there are also some which have a significant blueer colour which cannot be attributed to photometric errors. It would be interesting to study these clusters with other methods, for example very accurate CMDs or observations of the different abundances.
The diagrams from the Silva-Villa catalogue are very much alike for the five galaxies, so only the one for NGC 1313 is shown here. Unlike the other two catalogues the Silva-Villa data doesn't really match the model very good, because many clusters are redder than predicted. There are also some blueer ones for which the blue colour cannot be attributed to photometric errors, but the same holds for many redder ones, so this is not convincing.

There is however one very important critical note which has to be made and that is that the age is estimated using photometric (colour) data. So when the colours of a cluster are compared with the predicted colours for a certain age there is no new information added. Therefore it was a priori very likely that the colours in the model would fit data quite nicely, because the ages are estimated from the colours. Like explained in the section on how the model was made, comparing integrated colours of clusters with the modeled integrated colours for a certain age is more of a test whether the models to estimate the ages for clusters and the model to calculate the integrated magnitude are internally consistent. Therefore it might be a test for the use of the correct IMF, correct stellar evolution models and CMD-fitting on the other hand. This is however beyond the scope of this paper.

6 Data comparison with model 2

To compare observed data from star clusters with the colour-gradient model, there are two ways to obtain integrated colours versus the distance to the center. The first one is to measure somehow the magnitudes in different colour bands in rings around the clusters center. This way a (partially) unresolved image of a cluster can be used. The second way is by using photometry of individual stars together with their position-data to calculate the integrated magnitude versus the distance to the center.
This has been done for data for two clusters which already were mentioned in the section where data was compared with model 1, namely NGC 1569-B and NGC 1705-1. Larsen et al. (2011) studied the radial distribution of fainter and brighter supergiant stars in both clusters (both clusters are suspected to have multiple stellar generations) and altough the evidence isn't realy convincing, it seems that the younger populations are more concentrated in the clusters center. Using the photometric data for respectively 449 and 489 stars, the colour versus distance diagrams could be made, shown in figure 21. It should be noted that as the images are made by the Hubble Space Telescope, filters *F555W* and *F814W* are used in stead of V and I, but they are almost the same.
The distances to the center were calculated by first calculating the mean x- and y-position of the stars on the CCD image and defining that as the clusters center, and then use simple geometry to calculate the distance of the stars to the center. Once the data contained values for the distance to the center and the magnitudes in two different colour bands, DISKS.pro was used to create the integrated colour versus distance diagrams.

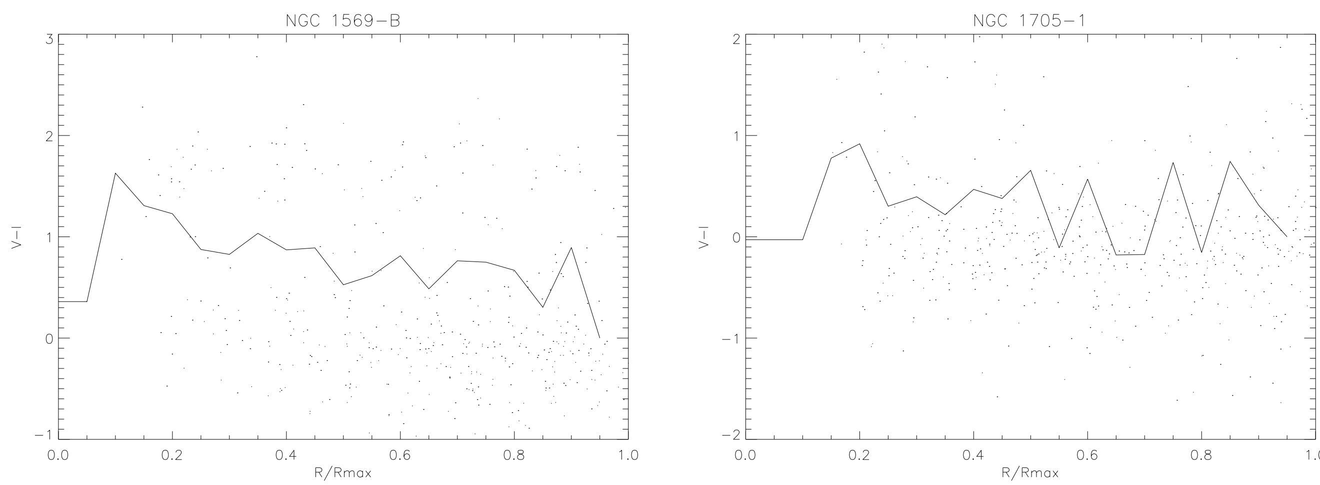

Figure 21: Left: $V - I$ versus distance for NGC 1569-B. Right: $V - I$ versus distance for NGC 1705-1. In the background the colours for the individual stars are plotted.

At first sight the diagrams in figure 21 don't seem to match the predictions from the model (a straight curve) very well. For the region between 0 and 0.2 R/R_{max} there is no data available because the cluster is unresolved there, while that should be the region where a second population concentrated in the center of the cluster is best visible in a colour to distance diagram. But given the fact that only data for 449 and 489 individually resolved stars is used, the integrated colours already fit a straight line quite well. The statistics are however way to poor to draw conclusions from these two diagrams, especially because there is no data in the interesting region near the clusters center. A nice additional feature of figure 21 is that the colours for the individual stars in the cluster are plotted as a background to the integrated colour to distance diagram. It can be seen that there are more stars with blueer colour than the integrated colour of the cluster, than there are stars with redder colours. This can be explained because stars with redder colours generally are larger, have more mass, high luminosity and therefore have more influence on the integrated colours.

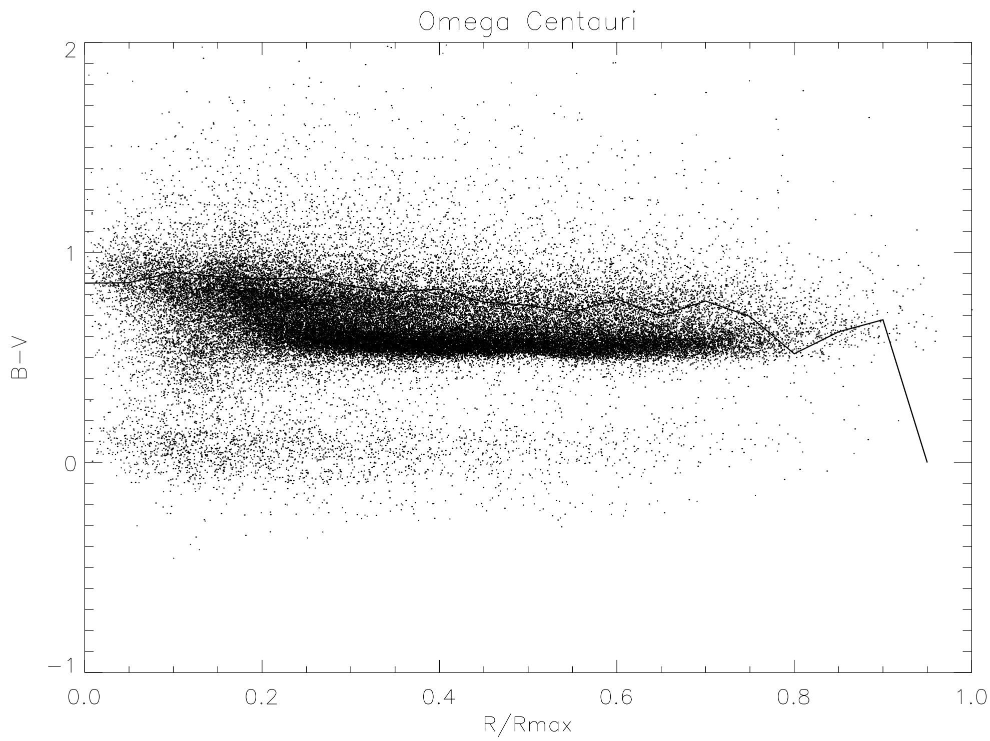

Figure 22: $B - V$ versus distance for ω Centauri. In the background the colours for the individual stars are plotted.

Finally it would be interesting to create a colour to distance diagram for a very massive cluster where many individual stars are resolved. The best candidate for this is of course ω Centauri, already mentioned earlier in this paper. The VizieR service contains many different catalogues with photometric data for stars in ω Centauri. The catalogue used is a quite recent one: *UBV(RI)cHalpha photometry in omega Cen* (Bellini et al. 2009). It contains photometric data for 350,000+ stars which are suspected to be part of the ω Centauri cluster. A nice feature is that the catalogue contains membership probabilities for all observed stars. The membership probability calculation is based on the proper motions of the stars. For the colour to distance diagram of ω Centauri only the stars with membership

probabilities higher than 90 percent were selected, this resulted in a sample of around 60,000 stars.
In figure 22 it can be seen that the statistics for ω Centauri are quite good in the region 0 - 0.9 R/R_{max}, the integrated colour matches a straight line very well. The integrated colours seem to be somewhat blueer near the edges of the cluster, but not convincing enough to make any statements on the spatial distribution of the different populations in the cluster.

7 Conclusion

The major goal of this paper was to study the influence of multiple stellar generations on the integrated colours of globular clusters. The main reason for this is that with the present-day telescopes it is not possible to get fully resolved images of clusters outside the Milky Way. It is possible to get partially resolved images of clusters in for example the near Magellanic Clouds, but when just the integrated colours of clusters are studied, the search volume is expanded enormously.

Two models were made to study the behavior of the integrated colours due to a second generation, based on stellar evolution tables. The first model looked at the evolution of the integrated colours in time. This resulted in a colour to age diagram which predicts the colours for a globular cluster with a certain age. The second model studies the effects of a concentric radial distribution of the second population of stars on the integrated colours. This resulted in a colour to distance diagram, which predicts the colours for globular clusters at a certain distance to the clusters center. From the diagrams in the colour evolution model it could be predicted that clusters become redder as they age with a reddening peak period around an age of 5 Myr, where the most massive stars evolve into giants, followed by a longer period in which they become redder more gradually. A second generation of stars with a younger age causes a blueening peak, but this doesn't last very long. A second generation could therefore possibly be seen when a young cluster has a much blueer colour than it should have according to its fitted age. This however involves an important problem, because the common method of determining the age of a cluster relies on CMD fitting and therefore the colour-data from the cluster is already used to determine the age. Therefore it is a priori likely that the integrated colours from clusters will match the colours predicted by the model, because the age is already fitted to the colours in some way.

The computations of the second model has shown two important things. First it could be seen that a young population concentrated in the center of a globular cluster causes a visible colour-gradient in the integrated colour to distance diagram. The second thing could however be seen as a constraint on the usage of the model, namely that the method of finding multiple populations of stars by looking for colour gradients is only valid for clusters with a very high number of stars. The stellar initial mass function combined with the Padua data allows a population of stars of a certain age to contain one or two very massive stars with very high luminosity, which influence the integrated colours grately. Especially red giants create various spikes and peaks in the integrated colour versus distance diagram and to solve for this (such that the integrated colours match a straight line as expected) simulations with more than 500,000 stars had to be used. Most globular clusters which can be observed are not that massive and therefore they cannot offer good enough statistics.

Both models have been compared to various observations. The first model was compared in two ways, namely firstly that clusters from which it was already suspected that they host multiple populations were used and secondly that catalogues with a high number of clusters were used. As expected the clusters colours in the catalogues matched the predictions by the model very well. Some of the clusters actually were much blueer than the model with a single generation of stars predicts, so it should be interesting to investigate them further.

Two clusters with possible multiple generations actually seem to lie on the blueening peak in the colour to age diagram, but of course the remarks made earlier must be kept in mind.

There are two approaches to obtain data for the integrated colour versus distance diagram. The first one is just by adding all luminosities of individual stars. This has been done for two clusters with a possible young population of stars in the center of the cluster, but because the statistics were far from good enough, they were of little use. The other approach would be to use direct images from the telescopes and measure the integrated colours in several rings around a clusters center directly, in this way also partially (un)resolved clusters can be used. It would be interesting to do this in future studies.

In conclusion it can be said that both models in theory show signs of multiple stellar generations in clusters, but that they are far from accurate enough to really confirm them. Other renowned methods should be used to really confirm multiple generations, but the methods based onintegrated photometry can be used as a first selecting procedure to find which globular clusters to focus on. It is easier to get integrated photometry than photometry of individual stars, so if a measurement of the integrated photometry of a cluster shows some signs of the features that a possible young generation causes, for example a blueer colour in the center, this cluster must be investigated further with other methods. What the models for integrated photometry have shown above all is that very massive stars like blue or red supergiants have great influence on the integrated colours and therefore are of great importance. Research should focus on these massive stars, because it is for example easy to compare their colours and luminosities with the predictions from the theory of stellar evolution. This could be used as an extra constraint for the age measurements of globular clusters.

8 Bibliography

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A Photometric System & Units

This section gives a short overview of the photometric system and the units that were used.

Photometric system: UBVRIJHK (cf. Maiz Apellaniz 2006 + Bessel 1990)

Filter	U	B	V	I
λ_{eff} (nm)	364.189	440.662	550.170	803.657
ω_{eff} (nm)	64.0	92.0	92.0	126.0

Photometric system: HST/ACS WFC

Filter	*F435W*	*F555W*	*F814W*
λ_{eff} (nm)	429.7	534.5	833.2
ω_{eff} (nm)	110.0	150.0	251.0

Units

pc = parsec = $3.08568025 \cdot 10^{16}$ m

$M_{sun} = 1.9891 \cdot 10^{30}$ kg

B IDL programs

B.1 INTMAG.pro

```
;provide the padua isochrone file with steps of log t = 0.05, the
;program gives a matrix Mag with as
;columns Mu,Mb,Mv,Mi

readcol,file,logt,mini,mact,logl,logte,logg,mbol,U,B,V,R,I,J,comment='#'
Mag=fltarr(5,80)
for q=0,79 do begin
w=where(logt eq (6.0+0.05*q))
Ui=U[w]
Bi=B[w]
Vi=V[w]
Ii=I[w]
M=Mini[w]
s=size(M)
dm=fltarr(s(3))
for j=0,(s(3)-2) do dm(j)=M(j+1)-M(j)
Uflux=10^(-Ui/2.5)
Bflux=10^(-Bi/2.5)
Vflux=10^(-Vi/2.5)
Iflux=10^(-Ii/2.5)
Mcorr=M^(-2.35)
Mnorm=M^(-1.35)
k=1/Total(Mnorm*dm)
Sumu=k*Mcorr*Uflux*dm
Sumb=k*Mcorr*Bflux*dm
Sumv=k*Mcorr*Vflux*dm
Sumi=k*Mcorr*Iflux*dm
Lu=Total(Sumu)
Lb=Total(Sumb)
Lv=Total(Sumv)
Li=Total(Sumi)
Mu=-2.5*alog10(Lu)
Mb=-2.5*alog10(Lb)
Mv=-2.5*alog10(Lv)
Mi=-2.5*alog10(Li)
Mag(0,q)=6.00+0.05*q
Mag(1,q)=Mu
Mag(2,q)=Mb
Mag(3,q)=Mv
Mag(4,q)=Mi
Ui=0
Bi=0
```

```
Vi=0
Ii=0
M=0

endfor
openw,1,'intmagoutput.dat'
printf,1,Mag
close,1

end
```

B.2 MULTIGEN.pro

```
;run intmag.pro first to calculate integrated magnitudes
;this provides a matrix Mag with columns: age,U,B,V,I
;provide n which is the fraction of mass in the second generation
;also provide tburst which is the time when second gen is born(in yrs)
;the output is in matrix Mmulti with columns: age(firstgen),U,B,V,I
age=Mag(0,*)
Uin=Mag(1,*)
Bin=Mag(2,*)
Vin=Mag(3,*)
Iin=Mag(4,*)

t=10^age
tsec=t-tburst
for i=0,79 do begin
if tsec(i) lt 0 then tsec(i)=0
endfor
age2=alog10(tsec)

U1lum=10^(-Uin/2.5)
B1lum=10^(-Bin/2.5)
V1lum=10^(-Vin/2.5)
I1lum=10^(-Iin/2.5)

Usec=interpol(U1lum,age,age2)
Bsec=interpol(B1lum,age,age2)
Vsec=interpol(V1lum,age,age2)
Isec=interpol(I1lum,age,age2)

U1l=(1-n)*U1lum
B1l=(1-n)*B1lum
V1l=(1-n)*V1lum
I1l=(1-n)*I1lum

U2l=n*Usec
B2l=n*Bsec
V2l=n*Vsec
I2l=n*Isec

for i=0,79 do begin
if U2l(i) gt 10 then U2l(i)=0
if B2l(i) gt 10 then B2l(i)=0
if V2l(i) gt 10 then V2l(i)=0
if I2l(i) gt 10 then I2l(i)=0
endfor
```

```
Utot=U1l+U2l
Btot=B1l+B2l
Vtot=V1l+V2l
Itot=I1l+I2l

Mmulti=fltarr(5,80)
Mmulti(0,*)=age
Mmulti(1,*)=-2.5*alog10(Utot)
Mmulti(2,*)=-2.5*alog10(Btot)
Mmulti(3,*)=-2.5*alog10(Vtot)
Mmulti(4,*)=-2.5*alog10(Itot)

openw,2,'multigenoutput.dat'
printf,2,Mmulti
close,2
end
```

B.3 SIMPOP.pro

```
;provide N, number of stars
;provide padua isochrone file
;the output is a 8xN table with columns:
;identification number, distance, mass, age, U,B,V,I
;adjust input parameters here:
mmin=2
mmax=20.1889
alpha=-2.35
eta=1.55
rc=0.05
Rmax=1
age=7

;create table and r(distances)
table=fltarr(8,N)

table(0,*)=findgen(N)
table(3,*)=age
r=rmax*findgen(N)/N

;generate pdistr:
Intensity=I0*(1+(r/rc)^2)^(-eta)
Intensity_normalized=Intensity/Total(Intensity)
pdistr=Intensity_normalized

;simulate distances
s=size(pdistr)
y=fltarr(s(3))
bak=fltarr(s(3))
for i=1,(s(3)-1) do begin
bak(0)=pdistr(0)
bak(i)=bak(i-1)+pdistr(i)
endfor
x=randomu(seed,s(3))
for i=0,(s(3)-1) do begin
if (x(i) lt bak(0)) then y(i)=r(0)
for j=1,(s(3)-1) do begin
if (x(i) lt bak(j)) and (x(i) gt bak(j-1)) then y(i)=r(j)
endfor
endfor

Table(1,*)=y

;simulate masses function powerlawdis, n, min, max, index
```

```
mass=powerlawdis(N,mmin,mmax,alpha)
Table(2,*)=mass

;find correct U,B,V,I magnitudes
readcol,file,logt,mini,mact,logl,logte,logg,mbol,Umod,Bmod,Vmod,comment='#'
w1=where(logt eq age)
Uage=Umod[w1]
Bage=Bmod[w1]
Vage=Vmod[w1]
Iage=Imod[w1]

miniage=mini[w1]

Usim=interpol(Uage,miniage,mass)
Bsim=interpol(Bage,miniage,mass)
Vsim=interpol(Vage,miniage,mass)
Isim=interpol(Iage,miniage,mass)

Table(4,*)=Usim
Table(5,*)=Bsim
Table(6,*)=Vsim
Table(7,*)=Isim

openw,1,'simpop1output.dat'
printf,1,Table
close,1

end
```

B.4 DISKS.pro

```
;simpop.pro first generates a cluster
;input data must be assigned here
dist=table(1,*)
Bm=table(5,*)
Vm=table(6,*)
m=table(2,*)
;the output is an array: BV_1pop
;----------------------------------------
;First create the disks:

disk=findgen(20)/20
Bmdisk=fltarr(20)
Vmdisk=fltarr(20)
mindisk=fltarr(20)
starsin=fltarr(20)

;Then create the integrated magnitudes for the single population
Bf=10^(-Bm/2.5)
Vf=10^(-Vm/2.5)

for i=0,18 do begin
wless=where(dist lt disk(i+1))
distless=dist[wless]
Bfless=Bf[wless]
Vfless=Vf[wless]
mless=m[wless]

wmore=where(distless gt disk(i))
distdisk=distless[wmore]
Bfdisk=Bfless[wmore]
Vfdisk=Vfless[wmore]
mdisk=mless[wmore]

Bfdisktot=Total(Bfdisk)
Vfdisktot=Total(Vfdisk)
;------------------------------------
;This section counts the number of stars in each disk

starsindisk=size(mdisk)
giant=where(mdisk gt 9.12,count)
mgiant=mdisk[giant]

mindisk(i)=count
starsin(i)=starsindisk(3)
```

```
;-----------------------------------
Bmdisktot=-2.5*alog10(Bfdisktot)
Vmdisktot=-2.5*alog10(Vfdisktot)

Bmdisk(i)=Bmdisktot
Vmdisk(i)=Vmdisktot

endfor

BV_1pop=Bmdisk-Vmdisk

plot,disk,BV_1pop

end
```

B.5 DISKS2.pro

```
;use input for old generation and young generation!
;the output is in BV_2pop

;Inputdata:

disty=table(1,*)
disto=table1(1,*)
Bmy=table(5,*)
Bmo=table1(5,*)
Vmy=table(6,*)
Vmo=table1(6,*)

Bfy=10^(-Bmy/2.5)
Vfy=10^(-Vmy/2.5)
Bfo=10^(-Bmo/2.5)
Vfo=10^(-Vmo/2.5)

disk=findgen(20)/20
Bmdisk2=fltarr(20)
Vmdisk2=fltarr(20)

for i=0,18 do begin
wyless=where(disty lt disk(i+1))
distyless=disty[wyless]
Bfyless=Bfy[wyless]
Vfyless=Vfy[wyless]

wymore=where(distyless gt disk(i))
distydisk=distyless[wymore]
Bfydisk=Bfyless[wymore]
Vfydisk=Vfyless[wymore]

Bfydisktot=Total(Bfydisk)
Vfydisktot=Total(Vfydisk)

woless=where(disto lt disk(i+1))
distoless=disto[woless]
Bfoless=Bfo[woless]
Vfoless=Vfo[woless]

womore=where(distoless gt disk(i))
distodisk=distoless[womore]
Bfodisk=Bfoless[womore]
Vfodisk=Vfoless[womore]
```

```
Bfodisktot=Total(Bfodisk)
Vfodisktot=Total(Vfodisk)


Bfdisktot2=Bfodisktot+Bfydisktot
Vfdisktot2=Vfodisktot+Vfydisktot

Bmdisktot2=-2.5*alog10(Bfdisktot2)
Vmdisktot2=-2.5*alog10(Vfdisktot2)

Bmdisk2(i)=Bmdisktot2
Vmdisk2(i)=Vmdisktot2

endfor

BV_2pop=Bmdisk2-Vmdisk2

end
```