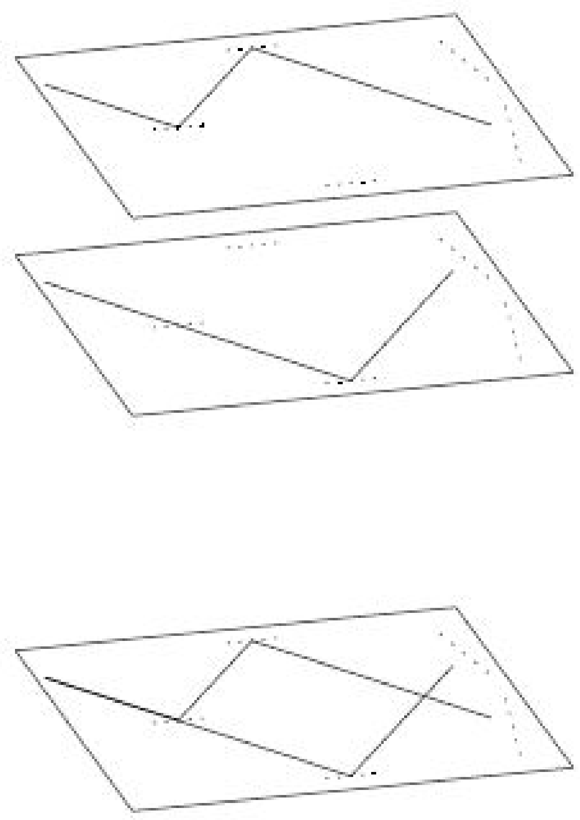
Surreal Trajectories and Intrinsic Position

On the Nature of Position of Objects in Bohm Theory



Utrecht University

Author:
Vincent Schoutsen

Supervisor:
Dr. Thomas Müller

June 23, 2010

Contents

1 Introduction

In this paper, which is supposed to end my career as a bachelor student, I will investigate the meaning of position in Bohm theory. We are looking for an ontological description of what happens in the quantum world. This is a particularly difficult quest. Because quantum theories are underdeterminated by the facts, there are multiple possible interpretations. I will take one interpretation for granted in order to reconstruct the world from that point of view. This is the Bohm theory.

In physics, the ontology of the quantum world is a subject on which there is a lot of disagreement. On the orthodox view, our predictions are always contextual, contextual in such a way that we can only make predictions on the possible measurement outcomes, given a certain measurement setting. In orthodox quantum mechanics there are a lot of individual processes that fail to be explicitly stated within the theory. We can, for instance, only attribute one of a pair of observables corresponding to a pair of canonically conjugate operators to an object, which leads to a view in which the classical terms are not fully applicable. This is detriment of the intuitive picture we can form of the quantum world.

The great merit of de Broglie-Bohm interpretation of quantum mechanics, is its visualizability. In the theory, particles and waves are postulated as ontological entities. This leads to a picture of the quantum world in which the individual processes seem intuitively graspable. There are some properties, however, that still lead to not immediately clear conceptions of the quantum world and our interaction with it.

In the Bohm theory, particles have a position. This position is a position in configuration space, guided by a wave packet; waves also propagate in configuration space. How can we interact with this entity and how is this translatable to real space? Can we form an intuitive objective picture of the quantum world? These are the questions I will try to answer in this paper.

In the first part I will describe a measurement setting in which we seem to measure trajectories that are "not there". In the second part I will go into the notion of position in Bohm theory. In Bohm theory, every particle is said to have the intrinsic property position. What does it mean in real space, to have a position in configuration space?

In the first part I will try to give the overview of the workings of the theory. I will first give a small historical introduction to the de Broglie-Bohm theory and its formalism in chapter 2. In chapter 3, I will investigate measurement situations in Bohm theory. We will see a couple of things happen to the trajectories in Bohm theory that we would not expect in advance. If we make a slow measurement, in which the value of the measurement is stored for some time, we see that there is a change in the trajectories only at the moment the value is read. This leads to strange situations in which one might wonder what it actually is we are measuring. The theory-ladenness of observation becomes very apparent. Is what we measure explainable outside the theoretical frame of Bohmian mechanics? It seems as if this is not possible. The first question that comes to mind is, can we talk about particle trajectories in real space?

In the second part I will investigate what it means for a particle to have a position in configuration space. Configuration space is a theoretical construct. If we know the theoretical trajectories and we know what it means in real space to have a theoretical position, we can form an image of the quantum world. In order to do this, I will in chapter 4 define the notion of intrinsic property to see if this position is an intrinsic property. In chapter 5 I will look into how a theory is connected to reality. Hilbert gave a very beautiful description of how we interpret objects to follow the axioms of the theory. To find the interpretation of the theory, would be to know how a particle in configuration space relates to its referent in real space. In chapter 6 I will therefore give a short outline of two possible positions on the ontology of real space. In chapter 7 I will try to project configuration space upon real space to see what it means in real space, to have a position in configuration space.

The trajectories Bohm theory predicts, are projectable on real space. There is a problem, however, that, when we project the trajectories outside of the theoretical configuration space, the explanation the Bohm theory gives of the trajectories is no longer tenable. The Bohm theory is only valid in configuration space. Outside configuration space, the position of the particle becomes an empty expression. We are confronted with the limits of the objective real world, for we are always restricted by the theory-ladenness in our explanation of the phenomena.

In the final chapter I will shortly recapitulate the paper and try to form a view concerning what we can say about the quantum world. I will describe some views from the literature on the ontology of configuration space. The projection on real space limits us in speaking about the world in an intuitive manner. From this we can conclude that the Bohm theory gives us just as the orthodox interpretation, a complicated world view. Not better or worse, but actually quite alike.

Part I

Trajectories

2 Causal Quantum Theory

2.1 De Broglie, 1927

In 1927, Louis de Broglie came up with a deterministic theory of quantum mechanics. He already thought up his distinction of matter in waves and particles as early as 1923 and also wrote his doctoral dissertation on this subject [2]. In his doctoral dissertation he coined the De Broglie hypothesis, claiming that all matter displays wave-like behavior. This is imaginable if the matter is guided by waves. This explains the wave-like behavior as well as the particle-like structures we encounter in measurement, both are existent.

This theory is a slight modification of the Schrödinger equation and it yields the same measurement outcomes. The particle's position is a hidden variable, the probability the measurements predict, merely reflects our ignorance of the actual, hidden, position. The particle is propelled by a so-called 'pilot wave'. The equations are equal to the Hamilton-Jacobi equations with an extra potential term. This allowed de Broglie to keep a causal view, which worked just as the old classical analogue did.

The effective wave packets, the wave packets without their 'tails', were postulated as physically real and existing. The particle's velocity is found by the gradient of the wave front, giving only trajectories perpendicular to the wave field.

There were some allegations by Pauli about a scattering problem. De Broglie's rebuttal was found inadequate by the audience, even though it was later shown that all necessary elements were present [2].

In 1928, de Broglie says that

> [p]ropagation in a configuration-space of purely abstract existence is, in fact, out of the question from the physical point of view. The wave representation of our system ought to involve N waves propagated in real space instead of a single wave propagated in the configuration space (p. 131)[20]

There is some friction between his idea about the ontology and the theory he founded. In his early theory he explicitly separated the particle and the wave and gave them both ontological existence. In his later views, after extensive discussions with Einstein, he opted for the interpretation in which the wave is not existent, but a mere theoretical entity [7]. In this interpretation we lose the causal effect of the wave on the particle. De Broglie failed his own visionary view.

De Broglie taught the orthodox quantum theory from 1932 for twenty years [12]. In 1932, von Neumann published a book in which every hidden-variable theory seemed to be refuted [44]. Later his refutation was proven to be based on false assumptions [28].

In 1952, Bohm published two papers setting forth a similar theory as de Broglie. De Broglie recognized Bohms ideas as his own and became one of the few early proponents of the theory.

2.2 Bohm, 1952

In 1951, David Bohm reinvented the causal quantum theory program. He was unaware of the fact that De Broglie already thought up this interpretation of the quantum formalism [9]. He suggests that it is possible to talk about particles just in 'completing' the Schrödinger equation by entering for ψ, the complex wave function, a complex number in its polar form, $Re^{iS/\hbar}$. The rest of the theory then follows without many problems, as long as one is willing to interpret the equations in a logical consistent manner.

We start from the Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + V(\mathbf{x})\psi \tag{1}$$

where $V(\mathbf{x})$ is an arbitrary potential depending on the position of the particle.

Filling in the polar form of the wave function, and then splitting the resulting formula in its real and imaginary parts, it yields two new equations. One for $R(\mathbf{x})$ and one for $S(\mathbf{x})$

$$\frac{\partial R}{\partial t} = -\frac{1}{2m}[R\nabla^2 S + 2\nabla R\cdot\nabla S] \tag{2}$$

$$\frac{\partial S}{\partial t} = -\left[\frac{(\nabla S)^2}{2m} + V(\mathbf{x}) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\right] \tag{3}$$

We see that the only term involving $\hbar$ is the second term in the equation for $S(\mathbf{x})$. If we set this term to zero, we are left with the classical Hamilton-Jacobi equation:

$$\frac{\partial S}{\partial t} = -\left[\frac{(\nabla S)^2}{2m} + V(\mathbf{x})\right] \tag{4}$$

S is interpreted as the 'pilot wave'. The rest term that we just set to zero, we call $U(\mathbf{x})$, the quantum potential. It is defined as:

$$U(\mathbf{x}) = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{5}$$

We see that the quantum potential only depends on R and $\hbar$.

R is interpreted as the square root of the probability density. That means that R^2 is, just as in the orthodox quantum theory, the probability to find a particle. The probability is about an ensemble of particles of which only one is actual.

We see that a particle has a velocity, namely $\frac{\nabla S}{m}$, thus contradicting the orthodox quantum mechanics on several places. Not only does every object have position even outside of the measurements, it is also possible to calculate the momentum if we know the position. This theory gives a completely different interpretation of the quantum world. In the orthodox quantum theory, there is Heisenberg's *uncertainty principle* [26]. The uncertainty principle states that two observables represented by two operators that are canonically conjugates, cannot be defined with arbitrary precision. Position and momentum are such a pair of canonical conjugates, ergo are not precisely definable in orthodox quantum theory.

The ontology of Bohm theory constitutes of wave packets and particles, both existent in configuration space.

Another important thing to notice is that, because the waves and particles live in configuration space, they are governed by first order differential equations. A first-order differential equation has, per input, one unique solution. This results in single-valuedness, therefore two trajectories cannot just cross, they have to coincide fully. The crossing of trajectories is not possible.

2.3 Observables other than position

There is, because of this great difference, a problem when interpreting objects from orthodox quantum mechanics within Bohm theory. Beware about naive realism about operators; in Bohm theory, there is only position [19]. In orthodox quantum theory, every self-adjoined operator is considered an observable[1] by postulate. In Bohm theory, every measurement of a non-position observable is considered a measurement of a contextual, ergo non-intrinsic, property.

A momentum measurement is only dependent on the S component of the theory, not always giving the same measurement results. A measurement of a particle in a box, subject to a potential

$$V(\mathbf{x}) = \infty \text{ for } \mathbf{x} = \pm a, \quad (6)$$
$$0 \text{ elsewhere} \quad (7)$$

will return no momentum at all. While in the orthodox case, there is a series of waves with different energies and therefore different momenta. In the Bohmian case, the particle is at rest. The momentum that is predicted in the orthodox case, is all in the quantum potential in the Bohmian prediction.

Bohm chose his theory's preferred variable to be a position variable. It would have been possible to put the whole through the Fourier machinery and we would be describing objects with momenta instead of objects with position. Bohm answers to this criticism on his theory, that

> ...it should be kept in mind that before this proposal was made there had existed a widespread impression that no conceptions of hidden variables at all, not even if they were abstract, hypothetical, and "metaphysical", could possibly be consistent with the quantum theory... it was therefore sufficient to propose any logically consistent theory that explained the quantum mechanics through hidden variables, no matter how abstract and "metaphysical" it might be. Thus, the existence of even a single consistent theory of this kind showed that whatever arguments one might continue to use against hidden variables, one could no longer use the argument that they are inconceivable. Of course, the specific theory that was proposed was not satisfactory for general physical reasons. But if one such theory is possible, then other and better theories may also be possible. And the natural implication of this argument is "Why not try to find them?" ([10], p. 360 and [11], p. 80)

The theory Bohm provides, is only provided to show that a theory with hidden variables is conceivable. whether these variables are position or momenta variables, does not matter to him. Pauli and Heisenberg said that Bohm creates an asymmetry by attributing reality to one of a pair of canonical conjugate variables [37]. By attributing reality to position, momentum becomes dependent on the context.

I will look into intrinsic properties in chapter 4. At the moment, we will take Bohm theory for granted, and allow position to be an intrinsic non-contextual property.

It is not difficult to incorporate spin in Bohmian mechanics. This incorporation shows very clearly what it means for a property to be contextualized. We will also need spin in Bohm theory in the next chapter to show what happens if we record a measurement in a non-macroscopic contextualized property such as spin in Bohm theory.

Spin in Bohm theory is nothing more than the configuration of the wave packet relative to the measurement apparatus. Spin is a contextual property, it is possible to change the measurement setting, while the trajectories remain the same, and measure up where we just measured down [**?**].

[1]This is a controversial subject in orthodox quantum mechanics, an observable corresponds with the property of an object through Heisenberg's "measurement is definition" and "measurement is creation". For more on this subject, see [26] and [28].

Spin is not an intrinsic property in Bohm theory. In practice it means that, if a particle starts in the upper half of a wave packet, before it is split in a Stern-Gerlach experiment, it will end up in the upper wave packet after the split, thus we will measure spin up. If, otherwise, the particle is in the lower half of the initial wave packet before the split, it will end up in the spin down wave packet, ergo we measure spin down.

3 Surreal Trajectories

3.1 The Persistence of Memory

In 2000, Jeff Barret writes a summary of the problems encountered in Bohm theory when a delayed measurement choice is made [3]. I will stay close to the original text, because it provides a welcome support in analyzing the theme.

Barrett starts by outlining Bohm theory, what I did in the last chapter.

> Since particle positions are always determinate on Bohms theory, this would guarantee determinate measurement records. (p. 683) [3]

This lets us conceive a measurement in the classical way, e.g. a measurement just gives us the value there is. Position is, in Bohmian theory, also a non-contextual property. So a position measurement is not influenced by the measurement setting, but only by the actual position of the particle. Notice that there is a very strong claim here; there is a particle and it has a position.

> Consequently, contextual properties are not intrinsic properties of the system to which they are typically ascribed. (p. 683) [3]

Here is something that will come in very handy later on in the paper. Because a contextual property is not intrinsic, we are led to believe that non-contextual properties are[2]. A contextual property is a property that depends on the method of measuring. So the outcome of a measurement depends on the method of measurement.

> While the language of contextual properties provides a convenient (but often misleading!) way of comparing the predictions of Bohms theory with the predictions of other physical theories, the predictions of Bohms theory are always ultimately just predictions about the evolution of the wavefunction and the positions of the particles relative to the wavefunction. (p. 684) [3]

In Bohm theory, a measurement is, in the end, always a position measurement. All other observables are only arbitrary arrangements of measurement settings and the position of the actual particle. A measurement in Bohmian mechanics always correlates the position of a particle to the position of a pointer on the measurement device. We can only speak about position as being "real". More precisely; we can only speak of the configuration of the particle relative to the wave function. So, if we regard the wave function as real, the position of the particle relative to the wave function is the only property that is measured. The measurement setting tells us how we call the property, not what we measure. When this is clear, we can go on and describe how this works.

A Bohmian particle lives in *Configuration Space*. The configuration space is of dimension $3N$, where N is the number of particles. Because this is a non-relativistic theory, we assume real space to have 3 dimensions. There is a difference between the theoretical configuration space and the assumed 'real' space which later will prove to be very problematical for the interpretation of what it is to be an intrinsic property.

The surreal trajectories were named surreal in 1992 by Englert, Scully, Süssman and Walther [23] in an article that was set out to prove Bohm theory wrong. The surrealistic element of the trajectories would be that the theory would provide different trajectories then what the measurement apparatus measures. This would deliver, if it is the case, a devastating blow to the theory.

[2]This is belief is founded upon a logical fallacy: $A \to \neg B$ does not imply that $\neg A \to B$

In their attempt to prove this, they use a delayed choice experiment in the context of a two-path. See Figure 1.

We start with a particle P with spin up in the z-direction. This particle's wave packet is split into an x-spin up component and an x-spin down component. The particle starts at position S

$$|\uparrow_z\rangle_P |S\rangle_P = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle_P + |\downarrow_x\rangle_P) |S\rangle_P \quad (8)$$

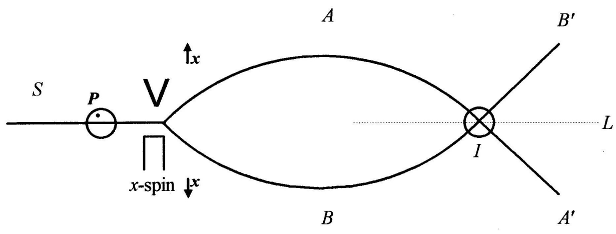


Figure 1: The set-up of the two-path experiment taken from [3]

After the wave packet splits, it will just look like this:

$$\frac{1}{\sqrt{2}}(|\uparrow_x\rangle_P |A\rangle_P + |\downarrow_x\rangle_P |B\rangle_P) \quad (9)$$

Then, at point I, the two particles will bounce, so to speak, the particle that at first follows the A path, will now follow the B' path. This is because, due to symmetry relations, there is a probability 0 for a particle at the line L. So if a particle reaches the line L, the quantum potential[3] will be infinite at this point, repelling the particle in such a way, that it produces the bounce [39].

This violation of the law of conservation of momentum is not observable, thus the final state is, just as we would normally expect:

$$\frac{1}{\sqrt{2}}(|\uparrow_x\rangle_P |A'\rangle_P + |\downarrow_x\rangle_P |B'\rangle_P) \quad (10)$$

Remember that this is possible because spin is not a property of the particle but of the wave packet. It is not the case that the particle flips spins at the moment of the bounce, the particle changes wave packets. In our example, the particle changes wave packets at point I. And as long as the symmetry will be intact, it will keep changing wave packets [43].

Imagine a particle as a dot inside a circular wave packet. The dot can be anywhere in the wave packet. At the moment the wave packets overlap, the particle will be in two wave packets; his original one, going from A to A' and the other one, going from B to B'. The moment that the particle is in two wave packets at the same time, it will be somewhere in A, but near the outer rim of B. It will continue to proceed horizontally until it exits the A packet. Then it will just follow the route of the B packet. When the particle is inside both of the wave packets, its horizontal velocity will be

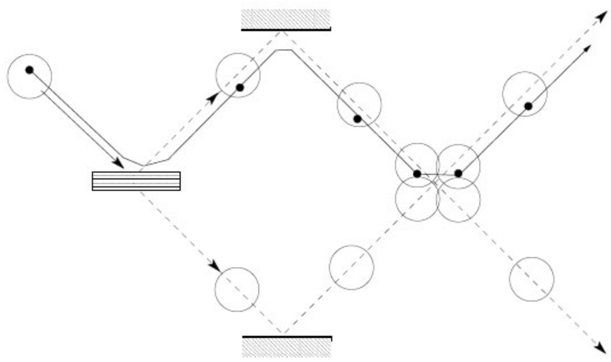


Figure 2: The change of wave packet, taken from [43]

$$v_{\text{hor}} = \frac{v_A + v_B}{2} \quad (11)$$

where v_A, v_B are the velocities of wave packets A and B.

The surreal trajectories would, according to ESSW, be observed if we measure the particle's position in one path, while Bohmian mechanics predicts the particle to have taken the other. This

[3]Remember that the quantum potential evolves proportional to $\frac{1}{R}$.

would mean that, if we measure a particle in A and it ends up in A', it did not bounce. It seems as if we have measured something different than we expected to measure. Let's see when trajectories bounce.

The bouncing behavior is only possible if the two particles are in the same dimension in configuration space and there is a line with probability 0. It is impossible to observe the bounce and show the violation of momentum conservation. To show this violation, we would have to make two measurements. The first measurement before the particle reaches I, to show which path the particle follows, and one measurement after, to show where it ends up. Let's see what happens if we make a measurement.

3.2 Trajectory Measurement

In making a measurement, we have to correlate the position of a particle with the position of a macroscopic object [6][5]. In following Bell in his article on delayed-choice experiments, we take an apparatus, with a flag. We name the flag F.

The detector will correlate the position of F with the position of the particle. The detector is put halfway path B. As Barrett puts it:

> More specifically, consider a single flag particle F whose position (as represented by the quantum-mechanical state) gets correlated with the position of P as follows: (1) if P is in an eigenstate of traveling path A, then F remains in an eigenstate of pointing at "No" and (2) if P is in an eigenstate of traveling path B, then F ends up in an eigenstate of pointing at "Yes". (p. 689) [3]

Even though this experimental set-up seems a lot like the earlier conducted experiment, there is a crucial difference. In performing a measurement in one of the two paths, the symmetry that prevented the particle from crossing the line L, is destroyed. Now, the particle can pass through I without performing the bounce.

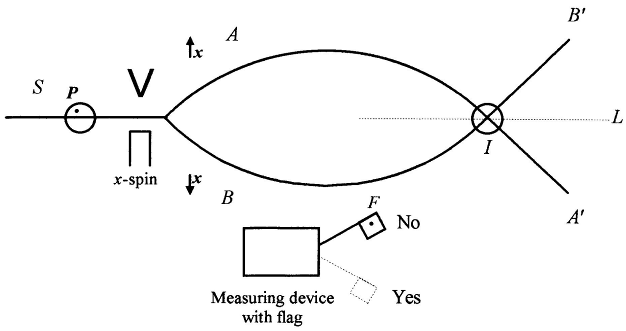


Figure 3: The set-up of the two-path experiment with F's position correlated with the position of P, taken from [3]

This is because, through the correlation of the particle's position to the flag F, the wave packets do not longer overlap in configuration space. Therefore the particle cannot perform his wave packet switching trick any longer. Even though the trajectories seem to intersect in real space, they merely pass in configuration space. In Dirac notation:

$$|\uparrow_z\rangle\,|S\rangle_P\,|\text{``No''}\rangle_F = |S\rangle_P\,|\text{``No''}\rangle_F\,\frac{1}{\sqrt{2}}(|\uparrow_x\rangle_P + |\downarrow_x\rangle_P) \tag{12}$$

$$P\text{'s wave packet splits }:|\text{``No''}\rangle_F\,\frac{1}{\sqrt{2}}(|\uparrow_x\rangle_P\,|A\rangle_P + |\downarrow_x\rangle_P\,|B\rangle_P) \tag{13}$$

$$F\text{ is correlated with P }:\frac{1}{\sqrt{2}}(|\uparrow_x\rangle_P\,|A'\rangle_P\,|\text{``No''}\rangle_F + |\downarrow_x\rangle_P\,|B'\rangle_P\,|\text{``Yes''}\rangle_F) \tag{14}$$

This shows that the particle, if it starts in the top of the initial wave packet, takes route A, crosses I and proceeds onto A', F stays at "No". If P had started in the lower half at S, it takes route B, P's position would be correlated with that of F, F goes to "Yes", P crosses I and proceeds into B'. And F will record the right paths every time; it will only record that P was on path A, if it ended up on path A'.

3.3 Slow Measurement

If we perform a slow measurement, we will reintroduce the surreal trajectories. We need an extra x-spin up particle M in which the trajectory of the particle is 'stored' until after the problematic point I. After this, the spin of particle M is correlated with the position of F and the flag will point up or down. If this would be possible, we can reintroduce the surreal trajectories problem. Barrett defines that the interaction between P and M is such that

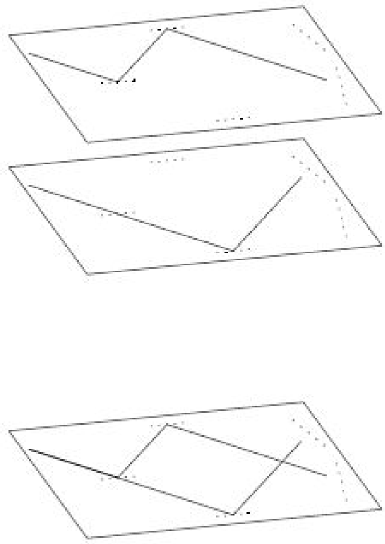


Figure 4: the top two planes show the Bohm trajectories, the lower plane shows their projection on real space, taken from [16]

If P's initial effective wave function were an x-spin up eigenstate, then nothing would happen to M's effective wave function; but if P's initial effective wave function were an x-spin down eigenstate, then the spin index of M's effective wave function would be flipped from x-spin up to x-spin down. (p.692-693)

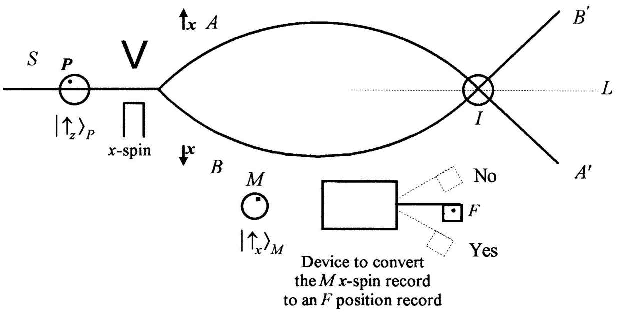


Figure 5: We try to capture the path with the spin of M, with F's position correlated with the spin of M, taken from [3]

Of course is there no other thing than position in Bohm theory. So the value of the x-spin record depends on the way it is read. It is possibly not a valid Bohmian account of recording information. So Barrett argues that we should keep in mind the orthodox collapse interpretation of quantum mechanics in which, if a wave function has collapsed and has eliminated the other terms in the superposition, the spin of M can be thought of as a which-path detector. We use this view to allow the measurement.

Next thing, there is the machine with the flag, F. The machine is designed to correlate the position of the flag particle F to the spin of particle M. So the flag particle will point to "No" if M is in the x-spin up state, and "Yes" if it is in the x-spin down state. The device will record the path taken by P in the flag position F.

The correlation of the spin of M and the flag position is only effectuated after P is long gone. Until then, the symmetry in the probability current is maintained, and the imaginary line L stays uncrossable. The paths bounce and the measurement device will be fooled. In Dirac-notation:

$$|\uparrow_z\rangle_P |S\rangle_P |\uparrow_x\rangle_M |\text{“No”}\rangle_F = |S\rangle_P |\uparrow_x\rangle_M |\text{“No”}\rangle_F \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_P + |\downarrow_z\rangle_P) \quad (15)$$

$$|\uparrow_x\rangle_M |\text{“No”}\rangle_F \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_P |A\rangle_P + |\downarrow_z\rangle_P |B\rangle_P) \quad (16)$$

And if we now correlate M's spin with the position of P:

$$|\text{"No"}\rangle_F \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_P |A\rangle_P |{\uparrow}\rangle_M + |{\downarrow_z}\rangle_P |B\rangle_P |{\downarrow_x}\rangle_M) \tag{17}$$

Then the wave packets pass through each other,

$$|\text{"No"}\rangle_F \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_P |I\rangle_P |{\uparrow}\rangle_M + |{\downarrow_z}\rangle_P |I\rangle_P |{\downarrow_x}\rangle_M) \tag{18}$$

and they separate again.

$$|\text{"No"}\rangle_F \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_P |A'\rangle_P |{\uparrow}\rangle_M + |{\downarrow_z}\rangle_P |B'\rangle_P |{\downarrow_x}\rangle_M) \tag{19}$$

After the separation we correlate M's spin with F

$$\frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_P |A\rangle_P |{\uparrow}\rangle_M |\text{"No"}\rangle_F + |{\downarrow_z}\rangle_P |B\rangle_P |{\downarrow_x}\rangle_M |\text{"Yes"}\rangle_F) \tag{20}$$

The wave functions do cross each other, but the particles change wave function. We end up with a particle that went from S, to A to B' while the flag points at "Yes". Or another particle that, if it started in the lower half of the initial wave packet, took route S, B, A' with F ending up on "No". It seems as if our measurement device did not work the way it was intended.

3.4 Surreal Trajectories

Finally we arrive at the surreal trajectories. Where the first two attempts merely illustrated the workings of Bohm theory in this experiment, we finally succeeded in creating a 'strange' situation. How to interpret this?

Because we interpreted M as a which path detector, or at least F, we thought that we would measure the path the particle took. And we did. This is in need of some explanation. It is possible to see the detector as reliable and Bohm theory as true. How can this be done?

The first thing we should notice, is that, up until now, we separated our notions of what the detector would measure and the trajectories Bohm theory predicts. However, according to DFGZ we have to

> [...] bear in mind that before one can speak coherently about the path of a particle, detected or otherwise, one must have in mind a theoretical framework in terms of which this notion has some meaning. BM provides one such framework, but it should be clear that within this framework the [test particle] can be detected passing only through the slit through which its trajectory in fact passes. More to the point, within a Bohmian framework it is the very existence of trajectories which allows us to assign some meaning to this talk of about detection of paths. (p. 1261-1262) [21]

So it seems that, in order to talk about trajectories of particles at the quantum level, we already need Bohmian mechanics. ESSW's original attempt, namely to show that Bohm theory is less clear than orthodox quantum mechanics, is deemed to fail for the incompatibility of orthodox quantum mechanics and trajectories. In other words; the Bohmian is the only one that is allowed to measure trajectories, so if Bohm theory measures "Yes" in this case, it means that the particle traveled path B.

But this is not how we intended the apparatus to work. So what happened?

This is a good example of the theory-ladenness of observation. Only the Bohm-theory is allowed to explain particle measurements in terms of trajectories. And only if a position measurement is made, does a *real* measurement occur. The Bohmian position is not a real position, only a position in configuration space. We cannot use another theory to talk about particle trajectories in configuration space, we need a theory that explains configuration space as well as particle trajectories, in order to legitimately talk about trajectories. The explanation of the measurements and the working of the measurement device are thus not given. They are part of the theory that has to explain, or interpret, the results. We ourselves have to explain, within Bohm theory, what happens during the measurement process and what the device is measuring. If we do this in a consistent manner, we see that there is nothing surrealistic about Bohm theory.

The recording of the position in spin, because it was no position measurement, is no determinate measurement. Only at the moment that M's spin is correlated with the position of F, can we speak of a definite measurement. Only at the moment the definite measurement is made, is the symmetry destroyed. Therefore, the particles do bounce. At the moment the correlation is made, the detector reliably tells us in which wave packet the particle is, *at that moment*.

This measurement is thus a non-local measurement of the spin-configuration of the wave packet that is occupied with the particle at the moment of the correlation. The non-locality of Bohm theory is a bit awkward from a physical perspective[4]. The measurement of the position of the particle in the wave packet can happen if the wave packet, and the particle in it, are already lightyears away. This does not matter, at the moment of the correlation, flag F will measure in which of the packets P is.

3.5 Questions

We found some very strange properties of objects in Bohm theory. The surreal trajectories are a great example of the theory-ladenness of observation. Our measurement device only measures in which effective wave packet the particle is at the moment of observation. It does not measure what we intuitively thought it would measure. The particle changes wave packets at point I.

If we would have correlated the position of a particle with a macroscopic object, as we did before, the trajectories would have intersected in real space, but not in configuration space. Then the question what happens in point I, is even more interesting. Remember that trajectories cannot intersect because of the first order differential equations.

In the beginning, the surreal trajectory problem was thought of as measuring the position of a particle, while the theory predicted the particle not to be there. Our talk about the measurement of position in Bohm theory seemed terribly theory-laden. While we constructed a device to measure the position of a particle at one particular moment, the device is explained in a completely unexpected manner in Bohm theory.

We did not measure the position of the particle, but the position of the wave packet guiding the particle. The trajectory of the particle lives in configuration space. A measurement of a particle's position, or its trajectory, creates a new dimension in configuration space for this trajectory. The possible trajectories pass each other in configuration space if there has been a correlation of the position of the particle with the position of a macroscopic object. This new three-space that is created upon measurement, does house a trajectory. We never experience the creation of extra dimension upon measurement, because, in the end, only one trajectory is actual. The correlation with the macroscopic object, makes sure that, because macroscopic objets can only have one position at the time, the two trajectories cannot be actual at the same time. For a causal explanation of the particle trajectories, possible and actual, we must assume the wave packets guiding the particles, as being real. They do exist in a multidimensional configuration space. So

[4]A thorough investigation of the possibilities of non-locality in Bohm theory is not within the scope of this paper

even though the trajectories can be possible trajectories, the wave packets exist in configuration space, making these trajectories possible.

This ultimately leads to the question, what does it mean to exist in configuration space? The particles are propelled by the wave packets and are also in configuration space. Is the position of a particle in configuration space on a given time a property of the particle? If this is a property, is it intrinsic or extrinsic?

Part II

Properties

4 Intrinsic and Extrinsic Properties

Bohm theory postulates position as a property. I am wondering whether the property, "having a position in configuration space", corresponds to an intrinsic property. In order to investigate this bold claim, we have to define the notion of intrinsic property. There is a widespread discussion in the literature on this subject, so I will give a summary of the most intuitively graspable ideas. At the end of this chapter we should have a clear view of in what aspect a property like position can be intrinsic.

4.1 Intrinsic/Extrinsic

Lewis gives a definition of Intrinsic that might just about cover all the intuitive aspects of the notion.

> A sentence or statement or proposition that ascribes intrinsic properties to something is entirely about that thing; whereas an ascription of extrinsic properties to something is not entirely about that thing, though it may well be about some larger whole which includes that thing as part. A thing has its intrinsic properties in virtue of the way that thing itself, and nothing else, is. Not so for extrinsic properties, though a thing may well have these in virtue of the way some larger whole is. The intrinsic properties of something depend only on that thing; whereas the extrinsic properties of something may depend, wholly or partly, on something else. If something has an intrinsic property, then so does any perfect duplicate of that thing; whereas duplicates situated in different surroundings will differ in their extrinsic properties. (p. 111-112)[35]

A property is intrinsic, if it says something on how the thing is, independent of anything else. A property is extrinsic if it says something on how the thing is in virtue of how other things are. An object's rest mass is intrinsic and its weight is extrinsic. The weight of an object is depending of the gravitational field in which the object is. So far the intuitive constituents of the notion intrinsic.

In Lewis's definition, position is not an intrinsic properties. In the last sentence Lewis states that the different surroundings of an object make sure that an object has different extrinsic properties. There are multiple possible ideas of space and position, as we will see in chapter 6, one of these ideas defines an object's position as relative to its surroundings. If we see position in this way, Lewis immediately disposes of position as an extrinsic property. If we think of the position of the particle in absolute space, however, it is possible to think of position as an intrinsic property.

In a later text by Lewis, with Langton, they give a set of requirements that have to be fulfilled for an object to have an intrinsic property[33].

- The property can be had or lacked independent of the object having it is accompanied or lonely.
- A property is intrinsic iff its duplicate has the same intrinsic properties.
- A property should not be the negation of a disjunctive property.

The first of these requirements, is pretty straightforward. It merely states that it has to be the property of the thing itself, not depending of its surroundings. The second of these requirements depends on the idea of a duplicate. A duplicate is defined by its intrinsic properties, all the properties that do not hold for the duplicate, but do hold for the original, are not intrinsic. We can immediately see that two duplicates should have the same position if position is an intrinsic property. The third is not needed in our application of the term, therefore I will not go into this distinction.

We also need a notion of a natural property. A natural property is a special case of intrinsic property, so "being lonely" is not a natural property. The distinction is not explicitly made in the text, in fact, the only requirement is that we have this subclass and that it belongs to the intrinsic properties. I would like to define the class of natural properties as the class of properties objects have, given by nature. We can assume that it is a subclass of the intrinsic properties. We can attribute "being made of wood" to a tree and say that it is a natural property of a tree.

This property is not always natural, because ashtrays can and cannot be made of wood. If we have a Newtonian substantivalist[5] view of space, a position is a natural property of a part of space. A position in configuration space can never be a natural property, because configuration space is a human, and not a natural construct. We want to define position in configuration space in terms of position in real space, hoping to find an intrinsic property. We should, therefore look to the property that defines the relation between the positions in these two spaces.

4.2 Relational/Non-Relational

In Bohm theory, almost all measurable properties are contextual properties and only depending on the position of the particle with respect to the wave function. A contextual property is relational in that it depends on the context of measurement. In the debate on properties, the intrinsic relations are often said to be non-relational where the extrinsic are relational. This is not always the case, for it is possible to have a relational intrinsic property such as "having longer legs than arms" [45]. This property defines a relation, but is still intrinsic.

For our purpose it would be best to define the set of intrinsic relations. We want to know if the relation between the position of an object in configuration space and real space is intrinsic. Is it possible to state that the relational property that gives a particle a position in real space, based on its position in configuration space, is intrinsic? We should first define the set of intrinsic relations, to see if our relational property can be a part of it.

The best way to define these intrinsic relations would be in exactly the same manner as intrinsic properties.

1. The relations between the objects are not merely relations because the objects are accompanied or lonely, both cases are contingent.
2. If we duplicate the objects between which the relation holds, the same intrinsic relations will hold.

A good example is a spatio-temporal distance relation between two events. This relation should be intrinsic to the events at hand. Given that every event has a unique spatio-temporal location, the relation between the two points is always the same.

We will need the notion of intrinsic relation in order to try and find the intrinsic relation between position in configuration space and position in real space. If an object has one property, does it have the other always in the same relation? We are looking for the internal relation that holds between two properties that might be attributed to the same object. Say

[5]This view will explicitly be stated in chapter 6.

$$Q(x) = f(P(x)) \tag{21}$$

Where P is the predicate position in configuration space and Q is the predicate position in real space. We are looking for the natural relation between the P's and the Q's. If this is a function, the relation will be intrinsic. In order to investigate this, I will define what a position in configuration space could be, then I will define what a position in real space could be. In the end I will try to see if this relation holds, by projecting configuration space on real space.

5 Position as a Property in Configuration Space

In this chapter I will show a bit of the history of the method of describing the world. If it can be a property to have a position in configuration space, then this has to relate to real space in a certain way in order for the theory to make sense.

In order to show which steps there have to be made to let the theoretical model refer to reality, I will give an historical account [32] of a similar point of view. In this point of view, some respectable scientists tried to justify the crossing of this gap. How can a theory talk about the world? There are multiple solution to this problem, and there is one which was stated by Hilbert in 1899 [27].

5.1 Hilbert and Einstein

In his 1922 paper *Geometry and Experience* [22], Einstein gives his view on the matter:

> To be able to make assertions about the relations of real objects, geometry must be stripped of its merely logical-formal character by the co-ordination of real objects of experience with the empty conceptual frame-work of axiomatic geometry. (...) Geometry thus completed is evidently a natural science. [22]

Einstein wants to do exactly the same as we do; describe the world with a mathematical geometrical framework. He succeeded in joining his description of space time to the objects existing around us. “Stripping of its logical-formal character” means that we need more than just the axioms of the geometrical space. We need to let items in the theoretical space refer to an object in the world.

In 1899, Hilbert uses the technique of reinterpretation of the terms in the axioms within another theory to prove their consistency [8]. This shows the logical consistency of the axioms. But this makes the meaning of the terms in the axioms empty until interpreted in a certain way. In a letter to Frege, the 28th of December 1899, he writes:

> Wenn ich unter meinen Punkten irgendwelche Systeme von Dingen, z.B. das System: Liebe, Gesetz, Schornsteinfeger ..., denke und dann meine sämtlichen Axiome als Beziehungen zwischen diesen Dingen annehme, so gelten meine Sätze, z.B. der Pythagoras, auch von diesen Dingen. (p. 13) [25]

What Hilbert means in this letter, and what Einstein very well understood, is that in order to let a geometrical system of axioms refer to anything in the world, we need an interpretation of the elements in the theory. So every object in the axiomatic theory has a counterpart by means of interpretation, in the world.

Without the interpretation, the set of axioms is merely theoretical. The theoretical entities have no reference or meaning. The interpretation of the geometry gives the theoretical constructs a meaning. This meaning does not depend on the axioms itself, but more on the interpretation of it.

It clearly follows that a theoretical framework is not complete without the objects in the framework having a clear interpretation. A theory consists of the theoretical description and the interpretation that gives the theoretical entities a meaning in real space. This interpretation is what we are looking for. What does it mean to have a position in configuration space?

5.2 What does it mean to have a Position in Configuration Space?

We have to be careful not to mix up the actual events happening with the theoretical entities describing them. Our intuitions tell us that something has to happen in real space, but what is it? In his book about Bohm theory, named the *The Quantum Theory of Motion*, Holland argues that configuration space is real and existing [30]. This would solve our problem, because if configuration space is real, a position in configuration space would also be real.

There are, however, no clues that point in this direction. Only our incompetence in explaining the quantum world more intuitively otherwise, does not mean that space has 3 distinct dimensions per particle. That claim is absurd. We encounter incoherent situations in the limit where $\hbar$ is negligible. In the classical realm, Holland still needs the $3N$ dimensions.

If there is a theoretical entity with a position in configuration space, this position can be a property. We do have to be careful on how we define position relative to a framework. There is a distinction we should make between position in configuration space and mere coordinates.

The coordinates assigned to an object within a theoretical framework are depending on the choice of the axes. It is possible to argue that the object within the framework does have an actual position within the framework. The coordinates only name the position. I will not go into a theory of naming, what will be clear, however, is that the referent of any element of a possible coordinate descriptions of space, should be a position within the space itself.

This is insightful if we keep in mind the construction of an affine space. An affine space is a vector space *without an origin* [15]. An affine space is as a vector space without a defined origin. So we have the coordinate system defined upon the configuration space, then, if we perform a translation or a rotation, the configuration itself stays intact. The structure that has been preserved in the transformation, is the affine configuration space. There is a space underneath the coordinates and we can define any arbitrary coordinate system on top of this space, but the space itself does have positions.

So we know how the coordinates and the position are related. But we want to define the position within the configuration space as well. Then we have to know what the positions in the configuration space means in the real world.

This relation can be characterized as a property of a particle (and of the wave packets) in Bohm theory. The next question that naturally arises is what this position in configuration space is in real space. Is the relation between a position in configuration space and a position in real space a relation that is, given the interpretation of the configuration space, always the same? If this is the case, we would have found our function f, and thus come to know an intrinsic property of the objects in the quantum world.

6 The Interpretation of Configuration Space

We are looking for the link between the quantum world and a description thereof. We have a position in configuration space. Now we are looking to what it would mean for a particle to have an ascribed position in configuration space. To give meaning to this theoretical notion, position in configuration space, we should try to find the relation between this position and real space.

In configuration space, there is a number of dimensions $3N$, where N is the number of particles. The number of particles in the visible universe is somewhere near googol, so there are three times as many dimensions in configuration space. Normally we may suffice by describing only a part of this enormous space, namely just the part describing the particles we need. It is possible for trajectories to be in the same configuration space, as we saw in the second chapter.

They are not in the same configuration space anymore, when we correlate them with something macroscopic. If we make a measurement to a particle, that is, correlate its position with the position of something macroscopic, we create an extra 3 dimensions. In Dirac-notation it is very clear, because one particle trajectory gets correlated with a macroscopic pointer state $|A\rangle$, while the other gets correlated with a macroscopic pointer state $|B\rangle$. This can never lead to overlapping wave packets in the same space, because the macroscopic pointer states cannot occur simultaneously. We saw this in detail in chapter 3.

Configuration space is $3N$-dimensional and real space appears to us as Euclidean 3-space. As I showed in figure 4 on page 11, there is a difference between the real space and the configuration space. Configuration space is not just $3N$ dimensional, it is $3N$ dimensional in groups of 3. Every third dimension denotes the same spatial direction [36]. Therefore, the most intuitive method of transforming configuration space to real Euclidean 3 space, is projecting every third dimension on one spatial direction.

Every particle has three degrees of freedom, so we need three dimensions per particle. Compare it with this example: if we are going to record the height of N different buildings, we can use a mathematical N-dimensional space. This doesn't make any differences to the space that the buildings are in. This would mean that there is a way that the configuration space holds the information about objects in real space, or gives information about the configuration of the particles. This leads us to expect that it is possible to find a way of how to project configuration space upon real space. That would be the interpretation that makes the configuration space be about the world.

To be able to define a projection of configuration space on real space, we need an additional definition of real space.

6.1 The Interpretation of Real Space

The first problem we encounter is that we have no formal definition of real space. There are a couple of different views on the matter [31]. These views originated in the time of Newton and Leibniz, and the projection of configuration space onto these different spaces, might have different outcomes. If we find similar outcomes, however, we can use the rule of disjunct elimination and we know that, if something has a property in configuration space, it certainly has a property in real space, whatever the shape of real space might be. In order to try and find this relation that a Bohmian particle has to real space, I will first outline the different views of space.

Remember that Bohmian mechanics is a non-relativistic theory, so I will only give definitions of non-relativistic space.

6.1.1 Substantivalism

In his 1687 masterpiece, the Principia, Newton defined space as absolute [**?**]. He argues that space and time can exist independent of anything. Here he also explicitly states the difference between 'relative' and 'absolute'.

> Absolute space, in its own nature, without regard to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is vulgarly taken for immovable space; such is the dimension of a subterranean, an æreal, or celestial space, determined by its position in respect of the earth. Absolute and relative space, are the same in figure and magnitude; but they do not remain always numerically the same. For if the earth, for instance, moves, a space of our air, which relatively and in respect of the earth remains always the same, will at one time be one part of the absolute space into which the air passes; at another time it will be another part of the same, and so, absolutely understood, it will be perpetually mutable. (p. 6) [38]

In this quote, Newton defines the difference between relative and absolute space. Absolute space is a space that can exist for itself, relative space is space as we observe it, relative to something else. On earth, we often use earth itself as the inertial frame. But while the earth and the relative space around it do not move relative to each other, they move through absolute space.

> Place is a part of space which a body takes up, and is according to the space, either absolute or relative. I say, a part of space; not the situation nor the external surface of the body. For the places of equal solids are always equal; but their [boundaries], by reason of their dissimilar figures, are often unequal. Positions properly have no quantity, nor are they so much the places themselves, as the properties of places. The motion of the whole is the same thing with the sum of the motions of the parts; that is, the translation of the whole, out of its place, is the same thing with the sum of the translations of the parts out of their places; and therefore the place of the whole is the same thing with the sum of the places of the parts, and for that reason, it is internal, and in the whole body. (p. 6-7)[38]

Places also have to suffer from the distinction of relative and absolute. When Newton uses the word "place", he means a position in space. An absolute place is a fixed position in absolute space, which will never ever change. No two places are equal in absolute space, they are all distinct. A place is not there because there is a body occupying it. The place will be there when the space is empty. There is also a famous thought experiment explicitly stating his ideas concerning absolute space.

Consider a bucket filled with water hanging from a rope in an otherwise empty universe. The bucket is set spinning around the vertical axis. Newton argues that, as we know from our experience with rotating buckets, the rotation will cause the water to move from the axis to the sides. This means that the surface of the water, which at first will be flat, is concave when the bucket is spinning. Newton supposes that the height of the water on the edge of the bucket corresponds with a unique rate of mechanical rotation. Because of this claim for uniqueness, the rate of rotation cannot depend upon a frame of reference, for a frame of reference is an arbitrary choice.

Ergo, the rotational motion is 'true' motion, motion relative to absolute space. This only works with acceleration, which is the change of absolute velocity. Absolute velocity itself is not visible. Because constant velocity is motion for which no force is needed, absolute space can, without us knowing, move in one direction relative to an inertial frame used on earth.

The ontological properties of absolute space are that it is non-material, because it has no causal properties. Nevertheless, it is a physical, as opposed to a mental, entity. This is awkward, for how can a physical entity be non-material?

That is a question that will remain unsolved. We do know what is meant with absolute position, but we are aware that we can only observe positions in relative space. Maybe we have enough information here to solve the puzzle.

6.1.2 Relationism

Leibniz defined space as being inferred from all possible spatial relations between bodily objects. He does this explicitly in his fifth letter in his conversation with Clarke between 1715 and 1717 [34]. Clarke was British and a proponent of Newton's substantivalism. Leibniz defines an object's position relative to its surroundings.

> [P]lace is that, which we say is the same to A and, to B, when the relation of the co-existence of B, with C, E, F, G, etc. agrees perfectly with the relation of the co-existence, which A had with the same C, E, F, G, etc. supposing there has been no cause of change in C, E, F, G, etc. (section 47)[34]

Leibniz first defines position, what he calls 'place'. Position is defined as that, what two arbitrary objects have in common if they are in exact the same relation to a set of other objects. This definition of place is therefore not an intrinsic property of the element, but it is part of the relation between the objects. For exact the same place, we need to take the set of all other objects.

Notice that for a place to be well defined, there should not be any change. We see that the definition of position is time dependent. This is very insightful, we normally use $x(t)$. But it becomes interesting when we see how Leibniz defines space:

> Lastly, space is that, which results from places taken together. And here it may not be amiss to consider the difference between place, and the relation of situation, which is in the body that fills up the place. For, the place of A and B, is the same; whereas the relation of A to fixed bodies, is not precisely and individually the same, as the relation which B (that comes into its place) will have to the same fixed bodies; but these relations agree only. For, two different subjects, as A and B, cannot have precisely the same individual affection; it being impossible, that the same individual accident should be in two subjects, or pass from one subject to another. But the mind not contented with an agreement, looks for an identity, for something that should be truly the same; and conceives it as being extrinsic to the subjects: and this is what we call place and space. (section 47) [34]

Here he defines space as the conjunct of all the places. And what is even more interesting is that he highlights the difference between the relations of A and the position of A. The relations of A to its environment, and the relations to B, which is at the same place, are different. This is because $A \neq B$. If the relations were completely equal, then $A = B$. This is why, he argues, our mind adds the notion place to the corresponding notion in the comparing of the relations of A and B, with their surrounding.

Space is the conjunct of all the possible conceivable positions. Because position depends upon the relations ascribed by the mind, space is formed in an 'idealistic' way. This means that we ascribe the space to the objects by identifying the relations. Empty space is therefore not conceivable, because we need the objects and the relations to speak of positions. Space is the order the mind forms in the application of relations.

A position is a point relative to this frame. Both position and space are extrinsic.

7 The projection of Configuration Space

Now that I have definitions of how real space could look like, I can see what happens if I project configuration space upon real space. It is interesting to see that the two images of real space are in great contrast with one another. Newton defines space as mind independent and Leibniz as mind dependent. If I can find a property that position in configuration space projected on real space has, independent of the view used of real space, then we can come to a greater understanding of the description Bohm theory gives of the world. In this chapter, if I talk about real space, I will mean something that is the case for both conceptions of real space. If it is about one of the two views, I will explicitly state this.

I think that the $N = 1$ case does not hold any problems for either of the two interpretations. In this case, the configuration space is equal to real Euclidean 3 space. This is relative[6] space in the substantivalist interpretation. In the relationist interpretation, however there is only one particle, there are two wave packets, as in the first case considered in section 2.2. If there is only one object, there is no perception of space, let alone motion, possible in the relationist perception of space.

In relationism we would rather have two or three objects in order to speak about positions in space (relative to those objects). I will assume that there are enough immovable objects for the relationist to define position and space. And I will consider the case that was introduced in the second chapter of this paper, the case in which there is a measurement made and the second wave packet will get its own 3 space, making it a total of 6 dimensions for two trajectories, both possible, one actual, and both through the point I in real space.

The most important process that is possible in the configuration picture, is that the wave packets can miss each other. So in order to keep this aspect in the projection on real space, the most natural thing to do (as a physicist) is to attach indices to the wave packets, saying when to and when not to interact. This will bring us back to configuration space, because this will introduce the extra degrees of freedom, and thus the extra dimensions.

Now we will encounter the biggest problem. As far as I can see, this will be a problem in any 3-dimensional space. Because every particle alone moves in Euclidean 3 space, the conjunct of multiple of these trajectories within Euclidean 3 space, destroys the single valuedness that is required to give the particles their unique trajectories. If we stick to our initial example, the trajectories that miss each other in configuration space, cross each other in projected substantivalist real space in point I.

In the relationist conception of space, at point I, both trajectories are in what is defined as 'the same place'. This definition is very strong in the relationist conception of space. We cannot deny that they cross in the projection on relationist space.

Outside of point I, we can attribute position to the trajectories. This is possible in both views, but it raises the immediate question on how this has to be interpreted.

7.1 Position as an Intrinsic Property?

Every particle alone lives in a 3 dimensional configuration space that is bijectively mappable on Euclidean 3 space. This gives us good hope to find the intrinsic property that would yield an interpretation of the position in configuration space, the relation between a theoretical and a real entity. However, in chapter 4, we saw that it is not allowed to ascribe an intrinsic property to objects that they have in virtue of being lonely. They should have the intrinsic property if they are accompanied as well. And with multiple particles, the particles in configuration space are not longer projectable on Euclidean 3 space.

[6] It may be absolute space, you never know.

As we have seen, the possibility of projecting on either interpretation of space did not work out well. It does not matter which interpretation of real space we use. There are two different possibilities in thinking about position in configuration space. Either, configuration space is real and existing, or, it is a pragmatic good description of reality in which there is no possible way of conceiving the particle trajectories to be located in real space and still be governed by the Bohmian theory. If this was possible, Bohm would have chosen Euclidean 3 space as the space for his theory to act on.

We have the possibility of projecting the trajectories upon real space, whether this is substantivalist or relationalist, that does not matter. By projecting, we lose information. We lose information necessary for the deterministic causal explanation, the centerpiece of Bohm theory. It is possible to project on real space, but by projecting, we cannot distinguish trajectory A from B at I. This is a great loss, and I think it poses an unsurmountable problem for us.

Within the scope of Bohm theory, there exists the possibility for a particle of having position x in its own 3 space, part of the bigger $3N$ dimensional configuration space. If there is another particle and the trajectories cross in the projection, the causal laws of Bohm theory conflict with the projected image. The solution to the equation for S, determines the speed of the particle. This is physically interpretable as the particle being propelled by a wave packet, always orthogonal to the wave front. If the trajectories, and thus the particles and the wave packets, all exist in 3 space, at the intersection, both wave packets should act on the particle. There is in our example from the 3rd chapter only 1 actual particle, but there are two wave packets, for the other trajectory is possible, so the wave packet should be there.

This leads to the insight that we cannot take the Bohmian trajectories outside the Bohmian framework. To give the trajectories another meaning, we would have to interpret them within another theoretical framework. We cannot have the wave packets existing in 3 space, creating possible crossing trajectories. We would lose the causal explanation of the trajectories, and therefore the trajectories would become meaningless. From within the Bohmian picture, we can talk about the particle trajectories, but then we are back to our initial question, what does this mean in real space.

We encountered a duality, we can either explain the trajectories in a real, 3-dimensional space, or we can explain the causality, and thus why the trajectories are there. This is Bohm's strongest explanatory power, the causal structure, and we don't want to give it up. We can, however, not find the intrinsic relational property f, we were looking for. But we can come to an interesting conclusion.

8 Conclusion

We saw that, objectively, we cannot speak about what it means to have a position in the configuration space related to real space. Real space is not defined in Bohm theory. We had the possibility of simply interpreting every particle's 3 space as Euclidean and adding them all up within the same 3. By doing this, we lose the beautiful insights Bohm theory gives us on how and why these particles move. It is not possible to maintain the view that the wave packets and particles exist in 3 space and that Bohm theory gives us the laws that govern their processes.

At first, the theory-ladenness of observation is beautifully apparent in the surreal trajectory-problem. The first intuitions on how our measurement devices work are wrong. The device should be interpreted within the theory. This means that there is no real measurement made before the correlation with a macroscopic object is performed. Upon correlation of a particle position with a macroscopic object, we create an extra 3 dimensions. That's why the particle trajectories miss each other in configuration space and do not bounce as we at first expected. because of the macroscopic objects that can only have one position, those two trajectories are never allowed to interact.

In a slow measurement, we do find surreal trajectories, and our measurement device does not measure until the actual correlation is made. Because of this late correlation, we measure something else then we expected. The trajectories are still in the same 3 dimensions, so the bounce will occur. Later, after the correlation, the extra set if dimensions is created. This act, the creation of the extra dimensions in configuration space, is something that makes us wonder what position in configuration space is.

As soon as we want to know how the particles would do in real space, we can only account for the non-problematic particle trajectories. That is, for 1 particle, without measurements. At point I we cannot explain the intersection of the trajectories if we made a measurement. The Bohmian laws governing the particle trajectories require single valuedness of the space, and thus do not hold. The trajectory description becomes empty and meaningless.

When we make a measurement, strange things happen. We can only meaningfully project particle trajectories on real space in the simple cases. By projecting the configuration space on real space, we lose information. This information was stored in the extra dimensions. This leads us to suspect that the objects in the quantum world have too many parameters to be described within a classical frame. The configuration of the particles, needs more information than can be stored in 3 dimensions.

In the literature there are different views about the nature of configuration space. I already mentioned Holland, because he takes a quite radical point of view. He argues that configuration space is real and existing, next to real space [30]. This makes way for a causal view, but it immediately raises the questions we have been trying to answer. We do not want to split the world in two and if we do, we need an interpretation of how the configuration space and the real space are related.

Someone who holds a contrasting view to that of Holland is Albert. Albert says that there is only a multidimensional configuration space. He even claims that "... whatever impression we have to the contrary (whatever impression we have, say, of living in a three-dimensional space, or in a four-dimensional space-time) is somehow flatly illusory" (p. 277) [1] in [7]. This is a very bold claim, claiming that the way we see the world around us is false. We would always be wrong, and if we are always fooled by this illusion, there is no way for us to test the claim of living in a $3N$-dimensional world. This claim is non-falsifiable and will therefore not hold.

In a recent article by Belousek, he gives another interpretation [7]. If we try to see the sub-spaces of configuration space as interpretable as Euclidean sub-spaces, then, we have to bare in mind that they are orthogonal in configuration space. This means that we can never properly project them on real space. Consider a system consisting of N trajectories, so N different particles guided by N different wave packets in $3N$ dimensions. Assume that the wave functions factorize, so $\psi(q_1, q_2, ..., q_N) = \psi(q_1)\psi(q_2)...\psi(q_N)$. We can never interpret the system as existing in Euclidean 3 space if we don't want the q_i to coincide. This leads to the same conclusion we arrived at. Belousek says that

> one can*not* simply regard the *total* quantum system as existing in ordinary 3-dimensional space, but rather must still regard it as existing irreducibly in configuration space, with each part existing in a separate sub-space. And that would undercut any sense of a *single* system existing *in one and the same* physical space, which is surely requisite for a coherent physical theory. (p. 155) [7] (emphasis in original)

Belousek solves this in quite an elegant manner. He takes a different approach towards the actuality of the trajectories. Because every actual particle trajectory takes place in 3 space, and most of the trajectories are only *possible* trajectories, Belousek lets the configurational part of the theory be a theoretical description about the possibilities. The particles are all in real 3 space. The equation for R gives the resulting force upon the particles. This is a completely different physical theory with the same mathematical framework. In this interpretation, the theory only has 1 trajectory per particle, and this eliminates most of the trajectories. In our example in the 3rd chapter, this would have solved our problem. Because there is only 1 particle, and thus also 1 real trajectory, this is always in 3 space. The other trajectory is not actual, but merely possible.

Still, there is a loss of information. The forces that result from the waves and the quantum potential are only part of the theoretical configuration space, and are thus primary forces. We cannot give a causal explanation of the origin of these forces. This is, of course, again a difficulty for this interpretation of the Bohmian framework. The causal explanation of the Bohm theory is one of its trademarks, by not giving the waves equal existence as the particles, this interpretation loses explanatory power.

Cushing has argued, in his book about the history of quantum mechanics [17], that the reason orthodox quantum mechanics is favored above the Bohmian interpretation, is not obvious. In 1927, when de Broglie gave his presentation on his causal quantum theory, what if he gave a clearer answer to Pauli's critique? What if the Bell-inequalities were already discovered at the end of the 20s? Einstein would not be able to cling to his locality desideratum, and the road to Bohm theory would be open. The fact that the Copenhagen Interpretation became the standard, is due to an historical coincidence. The order in which the different discoveries were made and certain events happened, created the Copenhagen monocracy.

In Bohm, we have now encountered that the causal view and the space time image of the theory are complementarily related. We can only see the space time picture, or have the causal explanation of this view, not both at the same time, though both of them give us information about the object[7]. This is one of the fundamental notions of complementarity argued for by Bohr [24], where Bohr sees the causal explanation as the conservation of energy and momentum. Though momentum is not conserved in the Bohm interpretation, the Bohmian causal explanation, which is quite similar to the causal explanation Bohr means, is complementarily related to a real space time picture of the trajectories in Bohm theory.

For $N \leq 2$, the theoretical world view given by the theory, is not more clarifying than the Copenhagen interpretation. At the end of this paper, I would like to argue that the Bohm theory and the Copenhagen Interpretation are very much alike in their views of the world. Only for the easy case $N = 1$, the Bohm description is more clear. For the multiple particle case, the Bohm description is just as theory-laden as the Copenhagen Interpretation and there is no unambiguous simple view of the quantum world. Every outcome is dependent on the measurement and the in-theoretical explanation of that measurement. We discovered that Bohm theory looks a lot

[7] I have supposed knowledge of the term complementarity, for more information on the subject, see [24], [13] and [14]

more like the Copenhagen Interpretation than we at first expected. I can therefore only appraise Cushing's insight that the Copenhagen Interpretation is only favored by sheer luck. From this we may conclude that we might have found an objective statement about the quantum world after all. Both interpretations give a description up until the same level of the world. They can give a spatial representation, or a causal interpretation, but never simultaneously.

List of Figures

References

[1] D.Z. Albert, *Elementary quantum metaphysics*, Boston Studies in the Philosophy of Science **184** (1996), 277–284.

[2] G. Bacciagaluppi and A. Valentini, *Quantum theory at the crossroads: reconsidering the 1927 Solvay conference*, Arxiv preprint quant-ph/0609184 (2006).

[3] J.A. Barrett, *The Persistence of Memory: Surreal Trajectories in Bohm's Theory*, Philosophy of Science (2000), 680–703.

[4] K. Bedard, *Material Objects in Bohm's Interpretation*, Philosophy of Science **66** (1999), no. 2, 221–242.

[5] J.S. Bell, *de Broglie-Bohm, Delayed-Choice, Double-Slit Experiment, and Density Matrix*, International Journal of Quantum Chemistry:*Quantum Chemistry Symposium* (1980), no. 14, 155–9, reprinted in [6].

[6] JS Bell, *Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy*, Cambridge University Press, 1987.

[7] D.W. Belousek, *Formalism, Ontology and Methodology in Bohmian Mechanics*, Foundations of science **8** (2003), no. 2, 109–172.

[8] Patricia Blanchette, *The frege-hilbert controversy*, The Stanford Encyclopedia of Philosophy (Edward N. Zalta, ed.), fall 2009 ed., 2009.

[9] D. Bohm, *A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I and II*, Physical Review **85** (1952), no. 2.

[10] ———, *Hidden variables in the quantum theory*, Quantum Theory, Part III: Radiation and High Energy Physics (D.R. Bates, ed.), 1962, p. 345.

[11] ———, *Wholeness and the implicate order*, (1980).

[12] D. Bohm and B.J. Hiley, *The undivided universe: An ontological interpretation of quantum theory*, Routledge, 1993.

[13] N. Bohr, *Causality and complementarity*, Philosophy of Science (1937), 289–298.

[14] ———, *On the Notions of Causality and Complementarity1*, Dialectica **2** (1948), no. 3-4, 312–319.

[15] M. Crampin and F.A.E. Pirani, *Applicable differential geometry*, Cambridge University Press, 1986.

[16] M.O.T. Cunha, *What is Surrealistic about Bohm Trajectories?*, Arxiv preprint quant-ph/9809006 (1998).

[17] J.T. Cushing, *Quantum mechanics: Historical contingency and the Copenhagen hegemony*, University of Chicago Press, 1994.

[18] J.T. Cushing, A. Fine, and S. Goldstein, *Bohmian mechanics and quantum theory: an appraisal*, Springer, 1996.

[19] M. Daumer, D. Dürr, S. Goldstein, and N. Zanghì, *Naive realism about operators*, Erkenntnis **45** (1996), no. 2, 379–397.

[20] L. de Broglie and L. Brillouin, *Selected Papers on Wave Mechanics*, 1928.

[21] D. Dürr, W. Fusseder, S. Goldstein, and N. Zanghi, *Comment on surrealistic Bohm trajectories*, Z. Naturforsch **48** (1993), 1261–1262.

[22] A. Einstein, *Geometry and experience*, Sidelights on relativity (1922), 27–56.

[23] B.G. Englert, MO Scully, G. Süssmann, and H. Walther, *Surrealistic bohm trajectories*, Zeitschrift für Naturforschung. A, A Journal of physical sciences **47** (1992), no. 12, 1175–1186.

[24] Jan Faye, *Copenhagen interpretation of quantum mechanics*, The Stanford Encyclopedia of Philosophy (Edward N. Zalta, ed.), fall 2008 ed., 2008.

[25] G. Frege, G. Gabriel, F. Kambartel, and C. Thiel, *Gottlob Freges Briefwechsel mit D. Hilbert, E. Husserl, B. Russell, sowie ausgew
"ahlte Einzelbriefe Freges*, Meiner Verlag, 1980.

[26] W. Heisenberg, *Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik*, Zeitschrift f
"ur Physik A Hadrons and Nuclei **43** (1927), no. 3, 172–198, English translation reprinted in [**?**].

[27] D. Hilbert, *The foundations of geometry*, The Open Court Publishing Company, 1950.

[28] Dieks D. Muller F. Uffink J. Hilgevoord, J. and M. Seevinck, *Foundations of Quantum Mechanics*, (2009).

[29] Jan Hilgevoord and Jos Uffink, *The uncertainty principle*, The Stanford Encyclopedia of Philosophy (Edward N. Zalta, ed.), fall 2008 ed., 2008.

[30] P.R. Holland, *The quantum theory of motion: an account of the de Broglie-Bohm causal interpretation of quantum mechanics*, Cambridge University Press, 1995.

[31] Nick Huggett and Carl Hoefer, *Absolute and relational theories of space and motion*, The Stanford Encyclopedia of Philosophy (Edward N. Zalta, ed.), fall 2009 ed., 2009.

[32] Klaas Landsman, *Newton en Hilbert*, class in the course "Geschiedenis van de Wis- en Natuurkunde" at the Radboud University, 15 april 2010.

[33] R. Langton and D. Lewis, *Defining'intrinsic'*, Philosophical and Phenomenological Research **58** (1998), no. 2, 333–345.

[34] G.W. Leibniz, *The Leibniz-Clarke correspondence: together with extracts from Newton's Principia and Opticks*, Manchester University Press, 1956.

[35] D. Lewis, *Extrinsic properties*, Philosophical Studies **44** (1983), no. 2, 197–200.

[36] P.J. Lewis, *Life in configuration space*, The British Journal for the Philosophy of Science **55** (2004), no. 4, 713.

[37] W.C. Myrvold, *On some early objections to Bohms theory*, Iinternational Studies in the Philosophy of Science **17** (2003), no. 1.

[38] S.I. Newton, A. Motte, F. Cajori, and R.T. Crawford, *Sir Isaac Newton's mathematical principles of natural philosophy and his system of the world*, University of California press, 1960.

[39] C. Philippidis, C. Dewdney, and B.J. Hiley, *Quantum interference and the quantum potential*, Il Nuovo Cimento B (1971-1996) **52** (1979), no. 1, 15–28.

[40] D. Platchias, *The veil of perception and contextual relativism*, Sorites (2004), 76.

[41] P.J. Riggs, *Quantum Causality: Conceptual Issues in the Causal Theory of Quantum Mechanics*, Springer Verlag, 2009.

[42] M.O. Scully, *Do Bohm trajectories always provide a trustworthy physical picture of particle motion?*, Physica Scripta Supplement T- **76** (1998), 41–46.

[43] L. Vaidman, *The Reality in Bohmian Quantum Mechanics or Can You Kill with an Empty Wave Bullet?*, Foundations of Physics **35** (2005), no. 2, 299–312.

[44] J. von Neumann, *Mathematische Grundlagen der Quantentheorie*, 1932.

[45] Brian Weatherson, *Intrinsic vs. extrinsic properties*, The Stanford Encyclopedia of Philosophy (Edward N. Zalta, ed.), fall 2008 ed., 2008.