# Thesis part II

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Which place did history of mathematics have in the field of mathematics education, 1950-2000?

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# I Foreword and acknowledgements

For a year and a half, I worked on the project below, going through a first job in education and a corona period. Time and again I returned to this file, to conclude it now at last. Thank you, Danny Beckers, for the excellent supervision, for all the conversations we had, which might or might not be related to history, mathematics or education. It was wonderful to see your fascination for the history of mathematics and your enthusiasm for research. Thanks to David Baneke for all your guidance and interest during the past one and a half years.

Thank you to my housemates for all the conversations we had about my thesis, but even more for the moments when we didn't have those conversations. I thank my parents and my grandmother for all their support. Above all, thanks to God, who teaches truth and enlightens us through His righteousness.

# I Introduction

Suppose an outsider starts exploring the field of mathematics education. He knows that traditionally mathematics education in daily practice looks like a classroom with students and a teacher – and a chalkboard. Behind this scene, a lot is going on. The ‘teacher’ is not just a random person but has been schooled in mathematics education, according to a certain curriculum. The subject taught is part of a well-discussed curriculum that has been changed through time. All the students have their own knowledge, background and level in school. When it is about the teaching of mathematics, teacher, teacher curriculum, student curriculum and student all have their opinions and ideas. Students mostly do not organize themselves to discuss that, but teachers the more so. Therefore, behind the classroom there exists a field of mathematics. In most countries, there is at least one journal or magazine for mathematics education and an association organizing congresses or in-service training. So, those will come along first when the outsider starts exploring this field. In the Netherlands, he will find the NVVW, an organization for mathematics teachers and *Euclides*, trade journal of this organization. Imagine this outsider takes one edition of this journal *Euclides* and let it be the second number of the 95th volume, published in October 2019. He or she starts in the back. First, an article about an annually held conference comes along, next a short article about the contribution of the Romans to the history of mathematics. Thinking about how that should be placed in a journal for mathematics teachers, the person turns more pages – finding an article under the heading ‘roots of mathematics’, which is already the 15th under this heading. Pictures of 17th century books are included in this piece about measuring distances using similarity. Looking it up, he or she finds that all articles appearing under these headings are about the possibilities to use primary resources in the classroom. The outsider thinks of it as remarkable and reads further. On one of the first pages, he or she then sees a message that all editions of the journal have been digitized and are available online. Now all first ingredients for this thesis are together: a surprising use of the history of mathematics in mathematics education and the possibility to research how this has developed in the past. This thesis will try to answer the following question: which place did history of mathematics have in the field of mathematics education from the 1950s until the 1990s of the preceding century?

In a theoretical framework this thesis discusses the historiography behind using history of mathematics for educational purposes. Fried states that the commitments of a mathematics teacher differ too much from the commitments of a historian, to let history of mathematics play a role in mathematics lessons. And indeed: nothing guarantees that history is always treated in a historiographical justified way when used to *support and explain* contemporary mathematics. Fried himself presents a few solving methods to this problem, for example to let history of mathematics be part of the curriculum for teacher education. However, this dichotomy plays a role throughout the history of using history of mathematics in education.

To be able to evaluate how the place of history of mathematics in education develops, concrete standards to compare different contexts and moments in time are indispensable. Therefore, a methodological framework is developed in section II. At the heart of it lies the categorization of Kjeldsen, who aimed to develop a framework for systematic analysis and discussion of uses of history of mathematics for teaching and learning mathematics. This consists of two pairs of concepts: actor – observer and pragmatic – scholarly. In this thesis, those concepts will be used to clarify the attitude towards history of mathematics. For the second layer of the methodological framework, it is important to make concrete what purpose is mentioned for the use of history of mathematics. Here I follow Blom and Gulikers, who listed four arguments in their review of the then recent literature on the use of history of mathematics in education. The third and last level maps on which level history of mathematics is used in education: the curriculum for mathematics teacher education, the curriculum for education of mathematics or at the level of separate lessons.

Both the theoretical and the methodological framework will make it more clear how history of mathematics is used. But what is meant by ‘place’ in the field of mathematics education? It indicates where and why historiy of mathematics is used, and how bigger the influence history of mathematics is given, the bigger its place within the field of mathematics education. In this thesis, there is searched for the extent to which the knowledge and the idea of applying history of mathematics in education did float around. As a main source to answer this question, the journal *Euclides* is used, which has since October 2019 all his volumes from the start in 1924 digitalized. As will turn out, history of mathematics was in 1950s-2000s not as prominent as nowadays. This becomes clear in *Euclides*. History of mathematics was however known and from various quotations it can be read what attitudes towards history of mathematics predominated. This of course changed over the period researched, which will be evaluated in this thesis by using the methodological framework mentioned above. Next to *Euclides*, other sources will be surveyed to get a more complete image. Topics of congresses for example show topics that were seen as relevant and other written sources, like a bulletin discussed in paragraph 5.4, also provide a new view on the whole field of mathematics education. The fifties are a good period to start with, because from 1953 on it is possible for pupils at an α-gymnasium to do history of mathematics as part of the curriculum. However, that did not give much of a debate, it is interesting to see how that was evaluated – also when it did not raise a debate. At the end of the nineties, in *Euclides* the idea of applying history of mathematics in education has become mainstream, at least the possibility of it. The development of the place of history of mathematics will be treated roughly by decades.

# II A theoretical framework

The goal of this section is to set up a theoretical framework. As it turned out, such a framework is crucial for getting useful information from the articles in *Euclides*. Useful in this case means contributing to the goal of this thesis: unravelling the thoughts and deeds of mathematics teachers concerning history of mathematics. The sources are first introduced and thereafter treated more in-depth.

An especially important source is an article written by Blom and Gulikers wherein they give an extensive overview of the literature published on the use and value of history of mathematics in education.[[1]](#footnote-1) Blom and Gulikers developed a structure in which all articles published so far seem to be contained. However, Fried throws some doubt on the whole issue, by stating that the integration of history of mathematics in education is impossible.[[2]](#footnote-2) The reason for this lies in the incompatibility of different commitments of mathematics educators and that of historians of mathematics. At least he points to an indispensable aspect of the theoretical framework set up here, which has not gained much attention in the articles discussed by Blom and Gulikers. This aspect is about the historical justifiability of the history used by the mathematics teachers. Only the word ‘use’ will already cause warning signals in the eyes of a historian. A historical layer thus has to be part of the theoretical framework. Fortunately, the historian Kjeldsen has written about different ways to apply history of mathematics in education, giving attention to the historical side of it.[[3]](#footnote-3)

## 2.1 Why and how should you use history of mathematics in education?

Blom and Gulikers have the goal to clarify the discussion on the role of history of geometry in education. Their choice of a focus on geometry does not make it impossible to use here, for it is not dominant in their categorization of articles. Moreover, it is most prominent in a paragraph that discusses concrete articles on how to use history of geometrical subjects in a classroom, whereas it is the earlier part, categorizing previous research on the use of history of mathematics in education, that offers suggestions that can be used in this thesis.

There is a broad variety of reasons why you would apply history of mathematics in education. Blom and Gulikers bring structure to this variety by making up four categories. Their first category consists of conceptual arguments. Various arguments are possible, like the importance of the historical-genetical line, or foreseeing learning obstacles; those arguments share the assumption that it is impossible both to teach and learn mathematics while neglecting its history. The second category is made up of (multi-)cultural arguments: history of mathematics crossed many national borders; it can be helpful in showing the place of mathematics in society; and it shows that mathematics is in fact a human activity, instead of a system of rigid truths. That last argument is remarkable, considering the value usually given to mathematics precisely for its rigor. Nevertheless, it might be useful for pupils to humanise mathematics instead of rigorizing it. Third are motivational arguments: using history can accrue interest of pupils, especially the use of resource materials. When performed in the right way, history can also help to understand modern mathematics. Of course, not everything can be positive and the *why not*-arguments are categorized fourth. Mentioned are: teachers do not have enough historical expertise, do not have access to the right materials and lack time.

Besides the question *why* you should use history of mathematics in education, there is the question about *how* you can do this. Although most research is done in the first part, Blom and Gulikers found enough on the second part to roughly divide it in the following three categories: giving historical information; follow a teaching and learning approach inspired on history while introducing mathematical concepts or methods; while unfolding such a teaching-learning approach, use primary sources or original instruments. When thinking on *how* to use history of mathematics in education and especially in the last of the three methods, it is essential to think of the relation between history of mathematics and the modern mathematics in the curriculum. Remarkable enough, it is on this place that Blom and Gulikers signalize a gap between the general articles, treating arguments as mentioned here, and practical articles, which contain suggestions for lessons or resources. In that gap, articles are desired that develop realizable plans for a lesson that integrates history of mathematics in daily education.

## 2.2 The integration of history of mathematics in education: problematic in origin?

According to Fried, it is no coincidence that this gap is not filled. A lot may have been written of the integration of history of mathematics in education, but this only disguises a basic assumption about the possibility of it. While Blom and Gulikers divide the content of their reviewed articles in a why- and a how-category, Fried’s statement thus needs a third if-category. Moreover, thinking about if it is possible to use history of mathematics in education, he answers negatively, using the following argument. When a mathematics teacher develops his lessons, he needs a didactical perspective. He also needs a focus on modern mathematics, because he is committed to teach the kind of mathematics that their students need in their later study of mathematics, science or engineering.[[4]](#footnote-4) Combining this with the history of mathematics presents a dilemma, for doing mathematics and doing history both bring their own distinctive, even nonconcurrent scientific methods. And because of the indispensable commitment to modern mathematics, the history used can only be Whiggish history: history of mathematics is not studied but used. The problem is not only that history is treated in a bad manner. Treating the history of mathematics Whiggish is also mathematical problem, for the mathematics of the past cannot be done justice as it was begotten then, except by avoiding Whiggish history.

Fried presents two solving methods for this dilemma. The first is a radical accommodation to the historiographical method and involves presenting a mathematical treatise from the past as historical text, seeing it as great in its own right. Within this solution, the whole curriculum or at least a part of it becomes historical. The second option is radical separation and includes adding some history to a modern curriculum, without any attempt to integrate. Still another option exists, however not mentioned by Fried: applying history of mathematics in a Whiggish manner. It might not be possible to justify – that does not make it impossible to do.

So, in the article discussed Fried adds to the discussion that putting together history of mathematics and mathematics education is problematic from scratch and there is no integration possible. However, in a later article, Fried reconsidered the case and decided he had overseen the difference between a mathematician and a mathematics teacher.[[5]](#footnote-5) The mathematics teacher has the ability to use both the commitments of a mathematician and a historian of mathematics. On the one hand, the mathematician sees mathematics as what is now, diachronically, while on the other hand the historian of mathematics sees mathematics synchronically, as a development over centuries. Those are not opposed but should be seen as two crossing axes. Modern mathematics and history of mathematics are now complementary.

Despite the later shift, the first article by Fried gives voice to a perfectly possible opinion in the discussion on the use or integration of history of mathematics in mathematics education. His statement that this is impossible from scratch can at least be used to explain some of the problems that pop up; and an explanation might even be the first step to solve it. It should be recognized that solving is not at all the intention of Fried but evaluating an attempt to do so may still be of worth for the theoretical framework that will be developed in the next section. Considering the attempt of Kjeldsen, this section proceeds with a search to find a historical justifiable manner of integrating history of mathematics, now different ways of integration are discussed.

## 2.3 Historical justified options for using history of mathematics

Kjeldsen presents a theoretical framework to look at how history is used both for teaching and learning mathematics. The analysis of this should occur with respect to purpose and didactical value.[[6]](#footnote-6) The historian Kjeldsen has a different perspective than Blom and Gulikers. She even starts with the difference between history as an academic research subject and history in mathematics education. It however is possible and needed, according to her, to make a didactical transposition between those. That can be done in different ways, chosen by a mathematics teacher. Two things are important to define that approach, based on Jensen[[7]](#footnote-7). In the first place, there is an important distinction between a pragmatic approach, guided by the idea that we can learn from history, and a scholarly approach, focusing on the distance between past and present and retaining. In the second place, the choice between an actor’s perspective and an observer’s perspective matters. When history is approached from an actor’s perspective, it is used to orient and act in the present world. An observer, however, uses history with the purpose to only enlighten his own time. Combining each of these gives us four possibilities for an approach in using history of mathematics in education. Which of those four is chosen by a teacher, should depend on the goal he has. If he wants to develop historical overview and awareness, he should use a scholarly approach from an observer perspective. But if a teacher wants to train specific mathematical competencies, he should choose a pragmatic approach from an actor's perspective. In this way, Kjeldsen (2011) connects different ways of treating history to various goals. By making those optional, her categorization opens the possibility to evaluate historical sensibility, for it has become a choice.

This section thus unfolded existing categorizations for the arguments that can be brought up in the debate about the use of history of mathematics. Furthermore, it appeared that it is also defendable to state that the integration of history of mathematics in education is impossible at all. The main problem is then to justify historiographically how history of mathematics is treated. If the possibility of *using* history of mathematics in education is accepted, different historiographical ways of applying history of mathematics in education can be distinguished, which all contain a different approach to history. Those results will be the main input for the method of this thesis, set up in the next section.

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# III A three-layered method for inquiring articles from *Euclides*

The method that will be developed in this section, will make it possible to investigate how the journal *Euclides* deals with history of mathematics. It will do so by building a framework, out of the theoretical sources discussed, and this framework makes it possible to categorize articles from *Euclides* that tell something about the place of history of mathematics from the 1950s to the 2000s. A following analysis will help to evaluate the place of history of mathematics had in education in this period. Besides setting up a framework, this section also reveals how out of thousands of articles in *Euclides* those that contain information about history of mathematics are chosen.

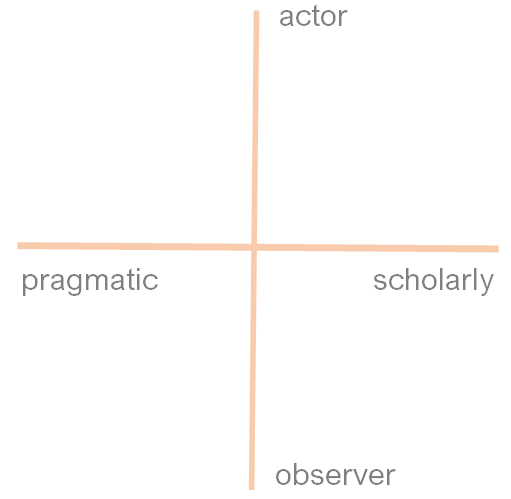
## 3.1 Three layers and a justification

Initial in this framework are the different approaches mentioned by Kjeldsen. Kjeldsen’s approaches are first, because they say most about the non-explicit presumptions, especially with respect to how history of mathematics is applied. Since the goal of this thesis is to discover potential changes in the position of history of mathematics in education, it is important to reveal the hidden presumptions about the topic. And after that, the arguments for using the history of mathematics in education are to be categorized, using the categorization of Blom and Gulikers. Thirdly, three possible levels of applying history in education contribute to the analysis.

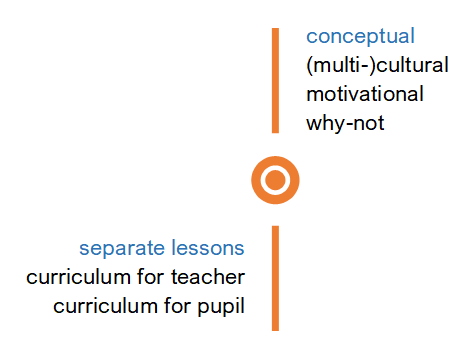
The third level exists to map de different possible levels at which history of mathematics can be applied in education, which has not been characterized until now. The first and most important level for using history of mathematics is the active use of concrete subject in a classroom. On this level, the teacher needs to take the initiative, possible supported by premade lessons on the internet or from another source. Besides this first level, there are other ways to give history of mathematics a place in education. An often-mentioned option is to make history of mathematics part of the schooling of a mathematics teacher. This is a far more indirect way but has the advantage of debilitating one of the major problems for applying history of mathematics in education: the lacking skills of a teacher to develop lessons in which history is profitable for learning mathematics. So, changing the curriculum for upcoming mathematics teacher is the second level on which an author can discuss the use of history of mathematics in education. Thirdly, there is also another curriculum which might be changed: that of pupils in high school. Applying history of mathematics in education on this level prevents randomness: the curriculum will be realized in every school, by every teacher.

Before this three-layered methodological framework is made concrete, some attention should be given to the question why it would be possible to use the categorization of Kjeldsen as well as that of Blom and Gulikers, given the source material this thesis uses.

The applicability of Kjeldsen’s categorization does not give a lot of problems since her approaches to history are already applied to mathematics education and moreover, are general in the sense they are made to fit everything between scholarly and pragmatic. The last range is indispensable, considering the character of the journal *Euclides*, which has teachers of mathematics as a target group.

With respect to Blom and Gulikers’s framework, showing that it is possible to use their theories for this thesis is a bit more complex, mainly because of a different type of source material. Four differences can be found between Blom and Gulikers and this thesis. In the first place, Blom and Gulikers researched scholarly articles, while *Euclides* consists of a different type of articles, even with a broad variety among those articles. Scholarly articles contain an explicit theoretical framework, in which assumptions are discussed that mostly remain hidden in other genres, also in articles of *Euclides*. Therefore, it is more difficult to categorize non-scholarly articles than scholarly articles and they demand more interpretation. In the second place, *Euclides* contains much more articles than can be read. Useful articles need to be chosen of this database. This is not inherently problematic, but since interpretation is needed, the justifiability of the categorization changes and that should be recognized. In the third place, related to the previous remark, the articles in *Euclides* are written by a huge number of different authors. Categorization of the articles can give an impression of the place history of mathematics has, but at the same time the thoughts of one author are of lesser meaning for the place history of mathematics had under all teachers between the sixties and the nineties, than the thoughts in one scholarly article say about the general scientific thoughts about the topic. In the fourth place, the purpose of Blom and Gulikers differs from the purpose of this thesis. While the first intended to clarify the discussion about the use of history of mathematics in education, the latter has the goal to analyse which place was given to history of mathematics by teachers of mathematics, between the fifties and the nineties. Clarification can be done by categorizing. Analysis, however, is not completed by categorization, and thus the method of this thesis needs to go further. Once a selected amount of the articles has been categorized, the outcomes need to be interpreted. 

## 3.2 The methodological framework itself

Kjeldsen’s four approaches to history can easily be set out on two axes, as beneath. Once an article is placed into this framework, which will be done by placing a point somewhere on the coordinate system. After that, the why-arguments named in the article should be categorized and attached. For this, the categories of Blom and Gulikers are used. The third and last layer of this framework evaluates how history of mathematics is applied in education: in a separate lesson, in the curriculum for educating mathematics teachers, or in the curriculum for pupils. Attaching the second and third part of this methodological framework will be done by colouring the why-arguments or levels that are mentioned. The example shows that a conceptual argument for using history of mathematics in education is mentioned, and that the separate lesson is mentioned as a possible way to actually use it.

## 3.3 Categorizing and interpreting

To show how this framework is used to categorize the different articles that are useful for the thesis, some elaborated examples are given here. The first two examples mainly analyse ideas about history and mathematics. The third example shows a case in which history of mathematics is used for a goal falling outside the scope of this thesis.

### 3.3.1. Not bringing his owls to their Athens

On December 10th, 1949, dr. G. Wielenga gave a talk about his journey to America. He went there to see the educational system at American High Schools and get some new ideas. His public consisted of the mathematical working group of the W.V.O. (cooperation for innovation of pedagogy and education), the Midlandgroep (a regional group with similar goals, but then concentrated around physics) and the Didactical Centre from the Pedagogical Institute of Utrecht University. With respect to history of mathematics, his talk is paradoxical. The first part of this paradox appears when Wielenga compares the American system to the Dutch system. Wielenga describes how in America, each pupil can choose various courses of mathematics. Mathematics teachers therefore take effort to make mathematics attractive. He contrasts this with the Netherlands, mathematics as a subject with a determined curriculum is indispensable for graduating. According to Wielenga, teachers rely on this fact and when a pupil questions the importance of mathematics, they call on Plato and talk about educational values, self-cultivation and more meaningless things.[[8]](#footnote-8) Within the Dutch system, he feels like it is impossible to see which goal there might be and he cannot see the wood for the trees. Going deeper into this metaphor, he states that all those trees will indeed lead to a dignified goal – but what does this goal consists of? Is it a delightful place, a temple, or a mausoleum? Reading the rest of his article, it becomes clear he is not as pessimistic as he might seem to be, and he admits he exaggerates. Unfortunately, it does not become clear to which extent Wielenga thinks of mathematics as a mausoleum, as worshipping past mathematics or mathematicians.

The other side of the paradox becomes clear when he says more about the freedom he wishes for in the Dutch system. He expresses how nice it would be when it was possible to say to a student taking α-courses who struggled with the discriminant of a discriminant: ‘how understandable this is difficult for you. Go and study history of mathematics and science and I will assess your knowledge about that subject.’ Including history of mathematics in mathematics education thus seems to be a plausible option for Wielenga. On what Wielenga has seen in America, he has a double-sided opinion. According to him, teachers are mathematically less competent. But the positive side of that is that in their education there is time for history, practical applications and the social and cultural values of mathematics. Applying the categorization of different levels of applying history of mathematics, the third level of the teacher curriculum is mentioned here and evaluated positive by Wielenga. On the reasons named by Blom and Gulikers on why you would use history of mathematics, only the third of motivational arguments can be found back. Next to this, history is predominantly important for broadening the free choice of topics in mathematics. History is not applied for being history but for not being part of traditional mathematics. Summing up the paradox mentioned: on the one hand, Wielenga has a negative view on history of mathematics when used to talk about educational values and he does not want to place the goal of mathematics in the past. On the other hand, history of mathematics can be used for motivational arguments and for having more freedom in the curriculum. Does Wielenga now want to convince his colleagues of his ideas? Interestingly enough, not, for he reckons they feel the same: and why would he bring *his* owls to *their* Athens?

### 3.3.2. Duparc as actor with a pragmatic view

In the summer of 1961, one of the biennial courses of the Mathematical Centre for mathematics teachers was held.[[9]](#footnote-9) The first speech was given by prof. dr. H.J.A. Duparc, about the changing position of mathematics in society. In those years, applied mathematics had become a field – time to discuss how mathematics education should relate itself to this field and how much attention applied mathematics should have in the curriculum. However, the second part seems to be only on education in algebra, specifically what is the best way to let pupils solve a quadratic equation. Nonetheless, Duparc gave his almost implicit conclusions while presenting a chronological overview of the various ways. The way used by the Greeks of course should be first. Interestingly enough, Duparc states that it would be offensive to explain their method further. Duparc apparently had reasons to suppose his listeners on this course for mathematics teachers knew the Euclidian methods. And if he was right, this means mathematics teachers had knowledge of at least the Greek period of the history of mathematics. When exploring the position of history of mathematics in education, this reveals a lot. But Duparcs reasoning did not yet stop: the modern method needs to be discussed. Quadratic equations were in his days, and still, solved by guessing coefficients, chosen to be easy, or by using the pq-formula. Both are not useful in practice – and therefore not useful in preparing pupils for jobs in which applied mathematics had a place. In practice, the solvation is never beautiful and therefore the pupil needs to be able to approximate. Therefore, they should learn the method used by Newton. Duparc thus uses history to provide various methods to explain one thing. As a consequence, he is an actor instead of an observer towards history and uses a pragmatic view. According to Kjeldsen this approach fits to the goal of training specific mathematical competencies. While that is recognizable, Duparc combines this with a more philosophical point of view. He does not think of using history in a specific lesson but expects that history of mathematics exists on the level of the teacher curriculum, for the teacher needs to be able to think about different solving methods used in history. This mental exercise should be performed with the goal to connect education to the place applied mathematics had in society, so Duparc brings in an implicit cultural argument.

### 3.3.3. History of mathematics as societal need

Three different levels have been discussed on which history of mathematics can be applied in education. However, this does not mean all articles in *Euclides* treat history of mathematics only on those three levels. This becomes apparent in the third issue of the 52th volume published in 1976, where H.J.M. Bos discusses success-stories, specifically in science.[[10]](#footnote-10) According to him, every success-story inherently has its ending and there should be attention for when and how this end will be. One of the success-stories alive in the seventies is the place mathematics has in society, at least according to this later professor of history of mathematics. Bos’s mathematics is not mathematics as an academic discipline or how it is taught at schools; but deals with mathematization. What Bos exactly meant with this concept will become clearer by the three aspects of mathematization. At first, mathematization brings power, for it makes it possible to act effectively and it gives information. The second aspect is the simplification, for standardization and automatization are necessary before mathematization can take place. The last aspect named by Bos is the diminishing of human involvement in labour.

For success-stories must come to an end, the positive appreciation of mathematics will also be overruled by new developments within time. According to Bos, one of those developments could be that the negative sides of a technocratic society become widely known, for example in atom bombs or prestige projects such as the moon race. Otherwise, the dismantling of this story could take place when the counterculture against objectification and rationalization of society would become influential. In both cases, the success-story of the mathematization of society would end because of becoming linked to negative consequences.

Mathematization of society and mathematics are two sides of the same coin, according to the later professor of history of mathematics and that is why mathematicians need to involve in the dialogue about mathematization of society and discuss the questions that are raised by it. Moreover, they need to teach about it and that seems to be the reason for Bos to write this article for *Euclides*, at that time presented as journal for didactics of mathematics, publication of the Dutch Assiociation for mathematics teachers (NVvW). In coaching pupils to develop an opinion on mathematization of society the history of mathematics and of science and technology in general is very important. Bos sees the usefulness of history of mathematics in education thus in getting a broader view on mathematics as a discipline and becoming able to judge how far its influence in society should go, while taking into account the negative and positive sides of mathematics on societal level. In the end, this article is written from an actor-perspective, using the first division of Kjeldsen.

Bos thus wants to use history of mathematics for a goal higher than high school: solving issues in society. This knowledge is most useful for mathematicians, for mathematicians are the persons who should deal with the place of mathematics in society. History of mathematics is therefore something to be taught *next* *to* mathematics education, not as a *part* *of* mathematics education. This thesis however focuses on the relation between history of mathematics and mathematics education while the first is placed into the second. Bos’ interpretation of history of mathematics needs such a different kind of analysis – because its goal lies in society instead of in education – that it falls outside the scope of this thesis. Thus, articles in *Euclides* sometimes deal with ways of using history of mathematics that are not treated in this thesis. They are not found often and are taken out for clarifying the goal of discovering the place history of mathematics has in mathematics education. Moreover, it shows that not everything found in *Euclides* will be categorized by the methodological framework and that it is developed to present what the most important ideas on the place of history of mathematics in education were, mainly how they changed in the period from the fifties until the nineties.

## 3.4 Using the methodological framework

This search to developing opinions of using history of mathematics in education will be fulfilled using the articles published in *Euclides* as a starting point. That is, *Euclides* will be the main source, but other sources will be discussed if they show certain relevant opinions. All information, if it comes from *Euclides* or from other sources, is interpreted by the methodological framework as laid out in the previous section.

In the first instance, this methodological framework serves to interpret a single article from a single author. The results of this are still a step away from tracing a development in a decade. This step is to select what kind of articles are prevalent, which comes down to selecting the most important ways of dealing with history of mathematics. Interpretation therefore took place in all different stages of this thesis and not only when openly discussed below. I strived to use the same characteristics as the methodological framework contains.

On an even further level, I interpret a period of fifty years by separating the period into five decades. This is not something that can be justified from inside the period, but is only chosen to choose a period. But in doing so, it is something artificial and imposed on time itself.

A last addition to this must be made. This thesis traces the developments of thoughts and attitudes to using history of mathematics in education. The moments when history of mathematics actually appear in education are a part of this; definitely an important part, but still a part. Tracing the whole development also has to contain a question for the underlying reasons to apply history of mathematics, which presumably change in time, or a search for reasons why educators of mathematics do not use history of mathematics in a certain time. Seen from the central question of this thesis, the first part is the core part, while the other parts offer explanations from extra scopes. This is why the methodological framework is about the actual uses of history of mathematics in education. It is also possible to map the reasons why history of mathematics is not used in education, to name one of the other parts, but those are only of second importance to this thesis. Being the core part, however, does not mean to be the most prevailing in each of the following chapters. To trace the development of using history of mathematics in education, it can be of more use to know of another development in that decade than to know in depth how it was used.

# 

# IV The fifth decade

In the fifties, the history of mathematics is not prevalent in education. Different developments take place in this first decade after the Second World War. Like everything in the Netherlands, education is trying to catch up after a period of little innovation. The attempts are bigger than the results.[[11]](#footnote-11) Within mathematics education, the same is happening, on the level of content and of organization. The existing habits are widely seen as outdated.[[12]](#footnote-12) History of mathematics does have its own, specific place within this context, but is not intertwined with mathematics education.

The generation of this situation will be tracked by first giving attention to causes why history of mathematics did not fit into the education of mathematics, assuming that the absence of a subject can be considered revealing as well. This will lead to recognition of a first approach to the history of mathematics, while in later decades other approaches will be discussed. Throughout a discussion of the developments between the fifties until the nineties, it will become that the place given to history of mathematics in education, is directly related to the way in which mathematics and the history of mathematics are seen.

This first chapter thus will start to explore the field of mathematics education, especially how mathematics and history of mathematics do have a place within it. This can be read from the way authors from this period write about mathematics and its history, or, as will be made understandable, in how he or she writes about didactics or uses a philosophical style or not. Reading and searching through the articles published in *Euclides* in all ten volumes of the concerning decade leads to the impression set out below.

## 4.1 An introduction to the fifties: issue 29.1

The first issue of volume 29, published in september 1953, was the result of my random draw out of the 69 issues that were published in the 1950s. I will use this issue to give a first impression of the sixth decade of the twentieth century. This first impression starts at the table of contents. At first, there is a new heading from Johan Wansink: under the title *Didactical revue* he discusses relevant articles that appeared in various journals on mathematics and education, Dutch and abroad. Some of those journals are more related to education than others, but all treat at least subjects that can be of relevance for education. Next in issue 29.1 is a research paper about the overloading of the program; not for teachers as discussed recently, but for students. This research is carried out by Dr. L.N.H. Bunt, who did more research with respect to the curriculum. Third, there is an article by Prof. Dr. Van Dantzig about the mathematical model in empirical sciences. With those empirical sciences he predominantly means physics. Van Dantzig thus discusses how to relate mathematical concepts to physics. According to him, this step is not self-evident. He warns against overrating mathematical models of a physical situation, which were in his time used in various fields of science and highly valued. Van Dantzig expresses his objections by explaining that trying to reach more accuracy in the determination of a certain value does not always have meaning within the physical context. For example, when one wants to determine the mass of the earth, it is meaningless to ask whether this number is rational or irrational. Besides, Van Dantzig attends his readers to the fact that a mathematical concept like ‘limit’ does not have a practical translation. In such a case, applying mathematics in physics does not increase the level of logical thinking, but instead it has a decrease as a consequence. This article shows at first the connectedness between mathematics and the other exact courses in school. It is only since issue 38.4 that the subtitle on the frontpage reads ‘journal for the didactics of mathematics’ instead of ‘journal for the didactics of exact courses’, a change that as a matter of fact took place imperceptible. But Van Dantzig does not limit his conclusions to physics as a school subject. Moreover, his article is about the relation between mathematics and physics in general and it is only in the end that he relates to education. This does not surprise, for the rest of this article has a philosophical approach more than an educational one. This becomes clear in the style of the article: it is not intended to be directly applicable in a classroom situation, in it being on-practical and in the scientific level of the article. Next to those longer articles, issue 29.1 contains an *In memoriam*, of Dr. G.F.C. Griss. Griss might have been partly a high school teacher, he had also made a name for himself at university with his work on invariant theory and intuitionistic logic. It is striking that in his *In memoriam* written in *Euclides*, the journal for education of exact sciences, Griss is honoured for his academic work, while only two sentences are given to his work at high school. There can be a variety of reasons for this, but in the fifties and even more so before the fifties, academic work was in general higher valued than teaching at high schools. Issue 29.1 ends with a few announcements, an advertisement and a one and a half page on a mathematical-philosophical consideration of homeostructurality in geometry, aesthetics and mysticism. Philosophy, according to author H.A.C. Roem, should fulfil the search for unity in those three quite different subjects. Philosophy and ideas about a metaphysics behind everything are not eschewed in *Euclides*.

The goal here was to grasp the character of the fifties. Looking at issue 29.1 showed till thus far a new interest in didactical considerations, a mathematical curriculum at issue in the fifties, and a guidance for how to apply mathematics to physics. But are those ideas typical for issue 29.1 only or do they give information on the fifties as a decade? The most important characteristics of the fifties are enlisted below, mainly using 29.1 as a starting point. When the landscape of the fifties in *Euclides* is sketched, it will be possible to draw the place of history of mathematics in this landscape.

## 4.2 An academic-philosophical character

The character of *Euclides* in the fifties is rather academic, especially compared to contemporary issues. This was visible in issue 29.1, first when Van Dantzig treated mathematics and physics as academic disciplines more than as school subjects and second because academic work became more renowned than teaching. The makers of *Euclides* seemingly did not stop at the border between mathematics as an academic discipline and mathematics as a school subject.

This approach cannot only be seen in the moments already mentioned, but is an indication of something bigger. The field of mathematics education appears to be intertwined with the academic field of mathematics. This can be seen in three ways. First, by looking at the authors who wrote the articles in *Euclides*es. Were they active in the field of mathematics education only or were they also related to the academic field? A second way to see to what extent *Euclides* can be called scientific, is to look at the editorial staff of *Euclides* and their thoughts on the scientific degree of *Euclides*. As a third, it is important to reckon with the most explicit measure of the character of a journal: the content of the concerning articles.

### 4.2.1. Scientific degrees of authors in the fifties

The titles of Van Dantzig, who wrote in 29.1 are Prof. and Dr. It is no exception that an academician like he writes an article for *Euclides*, in the fifties. On the contrary, a short research into numerical representation shows that most writers in *Euclides* have one or more scientific degrees. This is most visible in the first years of the concerning decade and slightly decreases at the end of the fifties. Still, more than half of all the authors in the fifties do possess an MSc or a PhD. Moreover, professors regularly take time to write articles to be published in *Euclides*. In those years, the journal was called *Euclides*: journal for didactics of exact courses. However its context is education at high school, not at university, this journal has many academically schooled writers.

All these scientific degrees contribute to the scientific character of *Euclides*. Having a scientific degree means you have access to scientific resources, which will influence the content of your articles both on how scientific it is and what themes are discussed. The second will be seen in more abstract or theoretical articles. Issue 29.1 already shows both manners of influence. Bunt’s article about his research on the overload of the mathematical school curriculum for pupils probably has not been thematically influenced by academic mathematics. In his study, Bunts discusses questionnaires, along other methods. In carrying out his research, however his method is not explicitly described in his article, Bunt probably has used the then available standards on quantitative research. In Van Dantzig’s article about the mathematical model in empirical sciences, the second way of influencing can be seen. His contribution cannot be understood without looking at academic discussions about epistemology and the ontology of models in science, so there is indeed a thematic influence.

### 4.2.2. Scientific degrees of editors

The preceding subsection has made clear that the academic schooling of the authors who wrote in *Euclides* influenced the character of the journal. Besides, it is also important to know who the editors were and if their background was also scientific. They, after all, have a lot of influence on the character of the journal. Namely, which topics get attention is not only a matter of authors that write about a certain subject, but also a matter of discussion by the editorial staff. They influence which subjects are discussed within the field of mathematics education and thereby what possible changes will get momentum. It turns out that the editorial staff has an even higher percentage of persons with an academic degree than the authors. Table 4.1 shows how these numbers developed during

|  |  |  |  |
| --- | --- | --- | --- |
| year | total number of editors|number of editors who have a scientific degree | number of regular contributors|  number of regular contributors with a scientific degree \* | number of people who left | number of people that are new |
| 1950-1951 | 18|18 |  |  |
| 1951-1952 | 16|16 |  | 0|0 |
| 1952-1953 | 15|15 |  | 0|0 |
| 1953-1954 | 15|15 |  | 0|0 |
| 1954-1955 | 15|15 |  | 0|0 |
| 1955-1956 | 15|15 |  | 0|0 |
| 1956-1957 | 6|5 | 15|14 | 6|12 |
| 1957-1958 | 7|6 | 15|14 | 0|1 |
| 1958-1959 | 6|5 | 15|14 | 1|0 |
| 1959-1960 | 7|5 | 15|14 | 0|1 |
| 1960-1961 | 7|5 | 15|14 | 0|0 |
| 1961-1962 | 7|5 | 15|14 | 0|0 |
| 1962-1963 | 7|5 | 15|13 | 1|1 |
| 1963-1964 | 8|8 | 15|13 | 0|1\*\* |
| 1964-1965 | 8|7 | 15|14 | 1|1 |
| 1965-1966 | 8|7 | 14|13 | 1|0 |
| 1966-1967 | 8|7 | 14|13 | 0|0 |
| 1967-1968 | 8|7 | 13|12 | 1|0 |
| 1968-1969 | 8|7 | 14|12 | 0|1 |

the fifties and the sixties. In 1956, a change took place in the organization of *Euclides*, for it was not anymore liaised to its founder P. Wijdenes. From this year on, there is a board of editors and a group of people who contribute on a regular basis. Both groups are mentioned on the second page of the journal. The table shows that there are only a few editors or contributors that did not obtain a scientific degree between 1950 and 1969, that is: five. A better look at those five shows that three of them did obtain a scientific degree later, leaving two editors that did not have a scientific degree at all. Their names were P. Wijdenes and G. Krooshof. P. Wijdenes was the founder of *Euclides* in november 1924. While he left in 1956, he returned in 1962 as one of the regular contributors. G. Krooshof, who is mentioned as editor from 1964 and would become chairman for eight years, thereby is the only significant exception on all editors from the fifties that have a place in the academic world. This however only takes place in the sixties. In the fifties, three editors got a scientific degree during the period they are connected to *Euclides* and the rest of them already possess one. The world of *Euclides* and the academic world turn out to be highly integrated. For the editors of *Euclides*, mathematics as an academic discipline is of as much importance as mathematics as a school subject, resulting in attention for academic insights and for topics that are under discussion in the academic world. It is likely that this has its influence on the themes that are discussed and how those are treated.

### 4.2.3. A scientific content

And this effect is indeed visible in the articles that are published in the fifties. It is not the practice of mathematics education that is dealt with, but the philosophical side of it. Van Dantzig’s article about the ontology of the mathematical model in the empirical sciences is a perfect example of this. Even if the subject itself might have a place in an educational context, the research style is not didactical at all. As has been pointed to above, it is a philosophical article about how a subsection of mathematics, namely mathematical models, might be used in physics. Van Dantzig’s article is not at all an exception in being written with an academic-philosophical approach instead of with an educational-didactical one, as might be expected from the title and subtitle of this journal. This article is a rule, no exception. A lot of issues show the same picture, for example the article in issue 33.1, written by Prof. Dr. A. Heyting. He writes about intuitionism and the value it can have for school mathematics. It reads as a contribution to a broader discussion about the subject, presuming knowledge about intuitionism and formalism and the relation it has to school mathematics now. As a matter of fact, Heyting thinks intuitionism can only be of use after a few years of training in it at university. But most important is how this article again shows an academic-philosophical approach. Last but not least, the content of this journal shows its relation to the academic world for it contains several inauguration speeches of professors.

The three preceding paragraphs show that the field of mathematics education, in which *Euclides* has an important place, cannot be understood without the academic field of mathematics. The academic character of the journal influences the field of the education of mathematics, however it must be taken into account that the field of mathematics education is much broader than the journal *Euclides*.

## 4.3 Rising didactics

There is one last thing we can see from issue 29.1, which can be taken as exemplary for showing characteristics of the fifties. I briefly mentioned a new heading, set up by Johan Wansink. Under this heading, articles from various journals and magazines are treated in short. Short is really short; mostly, only the title is mentioned. When Wansink introduces this new heading, he states explicitly that the heading serves to create more attention to didactics. His goal is to give an overview of the subjects in which de Dutch mathematics teacher is considered to have interest (29.1, Wansink, p. 1). But titles alone are not that inspiring. How would the average mathematics teacher get these journals to read more than a title? That is possible by subscribing for a portfolio with magazines from abroad. Once you received the magazines, you had a week or so to send them to the next subscriber and *Euclides* arranged for this. *Euclides* did so to make in-depth information of magazines from abroad more accessible and cheaper for the mathematics teacher. It has existed at least since 1950. There are not always enough participants to prevent financial trouble for the whole, according to the minutes of a members’ meeting in 1957.[[13]](#footnote-13). In 1961, there was again a call for participants, this time in *Euclides*. Despite this, the reading portfolio is successful enough to exist a few decades afterwards. The magazines in this portfolio are selected for treating mathematics and education. Apparently, the next step is to create a heading to mention and highlight the didactical articles. This shows there is an expected demand for deepening the information particularly on didactics; at the beginning of the fifties.

This is presented as a new movement at that time. However, when looking forward to the sixties, it turns out that authors or editors continue to state they, for the first time, have attention for didactics and the pupil. To understand how those statements relate to each other, a context different from that of the field of mathematics must be taken into consideration. This context is the context of changing ideas on democracy in the Netherlands in the fifties and the sixties. Wim de Jong has done research on this subject by looking at citizenship education, a course in Dutch schools, also in the previous century. He shows how the opinion of and attitude towards the citizen develops accordingly the ideas on democracy, and especially the opinion of and attitude towards the youth who have to be taught about democracy. As an historian, De Jong tracks the developments within this course and thereby shows how this statement concerning real attention for the pupil must be interpreted. It is here that a parallel with mathematics education is useful to understand the rising attention for didactics in this field.

### 4.3.1. Ideas on the democratic youth

In the fifties, the discussion about democracy was dominated by a view of the youth who needed to be educated and learn the basic ideas of a democracy. Therefore, youth parliaments and citizens' days were organised and the moment when young people voted for the first time in their lives was specially celebrated. This view contains the premise that before their political education, young people had no voice in the democratic process, and could not have one because they were attributed indifference and apathy.[[14]](#footnote-14) It is important to realise that young people are still taken seriously in this context, moreover, all sorts of things are organised to teach them the game of democracy. And in the sixties the same goal exists, but the opinions on the place of the youth in a democracy have changed. Within the main organisations in youth work, the conviction has then grown that young people should be given the choice to determine for themselves what they need for democratic education. The priority from the fifties on learning how democracy works then made place for a focus on educating young people.[[15]](#footnote-15) The consequence of this, for instance, is that within the libertarian repertoire of the sixties, as recognised by De Jong, attention is mainly given to empowered people, instead of to the empowerment of people that is more in line with the libertarian repertoire of the fifties.[[16]](#footnote-16)

Within the didactics as represented in the fifties and sixties, a similar development is visible. The subject of how the pupil should be educated recurs again and again and is therefore not new, but the way in which this should be understood differs. In the fifties, the way in which mathematics should be taught was relatively often expressed as 'the pupil must learn mathematics in order to become smarter', whereas in the 1960s 'the pupil must learn mathematics in order to benefit from it in the changing society in which mathematics has an important place' was more dominant. In these decades, *Euclides*repeatedly states that the focus will, from now on, be on the learner. It is clarifying to see how this statement in itself is part of a repertoire that was more widely present in society in the sixties. Moreover, it is plausible that also in the sixties, there was a pitfall to see everything behind as less successful and to forget that environments and social developments are also subjected to change.

Within mathematics education, therefore, there can be different ways of implementing didactics, but at the same time the changeability of the content of didactics does not deny an existing importance of didactics itself. A factor in this development is that didactics as a discipline and the structure of the field of mathematics education was much less developed in the fifties than in the sixties. This is reflected, for instance, in the training given to the mathematics teacher.

### 4.3.2. Teaching the mathematics teacher

In the fifties there are several ways to become a mathematics teacher. For every way the requirements concerning the pedagogic and didactic skills were low for a long time already in the fifties. Wansink mentions this explicitly in 1964 in Euclid, when he describes the Royal Degree of 1952 with the requirements regarding the pedagogic-didactic training of the teacher as “a timid attempt to make at least something of the actual teacher training in the Netherlands”[[17]](#footnote-17), but at the same time judges it as a modern formulation of the contents of the Royal Degree of 1828 and thus calls it rather outdated. In the fifties, a teacher thus began his career with little exercise in pedagogical-didactical skills. In general, there was still a lot of reorganization work to be done in education. Moreover, there were still few unifying organisations, although the Working Group for the Renewal of Upbringing and Education was an exception. From 1952 onwards, it publishes a Bulletin (in Dutch: Mededelingenblad) in which the rise of didactics is clearly visible. In the early years, a contribution by Van Hiele, for instance, shows how much the raison d'être of didactics is still under discussion, while at the end of the fifties there is a discussion about which didactics is best. In the Bulletin, this discussion is based on in-depth knowledge of the psychological thought process behind learning mathematics. More generally, by the end of the 1950s, in *Euclides*it also becomes unmistakable that knowledge of didactics is indeed necessary in the classroom and a discussion takes place about what various didactical approaches have to offer. Thus, a start is made with the development of a field for mathematics education and didactics. Within this thesis, and certainly within the methodological framework, the question then arises as to the place of the history of mathematics within it. As part of the teacher's education, however, the question is not treated.

There is however something else of particular interest within the education of mathematics teachers. However it takes too far to elaborate on how to get a qualification in the fifties[[18]](#footnote-18), it is remarkable that the focus was on making exercises, first to practise his own mathematical knowledge, but also to prepare for the later job as a teacher. That has to be related to the teacher that had to make the exercises for his pupils by himself. With this focus on the education of the teacher, there is only one obvious focus in how pupils should learn mathematics: making exercises. The act of doing exercises is what has an educational value in the education of mathematics. This does not bring one to have interest in the exercises that existed in earlier times or refer to history in another way. Therefore, it is in no way obvious to educate about the history of this means, the history of making exercises.

## 4.4 Bunt and Bruins

### 4.4.1. A curriculum change

Thus far, the chapter discussed why history of mathematics was not treated. That it does not have a prominent place in *Euclides*, does not mean it does not exist within mathematics education. In fact, from 1954 on, some teachers chose on their own initiative to teach the history of mathematics. This only happened at a part of the gymnasia, moreover only at α-gymnasium. In the Netherlands, there existed until 1968 two profiles within the gymnasia, one classically and humanities oriented (α) and one scientifically oriented (β). From 1960 on, history of mathematics was an elective topic within the exam for the α-gymnasium. However a pupil could only make this choice when the school or the teacher had chosen to place history of mathematics in the curriculum. At the same time, when that was done, every pupil doing α-gymnasium followed the lessons about history of mathematics. As an example, this happened in 55 schools in 1968-1969.[[19]](#footnote-19) This curriculum change must mainly be seen against a background of dissatisfaction with an old curriculum, especially for α-gymnasium, as explicated by different authors.[[20]](#footnote-20) Its start had to be placed in 1951, when five mathematics teachers started a trial with teaching both history of mathematics and statistics, under direction of Bunt. Bunt expressed his wish to make statistics part of the curriculum already in 1948.[[21]](#footnote-21) The choice for mathematics, however, might be related to the person of Dijksterhuis.[[22]](#footnote-22)

He held more than once a talk about the teaching history of mathematics.[[23]](#footnote-23) Two of them are published in *Euclides*. E.J. Dijksterhuis was a fervent promotor of the history of mathematics in general and was also known for that. In his talk, which is not just a talk but mainly a call for a specific change, he first excuses himself for not being familiar with teaching or even learning at α-gymnasium, for he only had experience with teaching at H.B.S., a somewhat more practically oriented type in the Dutch school landscape. Therefore, he only wants to timidly raise questions: would it be possible to organise it like this, or in that way? He proceeds by presenting the premise that it is possible to let the pupil become acquainted with an important aspect of Greek classical culture, through teaching mathematics in a specific way. To achieve this goal, the teacher needed to approach mathematics for pupils in the fifth and the sixth year as a historical phenomenon – which bears the inherent question of its origin. Dijksterhuis thus proposed, having let go of his timidity in the meantime,, that history of mathematics would get a place in the fifth and sixth year of the α-orientation of Dutch gymnasia, which means a pupil had less scientific subjects and more focus on learning languages. Possibly this idea was more widely known, but interestingly enough he brought it in 1952 as an idea never heard of before.[[24]](#footnote-24)

Two years later, two books treating history of Greek mathematics existed – intended public: pupil of an α-gymnasium. In the Netherlands, those two books were widely used in the period when the history of mathematics was part of the curriculum for α-gymnasium. The two books have an entirely different character. The first writer, Bruins, presents texts – the ones most important – from Greek mathematics.[[25]](#footnote-25) Bunt, the author of the second book, tells a history and explains how to use the techniques of Greek mathematicians, while giving examples and assignments.[[26]](#footnote-26) Bruins’s book is for three fourths in Greek, while Bunt’s book contains a few Greek words and sentences, mostly translated. Bunts book was, as might be anticipated, much more popular than Bruins’s book. Interestingly enough, Wansink mentions in a chapter about history of mathematics at the α-gymnasium, that several persons had pleaded to use Greek mathematical texts for this. Those people, and not only Bruins, apparently expected that this would work, while it was not successful.[[27]](#footnote-27)

### 4.4.2. History is Greek

Looking into the actual mathematics Bunt and Bruins deal with, strikingly one has to conclude that history of mathematics ends after the decline of Greek mathematics. They hardly treat the mathematics developed thereafter. Bunt’s *Van Ahmes tot Euclides* has a fitting title, but the subtitle ‘chapters from history of mathematics’ does not mention a choice for Greek mathematics. Still, the only reference to mathematics after Greek mathematics is a sentence mentioning the Arabs and North-Africans who brought us the Elements of Euclid. Bruins mentions his goal to present the main points of pre-Greek and Greek mathematical thought and therefore, one cannot expect he treats history of mathematics after the Greek period. But whether the choice is made implicit (Bunt) or explicit (Bruins), it is still the same choice.

Why does history of mathematics for Bunt and Bruins only mean Greek mathematics? An answer can be found in the talk given by Dijksterhuis, which strengthens the feeling that his talk indeed has a relation with the addition of history of mathematics to the α-gymnasium in 1954. Dijksterhuis held his talk for the association of teachers in classical languages and made the point that classical languages should include more than languages and culture. It had to incorporate classical mathematics as well. So Dijksterhuis had in mind not to add history of mathematics to the curriculum, but to add Greek mathematics in particular. The same is true for the actual change to the curriculum, made two years thereafter, looking at the school books used to bring this curriculum change in practice.

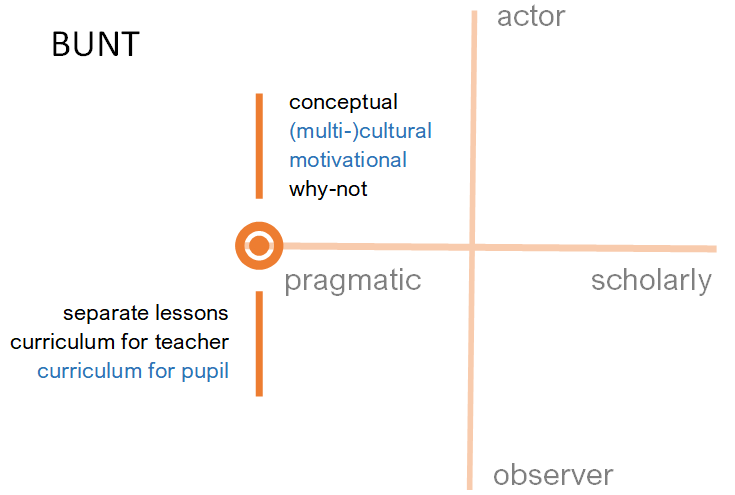
The awareness of the limitation to (pre-)Greek mathematics was however not always clear for everyone. When Bunt opens his foreword with the natural interest the pupil of mathematics will have in history of mathematics, he completely lets go of classical education. The most explicit sentence states that subjects from classical mathematics indeed arouses interest by pupils. The adjective would be superfluous if there were no other kinds of mathematics. This reference is all. Bunt thus possibly forgot or did not knew where this movement started. Bunt however was not the only one, Dijksterhuis himself also forgot. Between his speech and the publication of Bunt’s book, there had been a test in five gymnasia. The results were published in a book reviewed by Dijksterhuis in *Euclides*.[[28]](#footnote-28) The concrete situation is exactly the same as in his speech – the α-pupil gets acquainted with Greek mathematics – but now he mentioned a different goal. Instead of including mathematics in classical education, Dijksterhuis now mentions a long-expected improvement of mathematics education at an α-gymnasium. Probably, this shows Greek mathematics was more important for Dijksterhuis than the contribution of Greek mathematics to classical education. Whether those confusions are made coincidentally or not, they show that Greek mathematics was very prominent in the fifties when thinking about history of mathematics.

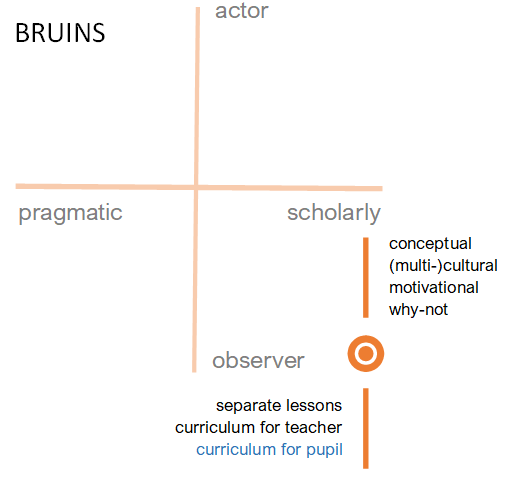
### 4.4.3 Applying the methodological framework

Until now, this example of history of mathematics has been treated by discussing its background and its actual mathematics. Those two aspects should be complemented by a discussion of how to categorize the manner in which history of mathematics is treated, using the methodological framework set out in chapter three. This will make it possible to compare different ways of dealing with history of mathematics and map changes over decades. The first layer of the methodological framework contains the two axes as the figure shows, and afterwards why-arguments and how-layers should be labelled to a point on these axes. This first layer is quite different for Bunt and Bruins, so they will be treated separately. The next layers follow from the context of the books as used for the curriculum and do not differ per book.

The writing style of Bunt’s book immediately shows its pragmatism, as the word is used by Kjeldsen: there is no distinction between past and present. This can be seen from often-occurring sentences like ‘apparently, the thought was as following’ or ‘one now had to use fractions’. To fulfil the first layer of the framework, it should now be determined if Bunt writes from an actor’s perspective, with the goal to use his discoveries in his own world, or from an observer’s perspective, only searching for an historical overview and awareness. A choice between those two does not follow from the actual sentences in the book, but can be read from the introduction and the goal of this book. It served as support for lessons in mathematics (‘an interesting background should help to understand’) and it should be a way to learn mathematics as well as to lead to more insight in the classical world. Apparently, both perspectives are combined.

Bruins’s book has a different character, as has been mentioned. Here, the style is scholarly, for the first chapters read as objective descriptions of what mathematics the Greek used and knew. Thereafter, Greek texts are literally written over, and this does not give information about Bruins's approach to history of mathematics. Moreover, the text is written mainly from an observer’s perspective. According to Bruins, Greek mathematics influenced the Greek world in all aspects. So the reader needs to make sure he understands the mathematics dealt with. Still, the goal is rather to get historical awareness and knowledge of the Greek, than to actually use this knowledge.

The second layer is about the arguments mentioned for why and at which layer to use history of mathematics. Before looking further at the arguments Bunt en Bruins mention, it has to be made clear that there is one reason which is given most credit to: mathematics in the α-profile was a mathematics only scaled down from mathematics in the β-profile and needed improvement and an own character. So, dissatisfaction with the current situation was in 1951 the first reason to start using history of mathematics in education, making this a case in which history was applied mostly for fitting into the overarching goal of the α-profile, which in this period principally was classical, Greek education and breeding.[[29]](#footnote-29) This wording is stated as general knowledge, but shows again that education, especially the α-profile, was interrelated with the Greek world, a lot more than after 1968, the year in which the educational system in the Netherlands was changed in many ways. 

Of the four categories of why-arguments: conceptual, (multi-)cultural, motivational and why-not, Bunt mentions first the motivational argument and also the cultural argument. These were also the main arguments used for this curriculum change. Bruins not really mentions any arguments. At which layer history of mathematics is aimed to use differs per book: Bunt has a more didactical approach, giving attention to a learning approach and working through the book, while Bruins just informs and presents primary sources. Maybe Bruins plotted a role for a teacher, maybe he did not care about didactics. At least, Bunt shows he maintains a didactical approach by often referring to the pupil. This attention for the pupil is not pretended; he has done an experiment with history of mathematics in various schools. 

It is possible to be short about the third layer: history of mathematics is here used as a part of the curriculum, not as a separate lesson or as background for the mathematics teacher.

Bunt and Bruins and the curriculum change are thus an exception in how concrete history of mathematics is given a place in the field of mathematics education in the fifties, and moreover show history of mathematics and a didactical approach indeed go together. Besides, the history of mathematics that is treated turns out to be Greek mathematics without giving a reason for this limitation. It would be interesting to know if this is true for the time afterwards, or that this is only true for the fifties, when the curriculum change maybe has an origin in Dijksterhuis’ talks or in the general significance of anything Greek back then.

## 4.5 Conclusion

We have now seen that in the 1950s an academic-philosophical character is typical for *Euclides*and that there is a lot of work being done in the field of didactics. Beforehand, I mentioned that a character sketch could clarify why the history of mathematics occupies a small but specific place within mathematics education. We have seen that this place is mainly taken within gymnasium education for α-pupils. In the discussion about this, it turned out that Dijksterhuis might have played a role in the establishment of the history of mathematics within gymnasia, and that Bunt and Bruins made this possible by writing a textbook on the subject. This means that an influence can be discerned from the academic-philosophical and from innovative didactics. Dijksterhuis was certainly in the academic-philosophical domain, having written articles and books on the history of science. Bunt, however, was part of the WVO. The programme itself seems to originate mainly from the academic knowledge of the history of mathematics, as its content follows Dijksterhuis' proposals and its emphasis is on Greek mathematics.

Furthermore, in the fifties the Euclidean way of teaching mathematics comes to the fore now and then, that is, axiomatically as in the Elements of Euclid. Within this way of thinking, inspiration is drawn from the past for the structure of mathematics education.

But now, how does the lack of history of mathematics in most places in the field of mathematics education relate to the background outlined above?

For this, we must start with Greek mathematics again. In the early 1950s, it was still often argued that Greek civilisation had influenced contemporary society to a large extent. That claim however lost force when througout the fifties the importance of mathematics in the society that was becoming increasingly technological and also more often referred to as technological, became more apparent.[[30]](#footnote-30) The fact that the history of mathematics remained an optional subject for α-gymnasium pupils until 1968 was mainly due to the fact that it was seen as one of the few ways in which α-pupils could still become interested in mathematics. After all, α-pupils would not continue with mathematics in their future profession.

Moreover, throughout this chapter, it became clear that including the history of mathematics in the pupils' curriculum was a poor match with the view of mathematics within education as a whole. The value of mathematics was seen more in doing the exercises than in knowing mathematics as a discipline. In many descriptions of the purpose of mathematics, an instrumental approach is noticeable, while it is less important how the field as a whole functions and how it has been developed. Within this view, there is a lot of freedom for the teacher to determine which parts of the field the pupil gets acquainted with. How often the history of mathematics appeared in specific lessons on initiative of a teacher, is quite difficult to track nowadays.

# 

# V The sixth decade

One topic stands out in mathematics education of the 1960s, and this is the curriculum change of 1968. This change in the curriculum was called the Mammoth Law because of its size and it brought about changes for the education system as a whole. For mathematics education in particular, this had to do with the fact that for years the character remained traditionally Euclidean, even though some changes had been made. The history of mathematics is, as it was in the 1950s, part of the whole field, but has no fixed place in the teaching of mathematics. However, things are shifting compared to the 1950s, so in this chapter an attempt will be made to explain from the 1960s background why history of mathematics has the place it has in the 1960s.

## 5.1 Background of the Mammoth Law

'À bas Euclide', said Dieudonné in Royaumont in 1959. This moment became a turning point for those who wanted to abandon Euclidean geometry. It probably contributed strongly as a first reason to the change of curricula in several European countries and America. In the Netherlands, this argument was also present. Yet it did not gain prominence in the speeches made at the installation of the CMLW in 1961. This committee was charged with the task of submitting a modernised curriculum, which was to take effect seven years later in the school year 1968-1969. The first speech, by the secretary of state for education, addresses the recent social developments that create a need for mathematics graduates. The renewed social role of mathematics and its consequences for mathematics education are topics often discussed in *Euclides*.[[31]](#footnote-31) These developments have now highlighted the gap between school mathematics and university mathematics. The elimination of this gap, and thus the modernisation of school mathematics, is an international problem, says Dr. G.C. Stubenrouch, who is then secretary of state for education.[[32]](#footnote-32) This choice of words shows how, at least from the influential position of the secretary of state, the forthcoming modernisation was viewed. Instead of the 'Euclidmust go', the tone of this speech is one of 'we now eventually should revise the curriculum and take the effort to solve a long-existing problem'. For that matter, this sentiment was not unique to this subject; Kennedy points out that it was widely present in the Netherlands in this period.[[33]](#footnote-33)

It is also apparent in the second speech, pronounced by Prof. H. Leeman, as chairman of the new committee. He provides two reasons why mathematics education has not kept pace with, once again, the rapidly changing society. According to Leeman, this change started at the beginning of the nineteenth century. People began to doubt all previously established truths and, inevitably, mathematics as well, even though in this field, certainty had been the goal for centuries. The mathematics that now emerged had a very different character from traditional mathematics, while the latter is still very influential. Leeman, as chairman of the committee responsible for introducing modern mathematics into secondary schools, suggested that this influence is present because traditional mathematics offers the best didactic possibilities, even for learning modern mathematics. Hence Leeman's striking question in this very speech: “whether the strong discrepancy between modern mathematics and school mathematics is a sufficient motive for modernising the school curriculum”.[[34]](#footnote-34) His position lies still close to the generally accepted view in the 1950s that the practice of mathematics is indispensable for the development of thinking skills. In the 1950s, it was also customary to add that the contemporary world was determined by the Greek world, from about two millennia ago. This view was bound to die down, especially in the 1960s. Yet it took quite some time and the change in Dutch mathematics education cannot be called revolutionary. One can already see this in a committee appointed seven years before its final aim. Also, for a long time mathematicians were unaccustomed to the fact that their profession suddenly had social prestige and had to offer something concrete. At the end of his speech, Leeman does not say: finally we have the chance to kick Euclid out and offer our pupils an education that really benefits them, but says: “we certainly believe that modernisation of the curriculum is desirable and possible.”[[35]](#footnote-35) In *Euclides*, he is not alone in using this wording.

In order to understand what was going on in the field of mathematics education in the 1960s, we first of all paid attention to what was changing in the field of education. Often the meeting in 1959 at the Royaumont Abbey near Paris is looked back on as the starting point for a radical turnaround in education. The following sentence from the report of the Dutch participants is interesting material for comparison with that vision: “The general opinion was, if we are not mistaken, that Dieudonné's views are very valuable on the one hand, but on the other hand should be taken with a grain of salt.”[[36]](#footnote-36) Seen from the contents of *Euclides*, this also seems to apply to the general Dutch opinion. However, it should be borne in mind that *Euclides* only gives one reflection of how opinions were divided at the time in the whole context of mathematics education. Concrete steps were already taken from 1963 onwards, when a first round of reorientation courses are held for first grade teachers of mathematics in the country. Such courses were well attended by, also in later years, at least 70% of the target group.[[37]](#footnote-37)

## 5.2 Renewal of didactics or renewal of content?

So, in the 1960s, much of the attention in mathematics education was directed towards the modernisation of the curriculum. The 1950s can be seen not only in how much the modernisers talked about modernisation, but also in how this modernisation was carried out. The CMLW of 18 members including the chairman had only 2 members without an academic title; apparently the renewal of the curriculum was something in which universities were important. It were professors who gave the necessary reorientation courses and took part in the CMLW.[[38]](#footnote-38) When Wimecos (the association for teachers at the HBS) was asked to have someone take a seat on the committee on their behalf, they refused, because it was a state committee just because the ordinary teacher knew too little about it. It was not until the end of the 1960s that questions were asked about this. On 30 October 1967, during the presentation of an interim report and some discussion notes about the teaching content, Van den Brom asks a question that ends up in the minutes as “it is dangerous that there are so many professors in the Commission and the teacher-aspect is so poorly represented.”[[39]](#footnote-39) The minutes also record an answer: the teacher element is supposed to be well represented. Also, an estimated two hundred teachers have input through experiments and discussions.

A few issues of *Euclides* later, the same Van der Brom writes a critical article in which he calls for redirecting attention from the content of the curriculum to renewing the didactics used to teach that curriculum. His criticism of the CMLW may of course have many sources, but the voice he expresses in these examples is one of the many times that in the 1960s, in the midst of the modernisation of the curriculum, attention is paid to the question of whether it is the curriculum that needs to be renewed, or rather the didactics. Both the curriculum and the didactics are clearly the subject of discussion in this decade, and undeniably the question emerges as to which is the more important. This, too, shows that the modernisation of the curriculum in the Netherlands did not have a natural importance before all else.

## 5.3 History of mathematics as an undercurrent

The first two sections of this chapter on the 1960s set out the main discussions at that time. How dominant were these discussions and what consequences did they have for the place of history of mathematics?

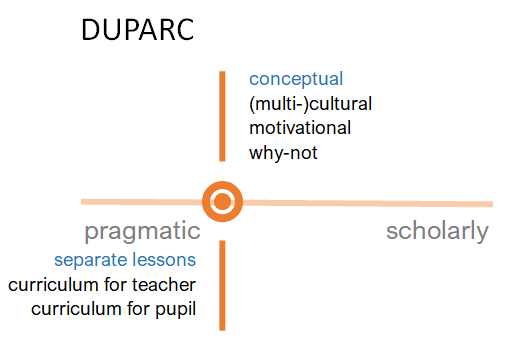
First of all: for the editors of *Euclides* those discussions were by no means dominant enough. In the 1960s, they regularly called for articles on the modernisation of the curriculum, preferably “articles that shed light on modern, possibly new material in a didactic way.”[[40]](#footnote-40) From 1962 until 1969, with the exception of 1963, all the editorial reports state that the editors would like to see more discussion of modern subject matter in the coming year.

From 1967 the curriculum is known and the editors, until that year led by J. Wansink, state that they hope to receive articles about the interpretation and didactic processing of the curriculum. In 1969, the editors finally are able to announce that this year more space than before could be devoted to “didactic problems or contributions to new programs, new curriculum or other forms of teaching.”[[41]](#footnote-41)

But what do the editors mean by 'too little'? Does this mean there is hardly anything written in *Euclides* about the modernisation of the curriculum? Not immediately, because there are authors who write about the modernisation of the curriculum, but at the same time, one can read from between the lines that there was no living debate among teachers of mathematics from 1963 onwards. Moreover, Vredenduin, who actually participated in modernising the curriculum, is the first person in *Euclides* to wonder whether mathematics will bring the pupil anything in practice. So even the critic is given a voice by someone who was already involved, instead of by an outsider. In addition, ample space is left for articles that deal with completely different issues. It is therefore needed to be aware that the reflection on the impending modernisation took place within the context of ordinary mathematics education, which continued as usual in those years between 1961 and 1969.

### 5.3.1. Ideas about the place of history of mathematics

That was also true of how history of mathematics was part of it. That small undercurrent continued throughout the 1960s. For example, in many volumes from the 41st onward, A.J.E.M. Smeur placed one or more articles on a historical figure that could be linked to mathematics, usually from the 18th or 19th century. The mathematical content of the contributions varies, while the historical is almost always present. A.J.E.M. Smeur was secretary of the Genootschap voor Geschiedenis der Geneeskunde, Wiskunde en Natuurwetenschappen. Additionally, in this decade, occasionally books are reviewed that are considered 'nice for the school library'.

Furthermore, this undercurrent becomes visible in a speech that Prof. Dr. H.J.A. Duparc held on a vacation course of the Mathematical Centre in 1968, about mathematics in antiquity.[[42]](#footnote-42) He began his speech by stating that it should come as no surprise that the organizers decided on a speech on this subject: a review of the long glorious history of mathematics is fascinating and provides insight, more so than with the younger natural sciences. In his wording, the academic-philosophical approach discussed in the previous chapter is occasionally recognizable. Duparc discusses, consistent with this, mostly Euclidean mathematics and some number theory. In his conclusion, he comes to the question of whether it is still useful to spend time on ancient mathematics now that modern mathematics already exists. Two components of Duparc's answer are noteworthy: firstly, his assertion that any reflection on the past makes sense, an assertion to which Duparc ascribes a self-evident validity. Secondly, he mentions that we can learn from history, from the major problems that existed and how they were tackled. Here he speaks of a deductive system that can be made useful to other disciplines from the point of view of mathematics and can thus be of importance for solving present-day problems. This way of seeing the usefulness for now of history of mathematics can best be categorized within the methodological framework as pragmatic on the scholarly/pragmatic-axis. The approach is predominantly pragmatic because Duparc is expressing the obviousness of the usefulness of mathematics from antiquity. At the same time, it is remarkable that Duparc apparently feels the need to put words to it. In addition, he discusses the system of mathematics and how it can help solve problems in other fields. This shows that a scholarly approach has also influenced Duparc's thinking, as he takes a particular part from that history and shows how it can be useful in the contemporaneous. With this method, he presupposes a distinction between the past and the present, the characteristic feature of the scholarly approach. When it comes to the vertical axis, an actor/observer-axis, it is not possible to tell from Duparc's description which of the two is more important to him. Both are briefly mentioned in his text. Nor are the reasons for engaging in history of mathematics identified here. However, Duparc does seem to want to apply history of mathematics primarily at the level of the individual teacher, rather than within the curriculum for teachers or students. 

### 5.3.2. Referring to locked history

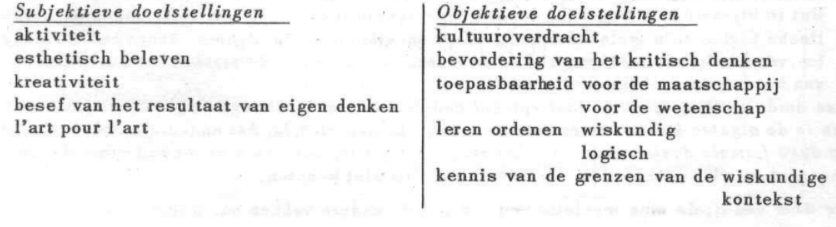
Although this undercurrent was small, it is valuable to spend some words on a last aspect of it. In 1965, an article on the helicograph appears in *Euclides* which illustrates this beautifully, by Dr. C.J. Vooys.[[43]](#footnote-43) It consists of a few lines mentioning the Greeks who mention the instrument or its use (for drawing a particular curve), a transmitted Greek manual for constructing it, and its Dutch translation. Not a single word is there that refers to anything contemporary, even the word helicograph only appears in the title. If we want to categorize this historical approach within the methodological framework, it is clearly an observer-perspective rather than an actor-perspective and there is a pragmatic attitude, because no distinction is indicated between past and present, in fact, the present is left out. In the 1950s, the point in the axis-system also tended to be in that corner, but there either knowledge was more actively deployed in the present (actor-perspective), or more thought was given to what history has to say in the present (scholarly-perspective). In the 1960s, people were still aware of the existence of history, but it had no context and did not acquire meaning in the present.

### 5.3.3. History of mathematics at the α-gymnasium

|  |  |  |
| --- | --- | --- |
| Pupils of α-gymnasia that did exam in history of mathematics out of five topics | | |
| 1961 | 257 | 25 |
| 1962 | 275 | 14 |
| 1963 | 263 | 10 |
| 1964 | 233 | 4 |
| 1965 | 319 | 10 |

Within the same way of thinking, there still exists the option for α-students in gymnasia to make the choice to study the history of mathematics, inherited from the 1950s. In the 1960s, this is completed with a final exam, as one choice out of five possible topics. This situation remains until the curriculum change in 1968. About the exams, there is a yearly report in *Euclides* by the committee that administers the central exam, which identifies salient features and areas for improvement. In the first half of the 1960s, it also mentions how many students took exams in history of mathematics. Were there to be equal interest in all subjects, between 47 and 64 students per subject were expected over these five years. History of mathematics can apparently count on significantly less interest, as the table shows. Nevertheless, history of mathematics thus remains part of mathematics education during most of the 1960s, at α-gymnasium.

## 5.4 History of mathematics in an innovative working group



The result of a 1959 meeting of the mathematics working group: the subjective and objective goals of algebra in modern mathematics education.[[44]](#footnote-44)

So far, about history of mathematics in education, it is clear that there is an existent small undercurrent. This however is not being activated and, moreover, it is not structurally part of what students learn in mathematics class or mathematics teachers in their training. This has been observed in particular by looking at the journal *Euclides*. It is useful to test this observation, by examining other sources from the same time. Fortunately, such sources exist, and one important source is the Bulletin of the Mathematics Working Group of the W.V.O., the Working Group for Renewal of Upbringing and Education. This working group lived up to the name of its organization and was indeed an innovating force behind mathematics education. It organized monthly meetings in Utrecht where the discussion about the desired renewal was lively. It also published a monthly Bulletin from 1952 to 1967 in order to give those who lived further away from Utrecht the opportunity to be involved in the renewal.

The question now before us is whether history of mathematics is seen within this innovative working group as a subject that should be given a permanent place in mathematics education. The first thing to mention is that the Bulletin certainly mentions subjects of which the Working Group wants to investigate whether they should be included in mathematics education. In the twelfth issue of the eighth volume, for example, Krooshof, who later became chief editor of *Euclides*, gives an account of this kind of discussion.

Yet it appears that in such reports there is no mention of history of mathematics. The only time the subject really comes up is when Van Hiele writes a review of Bunt's book, discussed in this thesis in section 4.4. In it, Van Hiele speaks quite positively about the use of history of mathematics in classes and expresses his intention to use this book for his classes. Yet the subject does not recur in his subsequent publications in the Bulletin. In addition, the word history recurs in an article about the founding of Pythagoras, the youth magazine on mathematics. One of their four goals at that time is to cover history of mathematics, using portraits of important mathematicians from the past.[[45]](#footnote-45)

In this issue of the Bulletin the topics that are indeed covered include the establishment of mathematics teacher training, didactics, film strips as an aid in mathematics lessons, the use of notations in mathematics lessons, the modernization of mathematics education in the 1960s, and of course didactics. Particularly in the first volumes, it is clearly visible how much the search for the meaning of didactics and its uniqueness has only just begun.

## 5.5 Conclusion

In the 1960s, much is about introducing a new mathematics into mathematics education, too much perhaps to look at that mathematics from a meta perspective. Moreover, didactics and content are competing for priority in attention. As a result, the approach of articles is one different from maintaining a discipline. However, they did of course convey a field of study and think about teaching mathematics. Moreover, authors in *Euclides* certainly discussed the goals of teaching mathematics. For example, teaching it was said to have a forming value. Sometimes even a cultural-historical value was attributed to possessing the knowledge, and being able to apply the various sub-fields of mathematics was seen as a requirement of the society of the time, but knowing the field of mathematics for the sake of it being a discipline is not addressed.

At the same time, it became clear in this chapter that in the 1960s the knowledge of earlier mathematics, especially that of the Greeks, was still present. In all the moments discussed above, however, the history of mathematics remains to exist without any translation or application. To what already exists is referred: “it would be an insult to you to expound...”,[[46]](#footnote-46) but the translation remains a literal translation, without processing or giving meaning.

Possibly the idea of teaching mathematics as a subject becomes more obvious when a program is already fixed for a number of years, and the methods used are becoming more and more uniform. After all, then there is a generally accepted framework of the mathematics being taught, making the move to the history of that delineation a possible step.

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# VI The seventh decade

At the beginning of the 1970s, people in the field of mathematics education are still processing the events of 1968. Writing a curriculum based on the New Mathematics is one thing, but then teaching students the mathematics of set theory in a formal way is a second step. During this decade, *Euclides* does not reflect much dissatisfaction with the program, but the next chapter on the 1980s will show that the failure of the 1968 curriculum for mathematics is then widely acknowledged. In 1975, the board of the Dutch Association for Teachers of Mathematics gives a rather negative first verdict in *Euclides*, precisely because the textbooks were written in haste, as information came much too late from the committee that wrote the program, the CMLW. “Some books stubbornly adhere to outdated traditions, others do not know how to deal with mathematical logic and contain sloppy formulations and improperly used symbolism. It also happens that the enthusiastic pursuit of new paths results in jettisoning too much of the old.”[[47]](#footnote-47)

Perhaps it contributed to the fact that in the 1970s the articles in *Euclides* finally seemed to become more connected to school practice, as the editors happily conclude in 1970.[[48]](#footnote-48) Incidentally, the editors in the seventies are clearly still looking for the balance between articles on mathematics education and those on mathematics content, as shown in the editorial report in 55.1.[[49]](#footnote-49)

Within this context, history of mathematics comes up very occasionally in *Euclides*. I will address this in two ways: first, by looking at how history of mathematics fits within the ideas on mathematics of this decade, and then by providing an explanation for why this history of mathematics, as it emerges in *Euclides*, remains with history of mathematics and seems to find no significant translation into teaching in this decade.

## 6.1 Diminishing history of mathematics

History of mathematics still appears in issue 46.7 (1971), when the then recently published third volume of Didactic Orientation by J. Wansink is discussed. In this book, which contains eleven independent treatises, room is made for the history of mathematics in education. An article by Dijksterhuis is included in its entirety, entitled *The place of history in the training of a mathematics teacher*. It was previously published in *Euclides*. The article is characterised by a large emphasis on Greek mathematics and relatively little content about all the developments that have taken place in the history of mathematics since then. Dijksterhuis certainly mentions the importance and influence of the Asian or Arab route that Greek mathematics followed after it became known in Western Europe. But that does not translate into the article on the level of content. Dijksterhuis' article is followed by a quite concrete overview from Johan Wansink of how the history of mathematics can be used in education. As a whole, the chapter seems to be rooted in ideas from previous decades. Only the focus for the level at which history of mathematics is useful is different. That focus is mainly on the curriculum for how the teacher is taught, instead of on using history of mathematics in seperate lessons.

History of mathematics is mentioned very seldom overall. One of the first times history is mentioned in this decade is in 1974, in an article by Van Hiele. In it, history of mathematics is not mentioned at all, yet the underlying view is exemplary precisely for how history of mathematics was viewed. In that article, Van Hiele describes how a new subject is learned. “A mathematician, for example,” Van Hiele states, “who reads for the first time a philosophical argument that comes from a historian, tends to judge that he is being presented nonsense. He is, after all, unfamiliar with this mode of reasoning.”[[50]](#footnote-50) Only when the terms of a field are familiar can it become meaningful. This phrase and Van Hiele's entire article, are closely aligned with New Math and fit within Van Hiele's level theorie. Van Hiele thus argues that the historian does something so essentially different that his work is impossible to follow for the mathematically trained. In doing so, Van Hiele does not rule out the possibility that mathematics itself has history, but it is certainly not obvious to bring history in as a subject within the field of mathematics. To do history, in his view, is entirely different from doing mathematics.

Yet it will eventually be seen that this lack of history of mathematics has not become the death knell for interest in history of mathematics. In the 1970s and even in the 1980s, it remained true that mathematics was approached as much as possible without culture and that history was therefore not a natural part of the field. However, other ways than that of culture are emerging to bring the history of mathematics back into the field of mathematics education, and Fred Goffree gives an impetus for one of these ways back in the 1970s. He writes from the perspective of mathematics didactics about mathematics as a human activity. Every human activity takes place at some point in history. Thus, if mathematics is characterized as a human activity, it carries history in it. In this way, this perception of mathematics opens the way for an exploration of history of mathematics.

In his role as a teacher of mathematics and didactics, Goffree visits classes and he takes that opportunity to accurately observe the teaching of mathematics. From there, he discusses mathematic-didactical notes in *Euclides* and in one of them he comes to the following observation: “Those who see elementary school students, P.A. students, and teachers at work experience mathematics as a human activity and learn with respect to the teaching and learning of mathematics.”[[51]](#footnote-51) In principle, this statement could have been written down at any point in time. Yet this sentence is exactly characteristic of how mathematics is seen at the time and is part of the New Math approach. Mathematics is then practiced when a single person makes an abstraction from the world around them, in the terminology of the Van Hieles: moving from one level of abstraction to another. Freudenthal, propenent of the Van Hiele-levels,[[52]](#footnote-52) gives the following example in an interview which shows that he observes this in the person who is thinking, thus bringing mathematics back more into concrete life:

“My grandson asks, 'What did that lady say to that lady pushing the cart?' ' After the second 'madam,' he hesitated, only to suddenly pour out the subordinate clause like a waterfall. Evidently he had noticed that something was wrong with the double 'lady'; the subordinate clause was a conscious (namely recognized as necessary) encore.”[[53]](#footnote-53)

This example, as Freudenthal himself also points out in the interview, contains no mathematics. What corresponds to mathematics, however, is thinking about reality (the two ladies are identified separately very easily) on an abstract level (the same word lady). Regarding Freudenthal's wording, it can be seen that he views this from the perspective of the thinking person. That position can also be taken within a different approach to mathematics education, but Freudenthal seems to see this as the essence of mathematics education: making abstract thinking happen. This is evident when he then moves without any transition to something that is indeed mathematical: perpendiculars.

The focus on the single person who practices mathematics is not only evident with Freudenthal. In issue 54.8, the results of a survey that examined how students experience mathematics are published.[[54]](#footnote-54) This is one of the few times since the early 1950s that *Euclides* has discussed the learner's attitudes, values, interests, and the like, summarized in the article as affective goals. Mathematics, then, seems to be understood more broadly and is approached less abstractly than it was in the early 1970s.

At this point it is necessary to make a comparison with mathematics in the 1950s, because then people spoke with terms that were close to what is used here from the 1970s. Similarly, in the 1950s there was a focus on the forming value of mathematics and so there was attention for the human level. Authors from that time, however, elaborate on that forming value by using cognitive objectives. The student is approached as someone who must be taught what is educational for him, while this survey shows attention to who the student already is and subsequently the effect, different for each student, that learning mathematics has.[[55]](#footnote-55)

The wording of mathematics as a human activity can also be read in the 1950s. Both approaches to mathematics thus pay attention to the human part of it, but it is important to see the difference, precisely because it is strongly expressed in the 1970s itself. That difference occurs when elaborating on what is meant by human activity. In the 1950s this is done by the Greek culture that shaped society, so with a clear historical line embedded in the culture. Within the New Math approach as it is present in the Netherlands in the 1970s, this happens precisely at the level of the individual human being, who relates to the world around him at different levels of abstraction. In the 1970s, the New Math intends to be independent of culture and thus it strongly contrasts with the culture-bound approach of earlier decades. Therefore, the description of mathematics as a human activity generally in the 1970s does not yet lead to a link with history of mathematics. But like any body of thought, this approach to mathematics had to reinvent itself over time, and the next chapter will make clear how precisely the interpretation of mathematics as a human activity, which had existed as an approach for some time, leads to a renewed interest in the history of that human activity, in the history of mathematics.

## 6.2 A reversal of the search for history of mathematics

History of mathematics thus remained remarkably absent in the 1970s. Above, we have discussed some of where history of mathematics did exist and how the then current conception of mathematics related to a historical view. Paying attention to mathematics in the 1970s was not a given from the prevailing climate within mathematics education. If it did not connect at that level, there remains the option that history of mathematics did connect to the field of mathematics education in other, more practical ways. I will therefore conclude this chapter by turning the question to the place of history of mathematics around, starting not from where history of mathematics is present, but from the field of mathematics education. How was it structured and was there a place where history could be embedded?

First, this could be the case at the level of the curriculum, a level within the field of mathematics education that has already been addressed. Between 1968 and 1987, however, there were no national curriculum changes and thus no changes take place there. But at levels other than the curriculum, there was little nationally organized at that time and there existed little between the level of the teacher and the level of a national curriculum. With that, the situation in the 1970s is really different from the present. In the meantime, for example, the first working group, Women and Mathematics, was formed in 1981 and organized national meetings. Incidentally, in the same year a working contact was established for History and the Social Function of Mathematics, which in the light of this thesis is an interesting initiative to pay more attention to the history of mathematics. The founders must have thought that there was an interest and a target group, or at least found it worthwhile to pay attention to the history as well as the social function of mathematics. At the same time, this working contact is hardly related to education, which is why no further attention will be paid to it in this thesis. Finally, it may be mentioned that from 1995 onwards annual National Mathematics Days (NWD) were organised.

What we here need to derive from these events in this chapter is that all these organisations and meetings were absent in the 1970s in the field of mathematics education. However, at such meetings contacts between mathematics teachers are developed and strengthened, while in addition all kinds of topics are addressed in lectures and the subsequent discussions and conversations. The long-standing absence of history of mathematics within education can thus perhaps be understood better because the historical component was absent from the curriculum, did not fit with the then prevailing intellectual-theoretical view of mathematics education, and there was little opportunity for new topics to emerge from various levels in the field of mathematics education. Although the number of (institutionalized) contacts has, beyond a doubt, increased dramatically in the decades since, this is not to say that there was nothing in the 1970s. The field of mathematics education is then more developed than it was in the 1950s.[[56]](#footnote-56) For example, the Mathematical Center began organizing vacation courses for teachers in 1946, which it continues to do to this very day. However, it is recalled at the 40th anniversary that in the early years of the Center there was still a desire to keep up with the field, while after that the focus shifted to didactics.[[57]](#footnote-57)

In fact, something else had yet recently made its appearance in mathematics education: statistics. At the beginning of the 1950s, statistics was written about in *Euclides* and experimented with, at the same time as the history of mathematics. The fact that statistics did indeed get a place in mathematics education can be traced back to the moment that it became part of the curriculum that became effective in 1968. Here the introduction is also linked to the introduction of the curriculum. The fact that statistics was chosen in 1968 and not the history of mathematics is, in terms of content, quite understandable: first of all, mathematics education had to be adapted to university content and statistics was especially important for further studies, while secondly the introduction of New Math was given priority.

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# VII The eight decade

In the preceding chapter it became clear that attention to the history of mathematics fitted poorly with the prevailing ideas about mathematics and the organization of the field of mathematics education in the 1970s. In the 1980s, it was signaled from different angles that mathematics education was lacking at that time. The development of a new perspective on how mathematics education should be designed, then, takes place slowly during the 1980s. At the end of it, the new curricula are launched, in which realistic mathematics education is paramount. In the meantime, not only the content of the curriculum has changed, but also a new approach has taken hold, which is related to the increase in scale that education has undergone. Numbers of pupils taking their final exams have long passed the hundreds, as was still the practice in the chapter on the 1960s. As a result, the conversations about education are cast in an entirely different framework. No longer does the question of providing ideal education have sole authority, but the question of measuring the success of the education provided also receives a great deal of attention. In this decade, history of mathematics finds its way back into the field of mathematics education from very different points of view. First, I will give some examples of historical contributions from the eighth decade of this century, bequeathed in *Euclides*. I will then elaborate on the main way in which history of mathematics is again related to education. This involves the conception of the practice of mathematics as an individual-human activity, which was present from the beginning within the New Math movement. I categorize this as important, not only in how influential it was, but also because of the manner in which it is a through-line with mathematics conceptions in the last decade of the twentieth century.

## 7.1 Kemme, Hogendijk en Bos

The 1980s, then, is one in which history of mathematics returns into *Euclides* in various ways. Actually, this development starts already in 1979, but since it is not otherwise visible in the 1970s and just in the 1980s, I discuss here two examples from 1979. Here, it is clearly visible that decade boundaries do not match practice. A total of three examples are discussed, which are the most striking or telling about the place of history of mathematics in the decade.

It is in 1979 that Sieb Kemme writes in *Euclides* about being linguistically engaged in mathematics education. Within the New Math movement from the 1960s, there was attention to the lingual side of mathematics education. Mathematics was brought as abstract as possible, and formulation is then of great importance. Kemme discusses all kinds of ways in which one is linguistically involved in mathematics education and in doing so he calls for awareness of the important role language plays, especially while mathematics is being explained. He recalls memories of his former mathematics teacher who used to have students write essays, memories of writing down mathematical experiences and discoveries with attention to the dialogue about them, and then also comes to terms and their history: “Where does the word come from? How does it fit together (content, area, circumference, addition, subtraction, parallel). Who will make a list of terms with their histories? Or does that list already exist?”[[58]](#footnote-58) With this Kemme shows that he is looking at a specific part of the history of mathematics from a completely new, probably not yet existing angle. Far-reaching influence of this remark cannot be recognized, but nevertheless it constitutes an example of how a historical approach is again being asked for.

A second example, one that involves a little more than a single remark, is a lecture on October 27, 1979. Jan Hogendijk then holds a lecture there about the number π.[[59]](#footnote-59) Without addressing the content of the lecture, it can be made clear that this lecture was not just mathematically inclined, because Hogendijk begins by briefly introducing how mathematics and science relate to Islam. Interestingly, Hogendijk mentions here as the reason for his choice of topic not a personal interest, but the request of the organizers that the topic be in the realm of the history of mathematics. So from the level of the organizers of the study days, there is a focus on history of mathematics. This is not an incident, but it is much more common during the 1980s that a historical topic is discussed at study days. The further content of the lecture into the number π fits within the subject of the study day and the link with Islam will have to be attributed to coincidence and personal interest. At least that is how he himself put it at the time.[[60]](#footnote-60) While this speech by Hogendijk is exceptional with respect to the academic field he belonged to, it is not so with respect to his own career, which combines history of mathematics and Islam. In this way, he contributed to the strengthening of history of mathematics, which, moreover, has not been limited to academia.

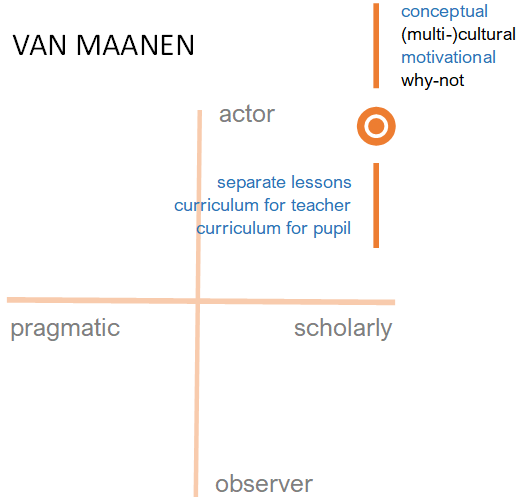
At the end of the 1980s, on 20 March 1987, Henk Bos accepted his position as extraordinary professor of History of Mathematics in Utrecht by giving a lecture.[[61]](#footnote-61) As a starting point he takes the alternation between surprise and recognition in the study of the history of mathematics. The elaboration of this corresponds quite well with pragmatic or academic approaches from the methodical framework with which this thesis opened. In his talk he pays no attention to mathematics education nor does he relate to the existing ideas there. With that, he seems too far removed from mathematics education to have any influence on it. In fact, Bos did write frequently in *Euclides*; one of his articles is discussed above as an example to apply the methodological framework. That article, too, was quite academic in nature and dealt primarily with topics and theories that are hardly relatable to mathematics education. Thus, Bos brings history of mathematics into mathematics education and sees relevance in it for proper reflection on mathematics and education, but his academic approach provides little replication in education.

## 7.2 Mathematics as a human activity

Among all the ways in which history of mathematics is reflected in *Euclides* in the 1980s, there is one most important in the line of history. It became clear in the previous chapter that within the New Math approach of the 1970s, mathematics was seen as a human activity. This was clearly distinguished from the cultural-historical line of the 1950s, which was almost always traced back to the classical, Greek world. Mathematics as a human activity was in the 1970s explained as the individual looking at reality on different levels. An abstract perspective on reality was to be learned through practicing abstract mathematics. It was precisely this abstractness that proved to lead to insurmountable difficulties in the classroom. In the 1980s, the discontent about this was complete: “We are now dissatisfied. We think it doesn't work. We think that set theory is not a good means of mathematical expression for the students. That it leads to blind knowledge, to copied half-understood notations.”[[62]](#footnote-62) So a lot of time and energy was put into renewing the mathematics curriculum, with the HEWET and HAWEX projects. With this, Dutch education wanted to move away from teaching mathematics in a structuralist way. These new curricula were introduced on the vwo in 1987 (HEWET)[[63]](#footnote-63) and for the havo in 1990 (HAWEX).[[64]](#footnote-64)

So what exactly is going wrong and what should replace it? The study day of 1979, the one on which Hogendijk gave his lecture on the number π, is introduced with two statements. The first comes from Sawyer's “Mathematician’s Delight”: “Mathematics is difficult when it is mistaken for something entirely outside everyday life.”[[65]](#footnote-65) The second is a Chinese proverb that can be translated as that one should not take on too much. So it is precisely attention to human activity, which has been part of the New Math approach from the beginning, that continues to be important. This is also apparent in an article written by Jan van Maanen. Van Maanen has been teaching mathematics for seven years when, in *Euclides*, he describes a lesson in which he teaches the subject matter not from the perspective of abstraction, but rather from a practical perspective. At the end of his article he says: “Mathematics should not be presented as something absolute. There were once people with problems, and the solutions to those problems live on today in our theorems and methods. That is precisely why I think it is important to know a little about the history of mathematics.”[[66]](#footnote-66) With this, Van Maanen, who incidentally was also concerned with the history of mathematics in his studies of mathematics, establishes a concrete connection between human activity and paying from there attention to the history of mathematics.

As a final step, I will place the approach to the history of mathematics used by Van Maanen in the methodological framework, again in order to be able to compare it with other times and places. Within this decade, this would also have been possible using the remarks of Kemme, Hogendijk or Bos, but all of them are in fact too fragmentary and too little influential within the field of mathematics education in the 1980s to make comparison meaningful.

First, there is the question of a pragmatic or scholarly perspective. On this axis, the scholarly perspective clearly has to be chosen, because Van Maanen always takes the effort to study the past, thus assuming that this is not evident from the present. In addition, Van Maanen shows clearly that mathematics should not only be observed, but should also 'live on in our theorems and methods'. The description of the lesson he gives in the article quoted above clearly shows its open and active character, so that an actor's perspective fits Van Maanen's approach. Reading between the lines, one notices that Van Maanen uses history mainly for its conceptual argument and its motivational argument. Other articles by him support this. As for the different levels on which Van Maanen wants to apply history, he does not become very specific. Nevertheless, he does want to actively deploy the history of mathematics in different places in education, as is also apparent from other articles he has written.[[67]](#footnote-67) He does not seem to want to limit himself to just one of those.

So a way is found out of the too far-going abstraction in an aspect that was present in New Math from the beginning, which is the human aspect of the practice of mathematics: everything human has a context and exists after all through history. From now on, realistic contexts are taken as the starting point, to teach mathematics from there. This is also what is at stake in the new curricula and is spread in the 1990s, when the new curricula are introduced. For Van Maanen this led to further study of the history of mathematics, also in the rest of his career. In the 1980s he is still one of the few interested in history of mathematics, but in the next decade he will meet more followers.

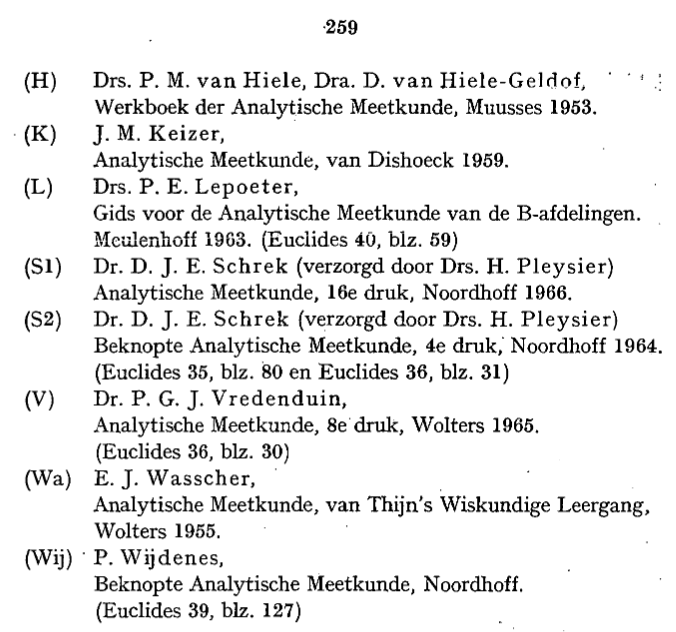
# VIII The ninth decade

In the 1990s, the path taken in the 1980s was further expanded. The history of mathematics was given attention, particularly as a result of the new mathematics programme A introduced in 1987/1990. During the 1990s this increased, against the background of a developing professionalism that had already begun in the previous decade. In the mid-1990s, articles also appeared in *Euclides* that explicitly addressed how the history of mathematics could be used in education, whereas in earlier decades that it was only present in a side note, or in Dijksterhuis' articles, who was a fervent proponent. In this final chapter, I will show how this development has taken place and what kind of thinking it is that gives the history of mathematics such an important role.

8.1 Mathematics textbooks

In recent decades we have seen some distinctive curriculum changes, and in the 1990s this continues. The HEWET- and HAWEX-projects were introduced in 1987 and 1990 respectively, and in 1993 de basisvorming (basic education) started for the entire Dutch education system. Within this new programme, every pupil, regardless of his or her level, follows the same education during a three-year period.

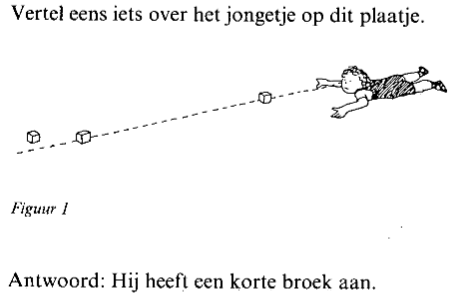
For both innovations, new textbooks were developed. In the 1990s, a development can be seen within the process of developing textbooks, which, incidentally, continued in later decades. The illustrations below show the representation of textbooks in 1969 (left) and 1994 (right).[[68]](#footnote-68) In 1994, more space is reserved for the single method, but that may also be because the various methods are discussed extensively in that issue. If we look at the presentation of a single method, it is striking that in 1969 the author of the textbook is mentioned first, followed by a title and then the publisher. In 1994 however, only the title and the publisher are mentioned and the author is missing. This example is illustrative of the way in which the books used in education are used and is also symbolic of a general change within education. In the middle of the twentieth century in particular, textbooks were written by a single author, who in the meantime had often built up a track record in education. Textbooks were then linked to the name of a person; when Piet Vredenduin was awarded the honorary membership of the Association of Dutch Mathematics Teachers, 'the books of Vredenduin' were once again praised.[[69]](#footnote-69) In 1994, this situation has changed to such an extent that in a review of nine different textbooks, the publisher is always presented as a characteristic. For that matter, in 1994 it was not yet the case that each individual textbook was a project of a publisher. The example given here, Mathematics Line, was set up by Anne van Streun. Despite contributions from others, the textbook is an elaboration of Van Streun's thoughts on mathematics education and is therefore certainly still linked to the person of Anne van Streun. A clear majority, however, namely six of the nine textbooks discussed in the 1994 article, do not have an author and the book is apparently a project of the publisher. The individual mathematics teacher who wrote a mathematics book in the evenings has made way for a group of authors who are selected for their ability to write a textbook and who are no longer necessarily in front of the class every day.



In addition, although I have been using the word 'textbook' up to now, the designation of the books used in mathematics education has also undergone a significant change: in the 1990s, there is no longer talk of textbooks, but of school books. The disappearance of the word 'teaching' still fits in with the educational visions of the 1990s, as the role of the teacher as a coach was introduced at the same time as basic education. The subsequent use of the word 'textbooks' seems to exemplify the institutionalisation of a new level in the field of mathematics education. Institutionalisation, because in the 1950s, as described above, the distance was already felt between the administrative tier that determined the programme and the teacher who had to put it into practice. It was certainly not the case that every random teacher wrote textbooks in those days, but there was still a link between the university and secondary education. That connection was rather diluted with the establishment of a special teacher education college, not connected to the university, and the disappearance of oral final examinations at the H.B.S. and the gymnasium, performed by those who taught at the university. In the gap that has now opened up, room is created for new institutions to take on tasks that are part of mathematics education but not part of everyday teaching. It is around the turn of the decade in the eighties and nineties that the word 'textbook writer' appears a number of times in *Euclides*, as, for example, in Jan de Lange's article on the introduction of basic education: “Further education will be essential, the textbook writers will have a great responsibility, the test makers will be watched with suspicion.”[[70]](#footnote-70)

This change is quite fundamental with regard to the field of mathematics education. It changes the whole context in which the history of mathematics has a place and is thus important for this thesis. Defining that relationship, however, is another step. I mention two ways in which this change in the field of mathematics education can affect the place history of mathematics has in it. First, the existence of a fixed discipline brings the step closer to thinking about the history of that discipline. Thus, it might increase awareness of the existence of the history of mathematics. Second, I mentioned in section 4.5 that the non-institutionalised nature of the discipline explicitly leaves open the possibility for individual teachers to do something with the history of mathematics. The existence of textbooks that can be followed blindly may lead to more uniform lessons.

## 8.2 Context-rich, realistic education

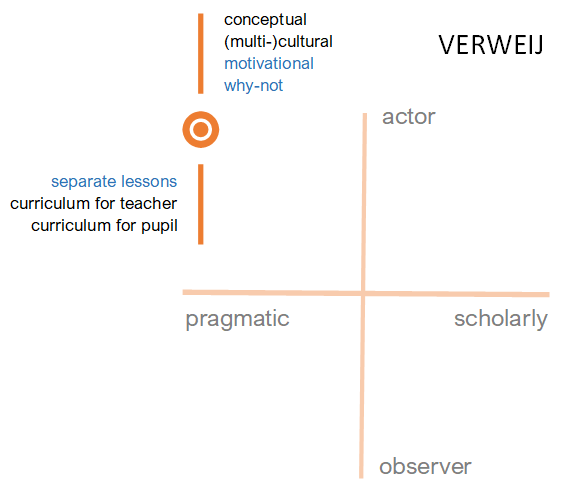
The article by Jan de Lange mentioned above discusses the developments that mathematics education is going through at that time. Both the mathematics programme A that has been taught at the havo-programma since 1987 and at the vwo-programma since 1990, and the W12-16-programme for the first three years of primary education, focus on realistic, context-rich teaching of mathematics. I have already pointed out that the word 'teaching' did not really fit in with the educational philosophy prevailing in the 1990s. Mathematics should not be taught, but learned from a given problem. Back in the early 1990s, *Euclides* raised the question of how 'realistic' this realistic teaching of mathematics could be in practice. The assignment pictured appeared with an article criticising the ambiguities and poor questioning of the packages used in the experimental schools.[[71]](#footnote-71)

### 8.2.1. As part of a curriculum

This context-rich teaching did, however, spark a new movement in the field of mathematics education. New, compared to all the curriculum innovations that had received full attention until the early 1990s. All these innovations first of all led to discussions about the content of the innovation and secondly, to a great deal of time and attention being devoted to further training. But once realistic mathematics education has really found its place in mathematics education, there is an important task for both teachers and the aforementioned textbook writers and test makers: find contexts. Contexts are sought in all areas and so the obvious option is to delve into the history of mathematics. The history of mathematics thus really started to emerge again in the 1990s, as one of the ways of giving content to context-rich education.

### 8.2.2. Als reden om geschiedenis in het onderwijs te gebruiken

In 1991, for example, Agnes Verweij wrote an article about an exhibition on the painters of the seventeenth century, especially Saenredam. This is a historical contribution in the field of mathematics education, but moreover she refers to its use in a mathematics lesson. She does this by referring to a winter symposium of the Mathematical Society, where Jan van de Craats gave a lecture on drawing a cube, which “could inspire many to include the work of the seventeenth-century architectural painters in the lessons about perspective, such as in mathematics B for havo and vwo.”[[72]](#footnote-72) Incidentally, in his lecture, printed in *Euclides*, Van de Craats does occasionally refer to the mathematics lesson, but without mentioning anything about adding a historical context to that mathematics lesson. Because remarks like Verweij's do recur in the rest of the 1990s and fit precisely within the context of mathematics education as described above, it is useful to place Verweij's remark within the methodological framework. For this purpose I include the then prevailing views. Presumably, that can be justified because she works at a teacher training college and in her articles in *Euclides* she appears not to be an opponent of realistic, context-rich mathematics education.

At the first level of the methodological framework it is clear that this approach to the history of mathematics is pragmatic, not scholarly. Nowhere, not even in Van der Craats' lecture, has attention been paid to the distinction between past and present. Moreover: when mathematics is here offered in a historical context, it serves to explain mathematics, and that goes beyond merely illuminating one's own time, as is the purpose of the observational perspective. The second level of the methodological framework categorises the reasons for doing history of mathematics in education. Of the four optional why's: conceptual, (multi-)cultural, motivational and why not arguments, especially the motivational argument and the (multi-)cultural argument seem to be important. To clarify why, however, it is necessary to go back to the first reason for using history of mathematics in this day and age: to offer mathematics in a context. This was done to give pupils a good basis for their later profession in society and to teach mathematics in an appealing way. So, indirectly, these are the arguments for applying history of mathematics in education. In general, there is another external reason for paying attention to the history of mathematics in education and that is a growing influence from the academic world of mathematics. Before I go into that, I would like to briefly mention the third level of the methodological framework in which I place Verweijs remark: the level of the mathematical field in which history is applied, the lesson, the curriculum or the curriculum for teacher training. Verweij's remark is quite specifically about the mathematics lesson, although the presence of the history of mathematics ultimately stems from the interest of the individual mathematics teacher, who was present at Jan van de Craats's lecture (or becomes interested through Verweij's article). Other such remarks in the 1990s seem to have to be characterised in the same way, which is also plausible within the realistic mathematics education that was prevalent at the time. 

## 8.3 From the academic world: a new reason for the history of mathematics in education

In the theoretical framework I outlined at the beginning of my research, I mainly cited sources from the beginning of the twenty-first century. One source in particular, Blom and Gulikers, is a reflection of all that has been written within the academic world about the use of history of mathematics in education. Their article clearly shows that in the 1990s the academic literature on the subject increased considerably. A special subgroup is the newly emerging Ethnomathematics movement, which, for example, devotes attention to Eurocentrism in science and explicitly addresses the fact that mathematics carries culture with it and therefore must not be imposed. This primarily academic movement is in line with *Euclides* and the Dutch situation on two fronts. Since the 1980s, every so often in *Euclides* one thinks about teaching mathematics to immigrants. In those years, the number of (families of) guest workers in the Netherlands grew considerably, resulting in a language barrier in education. Especially now that education had become more linguistic in the 1980s, this was causing problems.

In addition, within the Association of Dutch Mathematics Teachers special attention was paid to ethnomathematics, since member Hans Wisbrun in 1993 made the call for a contribution increase to raise money for mathematics education in, then, Third World countries. That call has been given an answer to date. At the time, Wisbrun posted a number of articles on mathematics education in Africa, including detailed coverage of ethnomathematics. In his words:

“The starting point of this movement is that each culture has developed its own mathematics to deal with the practical problems of everyday life and that you can use that genesis, in an abbreviated form, for your education. In this approach, mathematics is a cultural product, developed as the result of various activities.”[[73]](#footnote-73)

In addition, another noteworthy article has appeared in *Euclides* from this academic angle. This is a report of a thesis research with the central question: “What contribution can the history of mathematics make to the design of mathematics education?”[[74]](#footnote-74) Although it was already published in 1988, Van Breugel's article comes across as a tie-in to recent academic developments rather than something that emerged from practice. It is for this reason that the article is mentioned here. The core of the article consists of three examples, which at the same time are intended to give a reason for using the history of mathematics. It is notable that no attempt is made to answer the question in a theoretical way, but that three situations are mentioned that answer the original question in the affirmative. In this, it is particularly exemplary of the 1990s. His approach to the history of mathematics in education, as it can be categorised within the methodological framework, is almost identical to Verweij's categorisation given above. It does only differ at one point. Because Van Breugel introduces something new at this point, in 1988, and is aware of it himself, he does elaborate somewhat on what exactly he is doing and why he wants to use history. In his two-page article, the conceptual, the (multi-)cultural and the motivational argument can thus be discovered.

## 8.4 History of mathematics in the 1990s

It has become clear above that the history of mathematics is gaining a place in mathematics education from two sides, namely as a context example within realistic mathematics education on the one hand, and the influence from the academic world on the use of the history of mathematics, partly from ethnomathematics, on the other. It is now time to pay attention to the history of mathematics that exists in these years and what the story behind the categorisation tells. In *Euclides* this becomes visible, for instance through the articles by Ida Stamhuis. She writes about the history of statistics and probability in the nineteenth century. Stamhuis does not work in education herself, but investigates these subjects through her work at the university.

When in 1998 the study house was introduced in education, it made an appeal to pupils to work independently. Several reviews of historical books in *Euclides* remark that they are suitable for the study house.[[75]](#footnote-75) It is precisely the independent work that gives pupils the opportunity to do something that is not part of the curriculum or tested in the exam, and history of mathematics is apparently seen as suitable.

Moreover, Danny Beckers published in 1997 an article on history, which begins by clearly setting out a historiographical line for the application of history in mathematics education.[[76]](#footnote-76) And while historical contributions are thus becoming less and less the exception towards the end of the 1990s, presenting the history of mathematics with a clear historiographical line in order to apply it properly in mathematics education is.

This can be explained by the two ways in which history of mathematics entered education in the 1990s. As has become clear, the first way was through context-rich education. Within that context, the history of mathematics is one of the possible contexts for teaching mathematics in a realistic way. Context is the goal - history of mathematics is the means.This way of using history of mathematics gives no reason to delve into the backgrounds of mathematics or into the correct and truthful historical application of the history of mathematics. To the extent that the use of history of mathematics came from academia, it still found its application in the context of that realistic education. In itself, this movement from the academic world could be a ground for applying the history of mathematics historiographically in a correct way. In the theoretical framework, however, it already became clear that the reflection on this is not clear and that a clash easily arises between two starting points, namely that of the historian and that of the mathematician.

Something of this duality becomes apparent when Martinus van Hoorn, then editor-in-chief, reports in 1995 on a meeting about, indeed, a new mathematics programme for pupils from 15-18. This time in preparation for the introduction of profiles in the upper grades of havo/vwo, in 1998. Van Hoorn writes:

“Pupils who do not choose a heavy maths programme can take a history of maths, some say. This surprises me. Surely the history of mathematics is only interesting when it deals with essential matters? Measurability of line segments, calculating without logarithms, infinitesimal calculus, what should an alpha or gamma student do with all of that?”[[77]](#footnote-77)

For what reasons the teachers Van Hoorn refers to, noted this, of course, remains unknown. Perhaps some of them were familiar with the practice before 1968, when history of mathematics functioned as an optional subject for α-pupils. But it is equally possible that they were inspired by the growing interest in the history of mathematics for education in this decade. In any case, Van Hoorn is not very happy about it, arguing that pupils would then not learn the history of the subjects that are part of their mathematics curriculum. Van Hoorn seems to interpret the history of mathematics here as the complete development that has taken place. A brief look back to the 1950s makes it clear that this is not self-evident. At that time, the introduction of the history of mathematics appeared to entail a deepening of Greek mathematics. Introduction to the classical world was then seen as valuable for the pupil because of the influence it had on society at the time. In this respect, there has been quite a shift in the time between the 1950s and the 1990s, in the sense that the obvious link between the history of mathematics and the Greek world is really gone.

In addition, with this argument Van Hoorn draws attention to the fact that history of mathematics should be treated while retaining substantive knowledge on the subject at hand. This has been an important part of the discussion within academic articles on the use of history of mathematics within education and it now appears that this has been problematic from the start.

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# Conclusion

I will not here summarise the historical developments that have been discussed, but show first a few developments that are visible across the various methodological frameworks and then contrast two approaches to the history of mathematics. These two approaches arise from the general picture sketched above. Although the overview as a whole is obviously more complex, they can serve to map out many of the discovered views on the use of history of mathematics. It should also be mentioned that they do not exist in isolation from each other; it certainly happens that an author is influenced by both positions.

As mentioned several times, the methodological framework served to make a comparison over time of all the different approaches to the history of mathematics that were discussed. That comparison served to map developments, and was thus a tool for arriving at the above text. When we finally take a look at the methological categorisations for a single moment, it is noticeable that the pragmatic perspective is used more often than the scholarly one. This may well be due to the background against which the place of history of mathematics is examined here, namely the field of mathematics education. In addition, ideas about how to use the history of mathematics usually remain rather vague; the third categorisation, in levels of the mathematics field, was often absent or unclear. Thirdly, only in the 1950s there is any mention of the curriculum including the history of mathematics for pupils, whereas in the 1990s it were mainly the separate lessons that were mentioned.

I will now move on to the one position, which is mainly visible at the beginning of the 1950s, where the history of mathematics is part of mathematics itself and has its place within the field of mathematics itself. A genetic argument, that mathematics should be taught as it has grown, fits into this. This approach has shown its bankruptcy. The idea that the Greek world was the basis for the Dutch society of the 1950s was increasingly incompatible with the changing world of the time, but also with the abstract new mathematics that eventually made its way into the Dutch mathematical curriculum in 1968. In the following years, the history of mathematics became increasingly historicised and then history in *Euclides* is limited to a rare glance into the past.

In the other position, mainly visible in the 1990s, history of mathematics has a place within the teaching of mathematics. This approach is rooted in the 1990s in realistic mathematics education, where the view is held that mathematics is best taught from a realistic context. History of mathematics then turns out to be an easy source of contexts. Thus it is a means to something else. In *Euclides*, it becomes apparent that this is in itself a tricky situation, tricky because it carries with it an insurmountable question, namely how to do justice to the history of mathematics, which after all has a claim to existence outside of being a means for teaching mathematics.

Hence, an interpretation is given of how *Euclides* exhibits the two most influential positions given to history of mathematics within the field of mathematics education between 1950 and 2000.

Finally, I address a question that is not part of this thesis per se, and also falls outside the scope of the method used, but is nevertheless implicitly present throughout: what place would be ideal for the history of mathematics in mathematics education? After all, knowledge of the past helps to develop a perspective on the future. First of all, it is clear that the history of mathematics is given a place based on a frame of currently existing visions, about the pupil, about the position of the teacher, about certain theories that are in trend or not, and above all about mathematics in general. A vision of how the history of mathematics fits into this will therefore have to be developed anew each time. However, I have concluded my description of the two distinct positions with a problem that is linked to that position in history and thus warns us for the future. If we allow the history of mathematics to become too much a part of mathematics itself, that is not a very tenable position. In the last century, that approach eventually led to an over-reaching historicisation of the history of mathematics. After all, if the past is a closed story, it does not do justice to the open nature of the relationship between mathematics and history. The past must remain open to interpretation or it will lose its significance for the present. From another angle, too, attention should be paid to a historiographic position in the application of the history of mathematics. When history is used as a context for teaching mathematics, its independent existence and the importance of historical accuracy must not be lost sight of.

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# Discussion

In this thesis, a methodological framework was used to compare approaches to the history of mathematics despite different contexts and times. This was done on three levels: a historiographic axis system, four reasoning arguments why history of mathematics should be used, and three levels in the mathematical field where history of mathematics should have a place. The first was borrowed from Kjeldsen and was usually quite recognisable. The second, based on research by Blom and Gelikers who map out academic research, was also so, but its application was not always very insightful. Although in the period 1950-2000, history of mathematics did not have an overly fixed place in education, for some individuals it did. When they brought up the subject of the history of mathematics in their articles in *Euclides*, it was often to convince readers of its importance. They often succeed in coming up with most of the optional why's. Within the purpose of the methodological framework, however, it would be more interesting to know why this person was interested in the history of mathematics in the first place, rather than to tick off the reasons he uses to convince. At the same time, it is true that what he writes also influences others in the field of mathematics education and thus indirectly shows what is going on in that field.

This brings me to the next point that needs to be touched upon again, namely the relationship to the past that is being investigated. On many levels, the issue that lies hidden therein is addressed in this thesis. The past that is concretely examined in this thesis is the field of mathematics education in the period 1950-2000. I have investigated how the place of history of mathematics within it changed, mainly through written articles in *Euclides*. Some of the limitations that this creates include the existence of other ways and places where views have been expressed, the sheer volume of data that *Euclides* alone provides, the way in which the author and then an editor determines the content. An enthusiastic author can write a good story about using the history of mathematics in class, while in practice nothing happens with it, and the other way around can also happen: teachers do pay attention to the history of mathematics, while it does not appear in *Euclides*.

In this thesis, a conscious choice was made to examine the approaches to the history of mathematics in their context from a somewhat more theoretical angle. In this way, clarification can be given to the practice. At the same time, other research methods can be used to investigate the extent to which history of mathematics is actually used in practice, either in the past or in the present.

The results of using this method have come about mainly by reading, thinking, re-reading the database of articles provided by *Euclides*. Thus, relatively little use was made of digital methods as support. One reason for this was that the place of the history of mathematics appeared to be strongly linked to the context of the educational field, such as the prevailing view of mathematics. At the same time, those methods could possibly have increased efficiency.

A final point concerns the professionalisation of the mathematics teaching profession and, subsequently, the consequences that this has for the place of history of mathematics, as well as other possible effects on the development of mathematics education. I have made some suggestions in 8.1 regarding the first relationship, but quite a lot happened between 1950 and 2000 in terms of how the field was organised. Nevertheless, the field of mathematics education has been taken as a background in this thesis, while it exists in its own right and is undergoing all kinds of changes.

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19. Johan H. Wansink, *Didactische Oriëntatie Voor Wiskundeleraren, III*, vol. (Groningen: Wolters-Noordhoff, 1970), p. 329. [↑](#footnote-ref-19)
20. Wansink, *Oriëntatie III*, 327. [↑](#footnote-ref-20)
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23. In 1931 and 1954, presumably those were amongst others. [↑](#footnote-ref-23)
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51. Fred Goffree, “Vakdidactische Notities,” *Euclides* 54, no. 3 (November 1978): 81. [↑](#footnote-ref-51)
52. Sacha la Bastide-van Gemert, “Freudenthal and the van Hieles’ Level Theory,” in *All Positive Action Starts with Criticism* (Dordrecht: Springer, 2015), 179–204, https://doi.org/10.1007/978-94-017-9334-6\_7. [↑](#footnote-ref-52)
53. “De C.M.L.-Wiskunde,” *Euclides* 52, no. 3 (November 1976): 105. [↑](#footnote-ref-53)
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55. cf. 4.3.1. [↑](#footnote-ref-55)
56. cf. 4.3.2. [↑](#footnote-ref-56)
57. Korthagen, Th. J. ‘Zij Mogen Uiteraard Daarbij de Zuivere Wiskunde Niet Verwaarlozen’. *Euclides*, vol. 64, no. 4, Dec. 1988, pp. 110–11. [↑](#footnote-ref-57)
58. With reading signs Kemme emphasizes how the meaning of each word is visible in how it is written: in-houd, op-pervlakte, om-trek, op-tellen, af-trekken, even-wijdig; Sieb Kemme, “‘Talig’ Bezig Zijn Met Wiskunde-Onderwijs,” *Euclides* 54, no. 10 (June 1979): 397. [↑](#footnote-ref-58)
59. Jan Hogendijk, “Twee Vertellingen over Pi,” *Euclides* (May 1980): 395-408. [↑](#footnote-ref-59)
60. Bas den Hond, “De Meetkunde van Ibn Hûd, Koning van Saragossa,” *NRC Handelsblad*, November 14, 1985. [↑](#footnote-ref-60)
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62. Sieb Kemme, “Is Het Wiskundeonderwijs in Nederland Nou Nog Niet Af?,” *Euclides* 62, no. 7 (April 1987): 195. [↑](#footnote-ref-62)
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64. vwo: voortgezet wetenschappelijk onderwijs, which prepares to a further study at university; havo: hoger algemeen voortgezet onderwijs, which prepares for education in a certain profession. [↑](#footnote-ref-64)
65. Harrie Broekman, “Het Zijn de Kleine Dagelijkse Dingen Die Het Hem Doen,” *Euclides* 55, no. 9 (May 1980): 370 (quoted and translated to English from a Dutch translation of the book). [↑](#footnote-ref-65)
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68. L. van den Brom, “Als *l*1 ⊥ *l*2 Dan m1m2 = − 1?,” *Euclides* 44, no. 9 (June 1, 1969): 259; “Overzicht Brugklasboeken Voor de Basisvorming,” *Euclides* 69, no. 3 (November 1993): 67–77. [↑](#footnote-ref-68)
69. “Notulen van de Algemene Ledenvergadering,” *Euclides* 53, no. 6 (February 1978): 277; also Johan H. Wansink, “In Memoriam P. Wijdenes 1872-1972,” *Euclides* 47, no. 6 (February 1972): 202. [↑](#footnote-ref-69)
70. Lange, Jan de. ‘Nieuwe Curricula 12–16: De Basis Gevormd’. *Euclides*, vol. 67, no. 9, June 1992, pp. 259–62. [↑](#footnote-ref-70)
71. Translation of the exercise: say something about the boy on this picture. Answer: he wears short trousers. So within realistic mathematics, there is a challenge to hold on with mathematics instead of the context; Harthoorn-Postma, Nettie. ‘Een Impressie’. *Euclides*, vol. 69, no. 3, Nov. 1993, pp. 90–91. [↑](#footnote-ref-71)
72. Agnes Verweij, “Perspectiven,” *Euclides* 67, no. 1 (September 1991): 8. [↑](#footnote-ref-72)
73. Hans Wisbrun, “Wiskunde-Onderwijs in de Derde Wereld (Deel 2),” *Euclides* 71, no. 4 (February 1996): 136. [↑](#footnote-ref-73)
74. K. van Breugel, “‘Van Kleitablet Tot Overhead,’” *Euclides* 63, no. 4 (January 1988): 118. [↑](#footnote-ref-74)
75. For example: Gerdien Visser, review of *Van Rekeningh in spelen van Geluck*, by Christiaan Huygens and translated by Wim Kleijne, *Euclides*, March 1999. [↑](#footnote-ref-75)
76. Danny Beckers, “Historia Magistra Vitae. De Geschiedenis Als Inspiratiebron Voor Een Rekenles,” *Euclides* 72, no. 7 (April 1997): 259–62. [↑](#footnote-ref-76)
77. Martinus van Hoorn, “Verslag van Een Hearing,” *Euclides* 71, no. 3 (November 1995): 95. [↑](#footnote-ref-77)