# Formulas for Choosing the Most Economically Advantageous Tender - a Comparative Study

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**Abstract**

Choosing a supplier is a significant part of any procurement process. Usually, if the award criteria is not only price, it involves an evaluation of bids and ranking them by their score. The score is calculated according to a formula that takes as an input price, quality and their respective weights. The main objective of this research is to review and compare different formulas used in the procurement process. I study both relative and absolute formulas with particular emphasis on strengths and weaknesses of these 2 approaches.

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# Notation

| | | |
|---|---|---|
| $Weight_{Quality}$ | – | weight of quality |
| $Weight_{Price}$ | – | weight of price |
| $Q_i$ | – | quality of each individual bid |
| $Q_{Best}$ | – | highest quality of all submitted bids |
| $Q_{Ref}$ | – | reference quality |
| $P_i$ | – | price of each individual bid |
| $P_{Avg}$ | – | average price of all submitted bids |
| $P_{Best}$ | – | lowest price of all submitted bids |
| $P_{Worst}$ | – | highest price of all submitted bids |
| $P_{Ref}$ | – | reference price |
| $P_{Set\ Min}$ | – | lower end of price range |
| $P_{Set\ Max}$ | – | upper end of price range |
| $N$ | – | number of bids |

# 1 Introduction: What Are Procurement and Value for Money?

Procurement entails obtaining works, goods and services needed by the tendering entity to carry out its core functions. Obtaining, as it is meant here, includes the purchase, lease or rent of works, goods and services. Procurement is a process which can be divided into 6 phases: determining specification; selecting supplier; contracting; ordering; expediting and evaluation; and finally follow-up and evaluation[25]. As regards supplier selection, I assume that this is determined through competitive bidding.

Procurement is needed in both the public and private sectors. However, it must be noted that procurement in the public sector differs from that in the private sector in many ways. The 1st and most pronounced difference is that public procurement is funded by public funds. Furthermore, public procurement is subject to a number of regulations enacted to protect the public interest. One of these regulations is publication requirement. For example, in the EU, tenders above a certain threshold must be published on Tenders Electronic Daily, which is the online version of the 'Supplement to the Official Journal of the European Union', a publication dedicated to European public procurement.

Procurement is centralized when all relevant decisions are taken by the head office of a company or a central administration. It is decentralized when branches or local administrations are delegated the power to decide what and when to procure. There is also a hybrid model which is something in between full centralization and full delegation.

The importance of procurement has been elevated by several factors:

- Every operation relies on a supply of inputs which are in many cases obtained by procurement.
- Procurement can play a vital role in the delivery of strategic objectives.
- Efficient procurement can result in considerable monetary savings.
- Efficient procurement can help to achieve the best value for money.
- As regards the public money, poor procurement decisions or failure to comply with procurement legislation can result in a legal challenge.

3 key words for effective procurement are transparency, objectivity and non-discrimination. Transparency implies that all parties in the procurement process have all the information needed to make their decisions. The tendering entity should establish and announce conditions for participation, requirements specification and award criteria in advance. It should provide potential bidders as much information as they may need to submit their bids. The more precisely the tendering entity expresses its needs, the higher the chances that bidders will offer what it needs. On the other hand, bidders should provide the tendering entity with accurate and relevant information, so that it can make a wise and informed decision. Objectivity and non-discrimination mean that the procurement process is fair and all bidders are treated equally.

Another important feature of effective procurement is competition, which enables the tendering entity to get the best value for money. Competition is normally achieved through obtaining an appropriate number of bids. In other words, successful procurement heavily depends on a large participation of bidders. Therefore, the information on procurement opportunities should be disclosed as widely as possible.

Effective procurement should promote fair and honest competition among bidders. Each bidder should be treated equally and have equal opportunity to win the contract. All bidders must receive the same information. Fair competition and equal treatment are also important in the fight against collusion, bid rigging or corruption.

Finally, effective procurement is impossible without well-qualified, ethical procurement professionals.

Due to the limited resources, ensuring value for money in procurement is critical to the optimal allocation of available resources; thus value for money is the primary driver of procurement. It is a term used to assess whether or not an organization has obtained the maximum possible benefit from works, goods and services. It not only measures the cost of works, goods and services, but also takes into account quality; therefore value for money is not about achieving the lowest price, and is sometimes defined as the optimal combination of price and quality. Achieving value for money may be described in terms of the '3E's - economy, efficiency and effectiveness:

- Economy relates to minimising the cost.
- Efficiency means making the best use out of the available resources.

- Effectiveness concerns achieving desired results.

# 2 Problem Formulation

According to the European Commission the total expenditure of the government, public sector and utility service providers on works, goods and services accounted for 19.4% of GDP of the EU-27[1]. While in many cases, public contracts are awarded on the basis of the lowest price, hundreds of billions of Euros are being awarded based on the Most Economically Advantageous Tender criteria. The Most Economically Advantageous Tender is a synonym of the value for money and implies that other award criteria are taken into account, in addition to the price. Therefore, it is assumed that bids are evaluated based on price and quality. Whenever possible, price should be expressed as the total cost of ownership which is the true cost of works, goods or services over their lifetime. For example, the total cost of ownership of a car is not just its initial price, but also other expenses such as insurance or fuel. Quality is interpreted in a broad sense and includes all non-monetary factors such as technical merit, delivery conditions or aesthetic and functional characteristics. Measuring quality is not part of this research.

Evaluating bids based on price and quality requires an algorithm to choose the winner. Usually, it is a formula which assigns a score to each bid and then ranks the bids according to their score. These formulas are the crux of this research. I study formulas that come from our practice, Tenders Electronic Daily, which is the online version of the 'Supplement to the Official Journal of the European Union', dedicated to European public procurement, books, journals and websites. Most of the formulas are from the Netherlands, but I also include some formulas from other European countries. All formulas capture the best value for money by combining price and quality which have different magnitudes. In the setup of this research, quality is between 0 and 1 and price is a positive real number[2]. Therefore, most of the formulas provide a way to normalize the price so that the price score has a similar magnitude as the quality score.

[1]Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxemburg, Malta, the Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden and the United Kingdom.

[2]It may happen that the price is negative meaning that the bidder is willing to pay the tendering entity for the award of a contract. As an example, think of the demolition of a building. If the demolition waste, e.g. the copper used for electrical wiring can be recycled and is worth more than the actual cost of the demolition, then bidders may bid negative prices.

The most important issues for this research are:

- relative vs. absolute approach
- whether a formula provides protection against an extremely high price
- the shape of the indifference curve implied by the formula
- whether a formula reflects the preferences of the tendering entity regarding price and quality
- ranking paradox
- Pareto optimality

# 3 Relative vs. Absolute Approach

There are 2 main approaches to the evaluation of bids. One is the relative approach and the other is the absolute approach. In the relative approach, after all bid are submitted, each bid is evaluated using a formula that takes as input, for example, the lowest quality, the highest quality, the lowest price or the highest price. The absolute formula doesn't utilize the lowest quality, the highest quality, the lowest price or the highest price as input. In other words, the score calculated using an absolute formula depends on price and quality of a given bid. This is also why absolute formulas are sometimes called independent.

According to Albano, Dini and Zampino[2], applying an absolute formula leads to a lower submitted price on average. They compared average discounts for relative and absolute formulas and concluded that the average rebate was 21% lower when an absolute formula was applied. Rebate is the relation between reserve price and submitted price expressed as follows:

$$\frac{Reserve\ Price - Submitted\ Price}{Reserve\ Price},$$

where in competitive bidding, the reserve price is the maximum price accepted by the tendering entity. Another observation by Dini and Zampino was that when a relative formula was used, the submitted prices were more concentrated.

Some absolute formulas use, for example, reference price or price range. It must be noted that the resulting ranking of bids depends on these values. In other words, specifying different values of price range may yield different rankings. As for the price range, here is an example providing another reason for its great significance. Suppose that there are 2 buyers that both want to buy the same good. The 1st specifies the price range between 0 and 1000 Euros and the 2nd specifies the price range between 400 and 600 Euros. For the 1st buyer 10 Euros difference is 1% of their price range whereas for the 2nd buyer 10 Euros difference is 5% of their price range.

Absolute formulas entail the risk of a mismatch between reference price or price range and market prices, which may lead to the failure of the whole procurement process. This is because, in the end, works, goods or services are supplied by the market. Specifying the parameters of an absolute formula may sometimes be a difficult task. Although the market sets and is therefore aware of the set prices, and it is the job of the buyer to find these market prices, it may prove difficult regarding to works, goods or services that price is not widely available. Examples include real estate or new technology. If the tendering entity is unable to specify the parameters of an absolute formula in-house, it may need the help of external consultants which involves additional costs and makes the procurement process more expensive.

As regards price, relative formulas require that the bidder submits a price as close as possible to the expected lowest price. The more accurate this estimate is and the closer the submitted price is to this estimate, the more likely the bidder will achieve a high score on price. That is to say, the optimal strategy to win a tender is to submit the highest possible quality for the lowest price. This uncertainty about the lowest price may cause an overestimation of the lowest price and thereby an increase in the overall price level. On the other hand, the lowest price can be underestimated, causing the overall price level to decrease.

Another consequence of a relative formula is that by using the lowest price or the highest quality as a reference, the formula adjusts itself to the set of possible alternatives. This is a very intuitive property, and the following 2 scenarios will illustrate this point. Consider 2 scenarios. In the 1st scenario you compare prices in 2 fast food restaurants. In the 2nd scenario you compare prices in a fast food restaurant and a gourmet restaurant. Clearly, these 2 scenarios are different and anytime something is compared to something else, one should take into account what is being compared.

According to Telgen and Schotanus[22], relative formulas replace preferences of the tendering entity by a lottery, because the lowest price and the highest quality are determined by the market and not by the tendering entity. On the other hand, absolute formulas ensure that the value functions of the tendering entity are accurately represented, because the tendering entity can specify what it believes to be a good price or quality.

Finally, when an absolute formula is used, potential suppliers can either explicitly calculate their final score or only estimate it. The latter may be the case when the quality is evaluated by the tendering entity. Potential suppliers may not be able to exactly calculate the final score because they still face some uncertainty regarding the quality score, as it is given after the bid submission. When a relative formula is used, the estimate of the final score will be less precise as there is an additional uncertainty steaming from the fact that the final score depends on the highest quality or the highest price.

# 4 Ranking Paradox

A ranking paradox arises when after adding or removing one bid, rankings of individual bids are different than before. A price related ranking paradox will be defined as a ranking paradox caused by change of the highest or the lowest price of all submitted bids. A quality related ranking paradox will be defined as a ranking paradox caused by a change of the highest quality of all submitted bids.

An intrusive ranking paradox will be defined as a ranking paradox that involves change regarding the ranking of a bid that which initially ranked number 1. This occurs because a bid not ranked as number 1 entered or left the bid competition. Further explanation will be given below. Suppose there is some initial ranking of bids and another bid, that will not be ranked number 1 in the final ranking, enters the bid competition or a bid that is not ranked number 1 in the initial ranking is removed. If it happens that, for example, a bid ranked number 1 in the initial ranking becomes ranked number 2 in the final ranking and a bid ranked number 2 in the initial ranking becomes ranked number 1 in the final ranking, then it is an intrusive ranking paradox.

Non-intrusive ranking paradoxes are understandable. If number 1 pulls out, it may not always be the case that the number 2 bid becomes number 1, so their occurrence doesn't seem significant. Yet, what I refer to as an intrusive ranking paradox is less intuitive. Whether or not some other bid that will not be ranked as number 1 participates or not, makes a difference on which bid will be ranked number 1. This change is indeed significant.

Ranking paradox is only possible when a relative formula is used. The necessary condition for the ranking paradox is that the difference between the scores of 2 bidders depends on, for example, the lowest price or the highest quality. More formally, it means that the axiom of independence of irrelevant alternatives is violated. This axiom states that the preference between 2 alternatives should depend only on the individual preference between them. Relative formulas violate this condition and that makes the ranking paradox possible.

## 4.1 Ranking Paradox in Social Choice

Because ranking paradox is a well-known topic in the evaluation of bids based on price and quality, I will give some examples form other fields.

The Borda count is an election method named after Jean-Charles Borda. Under the Borda count, voters rank alternatives in order of preference. The least preferred alternative receives 0 points, the next-least preferred 1 point, and so on up to the most preferred alternative which receives m-1 points if there are m alternatives. Points for each alternative are summed across all voters and the alternative with the highest number of points wins.

A paradox in the Borda count may occur when the number of alternatives open to voters changes. For example, a paradox may arise in an election with at least 4 alternatives when 1 of them is removed from consideration after votes have been received, making recalculation necessary.

Consider the following example. Suppose there are 4 alternatives to choose from and 7 people are asked to rank these alternatives on their ballots. The most preferred alternative on each ballot will get 3 points; decreasing numbers of points will be given to alternatives ranked 2nd, 3rd and 4th. Here I use the notion of strict preference relation as defined in Appendix A.1. Suppose that there are 3 different preference rankings. 2 people choose $B \succ D \succ C \succ A$, 3 people choose $A \succ B \succ D \succ C$ and 2 people choose $D \succ C \succ A \succ B$. When the votes are tallied, alternative A gets $3 \times 3 + 2 \times 1 = 11$; alternative B gets $2 \times 3 + 3 \times 2 = 12$; alternative C gets $2 \times 1 + 2 \times 2 = 6$; and alternative D gets $2 \times 2 + 3 \times 1 + 2 \times 3 = 13$. Alternative D wins the election, followed by alternative B, A, and C in the last place. Now suppose that alternative C has been removed from consideration, which requires points to be recalculated, ignoring alternative C on the ballots. The top spot on each ballot now gets 2 points, while the 2nd and 3rd spots receive 1 and 0, respectively. Summing votes with the revised point system reveals that alternative A receives 8 points, alternative B receives 7 points and alternative D receives 6 points. Paradoxically, the winner has turned loser. No change in preference rankings accompanies this result. The only difference is the number of alternatives that are considered.

The Borda count is used to determine winners for Toastmasters International speech contests. Judges create a ranking of their top 3 speakers, awarding them 3 points, 2 points, and 1 point, respectively. All unranked candidates receive 0 points. The Eurovision Song Contest also uses a positional voting method similar to the Borda count, with a different distribution of points: Only the top 10 entries are considered in each ballot, the 1st entry receiving 12 points, the 2nd entry receiving 10 points, and the other 8 entries earning points from 8 to 1.

## 4.2 Ranking Paradox in Sport

Suppose 4 football teams are playing in a round-robin tournament. The scoring is 3 points for a win, 1 point for a draw and 0 points for a loss. 6 matches have been played. Team A beat team C and tied with teams B and D. That gave them 5 points. Team B defeated team D, tied with team A and lost to team C. That gave them 4 points. Team C defeated team B, tied with team D and lost to team A. That also gave them also 4 points. Team D tied with teams A and C and lost to team B. That gave them 2 points. The results are summarized in the table below.

| Team | Points |
|---|---|
| A | 5 |
| B | 4 |
| C | 4 |
| D | 2 |

Suppose that for some reason team C is disqualified. What should happen with the points that the other teams earned playing team C? If these points are considered valid, then the scores of teams A, B and D don't change. However, if these points are declared void, then the situation looks as in the table below.

| Team | Points |
|---|---|
| B | 4 |
| A | 2 |
| D | 1 |

As one can see, the score of team B is still 4 points, but the score of team A dropped down to 2 points. So now it turns out that team B is better than team A. Clearly, it is a ranking paradox.

Again, we see that the occurrence of a ranking paradox is not irrational. Competing football teams have, after all, parallels with competing bidders. In both instances, all competitors want to win and only 1 can win. In the procurement context, applying an absolute formula would be to determine the winner beforehand by measuring all teams against predetermined statistics[3]. On the other hand, applying a relative formula would be to let the teams compete against each other and determining the winner based on the results of this competition. Last year, UBS Wealth Management Research built a statistical model based on quantitative data including an objective

[3] In the procurement context these are, for example, reference price or price range.

quantitative measure assessing the strength of each team 3 months before the start of the World Cup. According to their model, Brazil had the best chance of becoming the world champion. This would be the outcome of the absolute approach in the procurement context and the relative approach would be to let the teams compete against each other. In the final, Spain beat the Netherlands and became the world champion. The issue is whether the winner should be determined by measuring all teams against statistics, or if the winner should be determined by tallying wins, losses and ties.

# 5 Indifference Curve and the Marginal Rate of Substitution

Using the properties of the weak preference relation introduced in Appendix A.1 and a few more technical assumptions, we can define what is known as an indifference curve. In theory of consumer behaviour, an indifference curve represents all combinations of goods that provide the same level of satisfaction to a consumer. Thus, a consumer is indifferent among combinations of goods represented by the points on the same curve.

In the context of this research I will consider combinations of price and quality of different bids. Furthermore, I assume that the indifference curve consists of bids that have the same score based on their price and quality that was calculated with some formula used in the procurement process. The price of each bid is measured on the horizontal axis and quality on the vertical axis. I also assume that economic agents prefer more quality for a lower price. This assumption corresponds to the basic assumption about consumer behaviour that consumers prefer more of any goods over less. It means that, for example, 20 units of food and 30 units of clothing is preferred over 10 units of food and 20 units of clothing because it is more food and more clothing.

Indifference curves are used to describe trade-offs that consumers face when choosing among 2 or more goods. For example, a consumer may be willing to give up 6 units of clothing to obtain 1 extra unit of food. There is a special measure to qualify the amount of one good that a consumer is willing to give up in order to obtain more of another good: It is called the marginal rate of substitution. The marginal rate of substitution of food for clothing is defined as the maximum amount of clothing that a consumer is willing to give up to obtain 1 additional unit of food. The marginal rate of substitution diminishes along the indifference curve. The more clothing and the less food a consumer has, the more clothing they will give up in order to obtain more food. Similarly, the more food a consumer has, the less clothing they will give up for more food. For example, if a consumer had 16 units of clothing and 1 unit of food, they would be willing to give up 6 units of clothing to obtain 1 extra unit of food. However, if they had 10 units of clothing and 2 units of food, they would be willing to give up only 4 units of clothing to obtain 2 extra units of food. In the setup of this research, the marginal rate of substitution of quality for price will be defined as the maximum price that an economic agent is willing to pay to obtain 1 additional unit of quality. The marginal rate of substitution either increases, decreases or remains constant along the indifference curve.

# 6 Formulas Used in the Procurement Process

In this section I will study formulas used in the procurement process. All formulas studied here satisfy 2 basic requirements. First, if bids are ranked in a descending order by their score, then the score diminishes as the price increases. Mathematically, if the formula were differentiable, then the 1st derivative of the formula with respect to the individual price would be negative. Likewise, if bids are ranked in an ascending order by their score, then the score increases as the price increases. Mathematically, if the formula were differentiable, then the 1st derivative of the formula with respect to the individual price would be positive. Second, if bids are ranked in a descending order by their score, then the score increases as the quality increases. Mathematically, if the formula were differentiable, then the 1st derivative of the formula with respect to the individual quality would be positive. Likewise, if bids are ranked in an ascending order by their score, then the score diminishes as the quality increases. Mathematically, if the formula were differentiable, then the 1st derivative of the formula with respect to the individual quality would be negative.

### Shape of the indifference curves

I have designed 6 experiments to compare the formulas. The aim of the 1st experiment is to examine the shape of indifference curves which have been introduced in Chapter 5. This experiment has been designed in such a way that it is possible to compare the indifference curves for different sets of weights of price and quality on possibly the widest quality interval. For each formula the indifference curves were constructed as follows. I defined the lower and upper ends of the quality interval which were usually 20% and 80%. Then, I picked a bid represented by a price of 10 Euros and a quality of 20% and I calculated the score of such a bid. Next, I found prices for each quality from the quality interval such that the score was equal to the score of the bid with a price of 10 Euros and a quality of 20%. Finally, I drew a line going through the points of the same score. This was done for 3 sets of weights of price and quality: 50%, 50%; 60%, 40% and 40%, 60%. If the indifference curve is a straight line, then the marginal rate of substitution of quality for price is constant and so every unit of quality is worth the same amount of money. If the indifference curve is concave, then the marginal rate of substitution of quality for price is increasing which means that consecutive units of quality are more and more expensive. If the indifference curve is convex, then the marginal rate of substitution of quality for price is decreasing which means that consecutive units of quality are less and less expensive. The results of this experiment are summarized in the table on page 91.

### Undesirable strategic aspect

The next experiment is designed to check whether a formula provides protection against an extremely high price. In certain situations, for example, when the weight of quality is very high and one bidder knows that they can offer a very high quality that gives them a sufficient advantage over other bidders, the ranking of a bid may become independent of its price. What is meant here is that a bidder will win the tender regardless how high their price is. For example, let the maximum quality score be 90 points and the maximum price score be 10 points out of 100 points. If one bidder knows that they can score on quality more than 80 points and that the quality score of the other bidders won't exceed 70 points, then they can charge anything they want and they will still be ranked number 1. In other words, a formula that doesn't provide protection against an extremely high price is one for which under certain circumstances ranking of a bid ranked number 1 doesn't depend on its price. That is to say, for increasing price, the price score stays the same. The results are presented in the table on page 94.

### Mistake of overvaluing the weight of price and undervaluing the weight of quality

The main idea behind the 3rd experiment is explained in the following example. Suppose the weight of quality is 99% and the weight of price is 1%. Suppose further that 4 bids have been submitted:

| | A | B | C | D |
|---|---|---|---|---|
| Price | 4 | 8 | 18 | 38 |
| Quality | 20% | 30% | 50% | 90% |

One may argue whether a good formula should rank bid B, C or D as number 1, but common sense would suggest that it is wrong when a formula ranks bid A as number 1 since the weight of quality is so much higher than the weight of price. The outcome of a tender should meet the preferences of the tendering entity regarding price and quality. For example, if it wants high quality, it should set a high weight of quality and a good formula should reflect this preference. Conversely, a high weight of quality means that the tendering entity wants high quality. Setting high weight of quality means also that the tendering entity is willing to pay for it. In the extreme case, if the weight of quality is 100%, then the tendering entity accepts to pay any price. Hence, if the weight of quality is high and any formula chooses the cheapest bid of not necessarily high quality, then one must consider this as its failure. This experiment was done using real data from 360 tenders. For each tender I set the weight of price to 99% and I counted the number of times a formula ranked as number 1 the cheapest bid with quality lower than the highest submitted quality in a given tender. The results of this experiment depend on the number of bids that have been submitted. For example, if there have been only 2 bids submitted, the choice is between the lowest priced and the highest priced bid. Therefore, I consider cases when at least 2 bids have been submitted, at least 3 bids have been submitted and at least 4 bids have been submitted. To conclude, this experiment is about finding formulas that choose the cheapest bid too often, regardless of the weights of price and quality. The results are displayed in the table on page 92.

### Likelihood of an intrusive and non-intrusive ranking paradox

The 4th experiment aims to give some idea about the likelihood of a ranking paradox. First, I applied every formula to 360 tenders in order to generate 360 initial rankings. Then for each tender I removed one bid at once and I compared the initial ranking with the final ranking. If the bid ranked as number 1 was removed from the initial ranking, then I compared the bid ranked as number 1 in the final ranking with the bid ranked as number 2 in the initial ranking. If these were 2 different bids, it meant that a non-intrusive ranking paradox occurred. If a bid ranked not as number 1 was removed from the initial ranking, then I compared the bid ranked as number 1 in the final ranking with the bid ranked as number 1 in the initial ranking. If these were 2 different bids, it meant that an intrusive ranking paradox occurred. The results of this experiment are presented in the table on page 93.

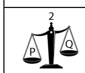
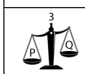
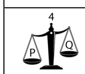
### Pareto optimality

This experiment is based on the concept of the Pareto optimality. As in the previous 2 experiments, it was also done on real data. An economic outcome is said to be Pareto optimal if it is impossible to make an economic agent better off without making some other economic agent worse off. In the setup of this research, if a formula ranks any bid as number 1, but there was another bid that was both better quality and cheaper, then such formula can't be considered as Pareto optimal. The reason is that the tendering entity would be better off if a formula ranked as number 1 a bid that was both cheaper and better quality. The results of this experiment are summarized in the table on page 94.

### Zero score and symmetry of the score

In the last experiment, I check 2 properties. The 1st property, called the zero score property, is satisfied if a bidder who submits the bid with a price of 0 Euros and a quality of 0% receives a score of 0. The 2nd property, called the symmetry of the score, is satisfied if 2 bidders submit the same bids and they both get the same score. The results of this experiment are summarized in the table on page 95.

In this chapter I will use the following pictograms to describe specific features of the formulas.

| | |
|---|---|
| A | An absolute formula. |
| R | A relative formula. |
|  | If the indifference curve is concave. |
|  | If the indifference curve is convex. |
| / | If the indifference curve is a straight line. |
| ∞ | If a formula provides protection against an extremely high price. |
|  | Mistake of overvaluing the weight of price and undervaluing the weight of quality when at least 2 bids have been submitted. |
|  | Mistake of overvaluing the weight of price and undervaluing the weight of quality when at least 3 bids have been submitted. |
|  | Mistake of overvaluing the weight of price and undervaluing the weight of quality when at least 4 bids have been submitted. |
| ! | Likelihood of a non-intrusive ranking paradox. |
| !! | Likelihood of an intrusive ranking paradox. |
|  | Pareto optimality. |

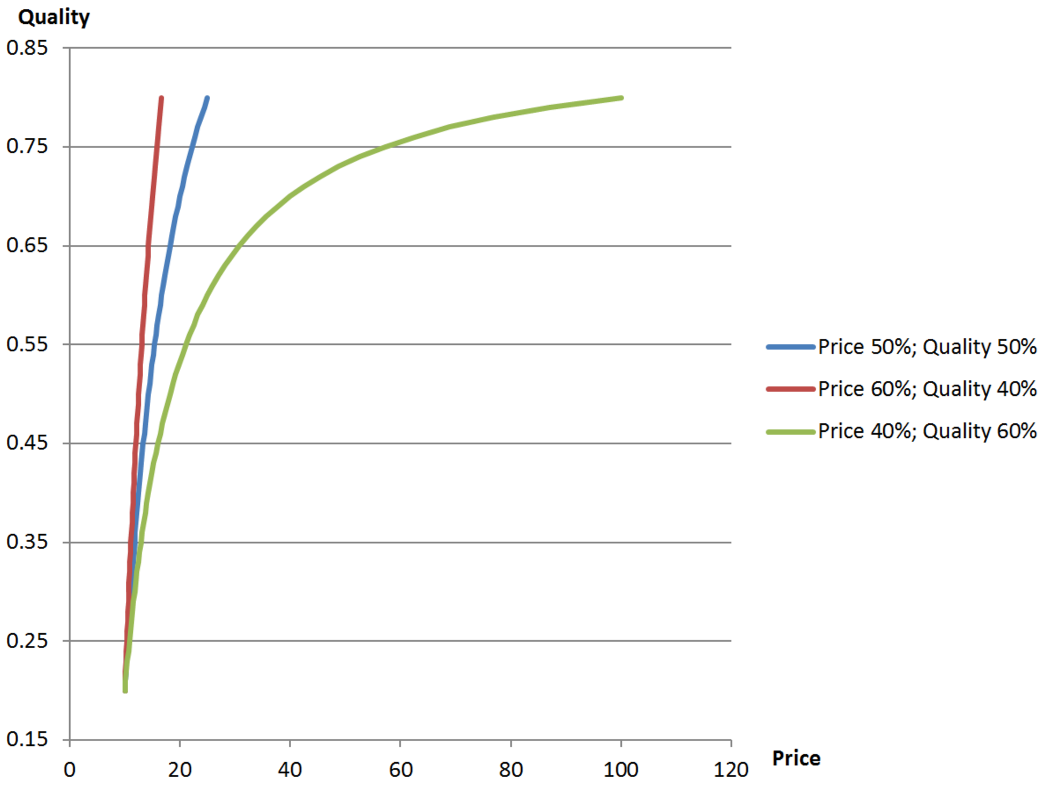
## 6.1 Lowest Bid Scoring

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| R |  | |  |  |  |  |  |  |
| | | | 0.00% | 0.00% | 0.00% | 0.00% | 0.08% | 0.00% |

This formula comes from[9] and has been modified to include the quality score as well. The bid ranked number 1 is the one with the highest score. Lowest bid scoring is derived through the following equation:

$$Score = W_{Quality} \cdot Q_i + W_{Price} \cdot \frac{P_{Best}}{P_i}.$$

It is a relative formula since the score of each bid depends on the lowest price of all submitted bids. It encourages bidding low prices because bidders, by bidding low, can increase the likelihood of getting the highest price score and reducing the price score obtained by other bids. This formula doesn't provide protection against an extremely high price. Let me illustrate this idea with an example. Suppose that the tendering entity wants a very high quality and therefore sets the weight of quality to 90%. That is to say that the maximum quality score is 90 points and the maximum price score is 10 points out of 100 points. If one bidder knows that they can offer a very high quality that gives them more than 80 points and they have a sufficient advantage over other bidders, meaning their score on quality won't exceed 70 points, then they can charge whatever amount they see fit and will still be ranked number 1. As is proven directly above, this formula doesn't provide protection against an extremely high price. Finally, it is impossible to calculate the score of a bid with price equal to 0 since it would involve division by 0, which is undefined.




As one can see, the shape of the indifference curves is not a straight line. The indifference curves are concave, which is very pronounced when the weight of price is 40% and the weight of quality is 60%. The marginal rate of substitution of quality for price is positive and increasing which means that consecutive units of quality are more and more expensive. Another observation is that as the weight of quality increases, the indifference curve becomes more concave.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 360, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox didn't occur and an intrusive ranking paradox occurred 1 time out of 1221, which is 0.08%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

Lowest bid scoring is relative and therefore a price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. The necessary condition for a ranking paradox to occur is the change in the lowest price of all submitted bids.

## 6.2 Highest Bid-Lowest Bid Scoring

| R | | | 2 P Q | 3 P Q | 4 P Q | ! | !! | |
|---|---|---|---|---|---|---|---|---|
| | | | 1.76% | 1.81% | 0.57% | 1.10% | 0.91% | 0.00% |

This formula also comes from[9] and has been also modified to include the quality score. The bid ranked number 1 is the one with the highest score. Highest bid-lowest bid scoring is derived through the following equation:

$$Score = W_{Quality} \cdot Q_i + W_{Price} \cdot \left( \frac{P_{Worst} - P_i}{P_{Worst} - P_{Best}} \right).$$

This is another example of a relative formula since the score of each bid depends on both the lowest and the highest price of all submitted bids. Highest bid-lowest bid scoring doesn't provide protection against an extremely high price. This can be illustrated with the same example as in Section 6.1. To summarize the example, if the weight of quality is very high, a bidder who can offer a very high quality while having a sufficient quality advantage over other bidders can charge anything they deem appropriate and still be ranked number 1. Finally, it is impossible to calculate the scores if all submitted prices are the same since it would involve division by 0, which is undefined.

As for the indifference curves, it is impossible to plot them according to the method used in this paper. Note that the lowest priced bid receives a score equal to its quality score plus $W_{Price}$ and the highest priced bid gets the final score equal to its quality score. Thus, if the weight of price is 60% and the weight of quality is 40%, it is impossible to find such values of quality, which by definition are between 0 and 1, that the score of the lowest priced bid will be equal to the score of the highest priced bid.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 6 out of 341 times, which is 1.76%. When at least 3 bids have been submitted, it was 5 out of 277 times, which is 1.81% and when at least 4 bids have been submitted, it was 1 time out of 175, which is 0.57%. In the 4th experiment, a non-intrusive ranking paradox occurred 3 times out of 273 times, which is 1.10% and an intrusive ranking paradox occurred 11 out of 1208 times, which is 0.91%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $W_{Price}$[4].

Highest bid-lowest bid scoring is relative and therefore a price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the lowest or the highest price of all submitted bids change. The dependence on both the lowest and the highest price makes this formula more vulnerable to ranking paradoxes than a formula that depends only on the lowest price. It is true, because the ranking paradox can be caused by the change in the lowest price or the highest price of all submitted bids.

[4] Assuming that $P_{Best} = 0$.

## 6.3 Average Scoring

| R | | |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| | | | 1.17% | 1.08% | 0.00% | 8.79% | 7.53% | 0.00% |

This is the 3rd formula from[9] that has been modified to include the quality score as well. The bid ranked number 1 is the one with the highest score. Average scoring is calculated by utilizing the following equation:

$$Score = \begin{cases} W_{Quality} \cdot Q_i + W_{Price}, & \text{if } P_i < P_{Avg} \\ W_{Quality} \cdot Q_i + W_{Price} \cdot \left(\frac{P_{Worst} - P_i}{P_{Worst} - P_{Avg}}\right), & \text{otherwise.} \end{cases}$$

As one can see, this is a relative formula. The score of each bid depends not only on the highest price, but also the average price of all prices that have been submitted. Another feature of the average scoring is that all prices below the average price obtain the maximum price score. Particularly low prices are not rewarded in terms of a price score since such bids get the same price score as a bid, which is just below the average price. Moreover, if one of the submitted prices is much larger than all other submitted prices, it can have very serious effect on the final ranking. I will illustrate this with an example. Suppose that one of the submitted prices is 10000 Euros, the other is 1000 Euros and there are also 8 prices between 10 and 20 Euros. Clearly, prices between 10 and 1000 Euros are below the average price, which is above 1100 Euros. Hence, these bids receive the maximum price score irrespective of their price. If the bid with price 1000 Euros is of a slightly better quality than that of the bids with prices between 10 and 20 Euros, it will be ranked number 1. Average scoring doesn't provide protection against an extremely high price and this can be shown with an example introduced in Section 6.1. In short, if the weight of quality is very high, a bidder who can offer a very high quality while having a sufficient quality advantage over other bidders can charge whatever amount they wish and still be ranked number 1. Finally, it is impossible to calculate the scores if all submitted prices are the same since it would involve division by 0, which is undefined.

As for the indifference curves, assuming that the lowest priced bid is below the average price and the highest priced bid is above it, then it is impossible to plot them according to the method used in this paper. Note that the lowest priced bid receives a score equal to its quality score plus $W_{Price}$ and the highest priced bid gets the final score equal to its quality score. Thus, if the weight of price is 60% and the weight of quality is 40%, it is impossible to find such values of quality, which by definition are between 0 and 1, that the score of the lowest priced bid will be equal to the score of the highest priced bid.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 4 out of 341 times, which is 1.17%. When at least 3 bids have been submitted, it was 3 out of 277 times, which is 1.08% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox occurred 24 out of 273 times, which is 8.79% and an intrusive ranking paradox occurred 91 out of 1208 times, which is 7.53%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $W_{Price}$[5].

Average scoring is relative and therefore a price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the average price changes. Also, the dependence on the average price makes this formula much more vulnerable to ranking paradoxes than a formula that depends only on the lowest price or the lowest and the highest price. The reason is that the average price is calculated as the sum of all submitted prices over the number of submitted bids.

[5] Assuming that $0 < P_{Avg}$.

## 6.4 Based on Bid Spread

| R | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 1.76% | 1.81% | 0.57% | 1.10% | 0.91% | 0.00% |

The formula based on bid spread comes from[24] and has, as the previous formulas, been modified to include the quality score. The bid ranked number 1 is the one with the lowest score. The formula based on bid spread is:

$$Score = W_{Price} \cdot \left( \frac{P_i - P_{Worst}}{P_{Worst} - P_{Best}} \right) - W_{Quality} \cdot Q_i.$$

This is another example of a relative formula. The score of each bid depends on the lowest and the highest price of all submitted bids. The highest price bidder plays a key role in this formula. Denominator of the price score determines the price range, and in the numerator of the price score each bid is measured against the highest price of all submitted bids. Using the example of Section 6.1, it can be shown that the formula based on bid spread doesn't provide protection against an extremely high price. In brief, if the weight of quality is very high, a bidder who can offer a very high quality while having a sufficient quality advantage over other bidders can charge whatever they wish and still be ranked number 1. Finally, it is impossible to calculate the scores if all submitted prices are the same since it would involve division by 0, which is undefined.

As for the indifference curves, it is impossible to plot them according to the method used in this paper for the same reason as in Section 6.2.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 6 out of 341 times, which is 1.76%. When at least 3 bids have been submitted, it was 5 out of 277 times, which is 1.81% and when at least 4 bids have been submitted, it was 1 time out of 175, which is 0.57%. In the 4th experiment, a non-intrusive ranking paradox occurred 3 out of 273 times, which is 1.10% and an intrusive ranking paradox occurred 11 out of 1208 times, which is 0.91%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $-W_{Price}$[6].

The formula based on bid spread depends on the lowest and the highest price. Therefore price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the lowest price or the highest price of all submitted bids change. The dependence on both the lowest and the highest price makes it more vulnerable to ranking paradoxes than a formula that depends only on the lowest price.

[6] Assuming that $P_{Best} = 0$.

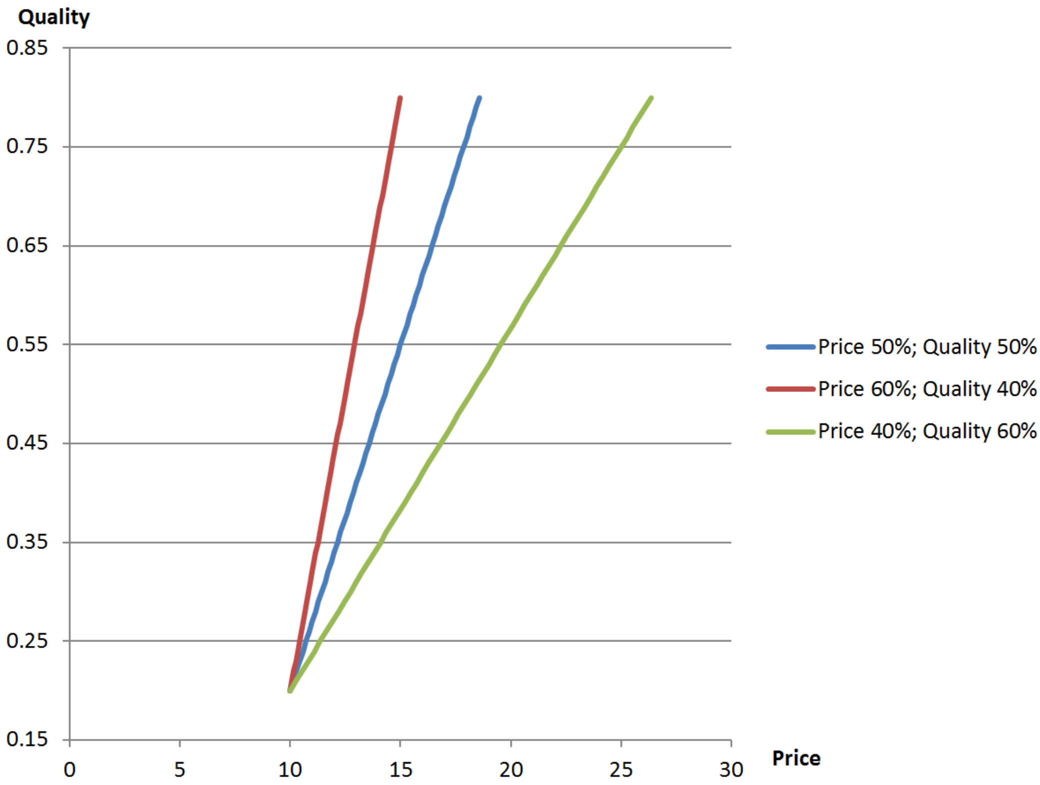
## 6.5 Based on Average Bid

| R | / | ∞ |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| | | | 0.00% | 0.00% | 0.00% | 0.00% | 0.57% | 0.00% |

The formula based on average bid also comes from[24], and has been also modified to include the quality score. The bid ranked number 1 is the one with the lowest score. The formula based on average bid is:

$$Score = W_{Price} \cdot \left(\frac{P_i}{P_{Avg}}\right) - W_{Quality} \cdot Q_i.$$

This is yet another example of a relative formula. The score of each bid depends on the average price of all submitted bids and thus on all prices that have been submitted. It provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. Finally, it is impossible to calculate the scores if all submitted prices are 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 360, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox didn't occur and an intrusive ranking paradox occurred 7 out of 1221 times, which is 0.57%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies both the symmetry of the score and the zero score property.

The formula based on average bid depends on the average price and therefore a price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the average price changes. The dependence on the average price makes this formula extremely vulnerable to ranking paradoxes because the average price is calculated as the sum of all submitted prices over the number of submitted bids.

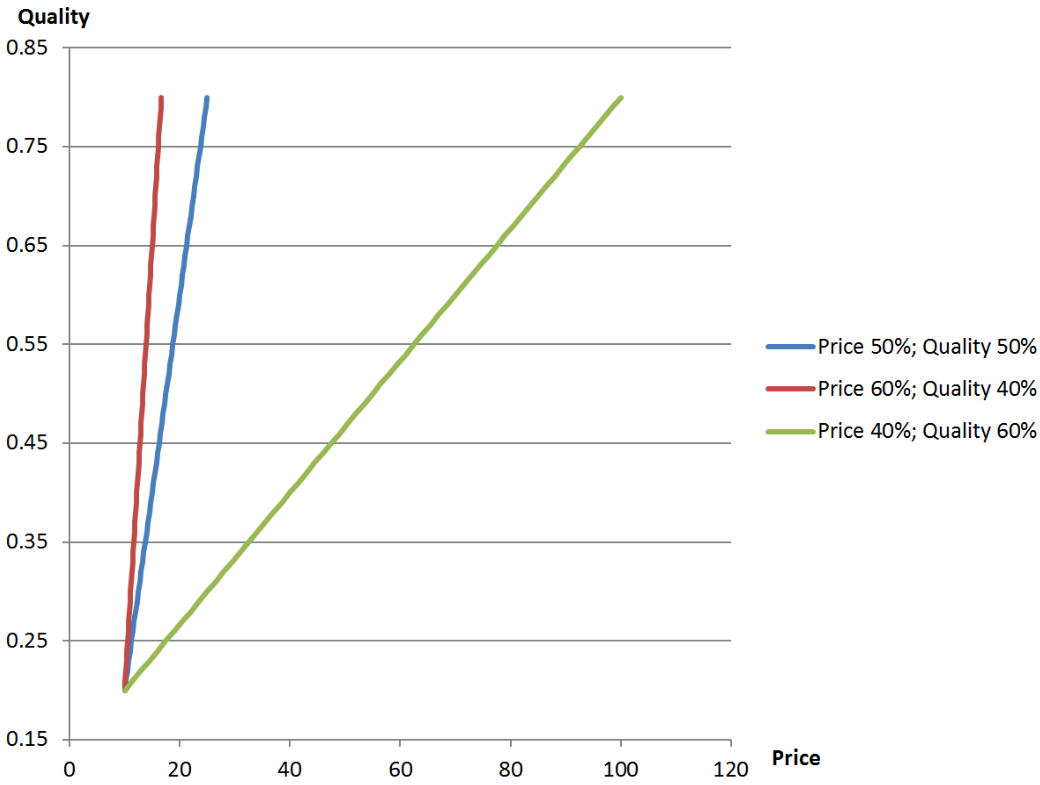
## 6.6 Maximum Price Deviation Model

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| R | / | |  |  |  |  |  |  |
| | | | 0.00% | 0.00% | 0.00% | 0.00% | 0.08% | 0.00% |

This is the 3rd formula from [24] which has been modified to include the quality score. The bid ranked number 1 is the one with the highest score. It is calculated according to the following formula:

$$Score = W_{Price} \cdot \left(1 - \frac{P_i}{P_{Worst}}\right) + W_{Quality} \cdot Q_i.$$

This is a relative formula since it is based on the highest price of all submitted bids. It doesn't provide protection against an extremely high price. Note also that the score of the most expensive bid is equal to its quality score because each bid is compared against the highest price of all submitted bids. In the situation described in the example in Section 6.1, a bidder who can offer a very high quality while having a sufficient quality advantage over other bidders can charge whatever they wish and still be ranked number 1. Finally, it is impossible to calculate the highest price of all submitted bids is 0 since it would involve division by 0, which is undefined.




To plot the indifference curves, the highest price has been set to 25 Euros for the weights of price and quality equal to 50%, 16.67 Euros for the weights of price and quality equal to 60% and 40% and 100 Euros for the weights of price and quality equal to 40% and 60%. The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 360, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox didn't occur and an intrusive ranking paradox occurred 1 time out of 1221, which is 0.08%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies both the symmetry of the score and the zero score property.

The maximum price deviation model is based on the highest price and therefore a price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the highest price changes.

## 6.7 Negometrix

| R | / | $\infty$ | 2 | 3 | 4 | ! | !! | |
|---|---|---|---|---|---|---|---|---|
| | | | 0.00% | 0.00% | 0.00% | 2.13% | 0.00% | 0.00% |

This is a formula used by Negometrix. It can be summarized in 3 steps:

Step 1
Calculate Negometrix Utility Index given by:

$$U_i = \frac{\left[1 - (Q_{Best} - Q_i) \cdot \frac{W_{Quality}}{W_{Price}}\right] \cdot P_{Best}}{P_i}, \text{ where } W_{Price} \neq 0.$$

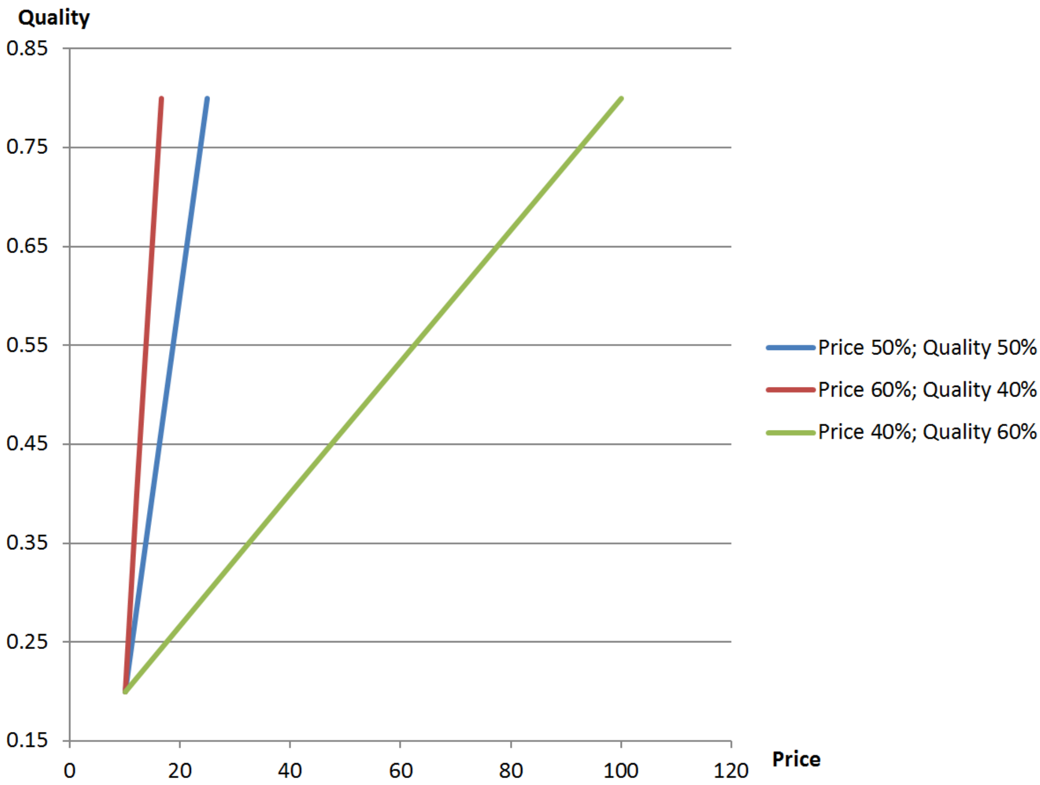
Step 2
Calculate Best Buy which gives price at which the each bid would give the same Utility as the best bid:

$$BB_i = \frac{U_i}{\max\left(U_1, \cdots, U_N\right)} \cdot P_i.$$

Step 3
Calculate the score expressed as a price discrepancy:

$$Score = P_i - BB_i = P_i \cdot \left(\frac{\max\left(U_1, \cdots, U_N\right) - U_i}{\max\left(U_1, \cdots, U_N\right)}\right).$$

It is a relative formula since it is based on the highest quality and the lowest price of all submitted bids. The economic interpretation of the Best Buy is that it gives a quality-adjusted cost of a given bid. An interesting feature of this formula is that it ensures a very precise relation between price and quality between any 2 bids. A simple proof of this statement is given in Appendix A.2. As an example, imagine a tender where the weight of the price is 50% and the weight of the quality is also 50%. In such a tender, 1% price difference is equal to 1% quality difference. If the ratio is 40% quality and 60% price, then 1% price can be traded for 1.5% quality because price is 1.5 times more important than quality. This formula also provides protection against an extremely high price. Finally, it is impossible to calculate the score of a bid with a price equal to 0 since it would involve division by 0, which is undefined.




To plot the indifference curves, the highest quality has been set to 80%. The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 360, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox occurred 6 out of 282 times, which is 2.13% and intrusive ranking paradox didn't occur. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

It is a relative formula and therefore price and quality related ranking paradoxes are possible, including also the possibility of quality related intrusive ranking paradoxes. Although it is a relative formula, a price related intrusive ranking paradox is impossible. More formal proof of this statement is to be found in Appendix A.3.

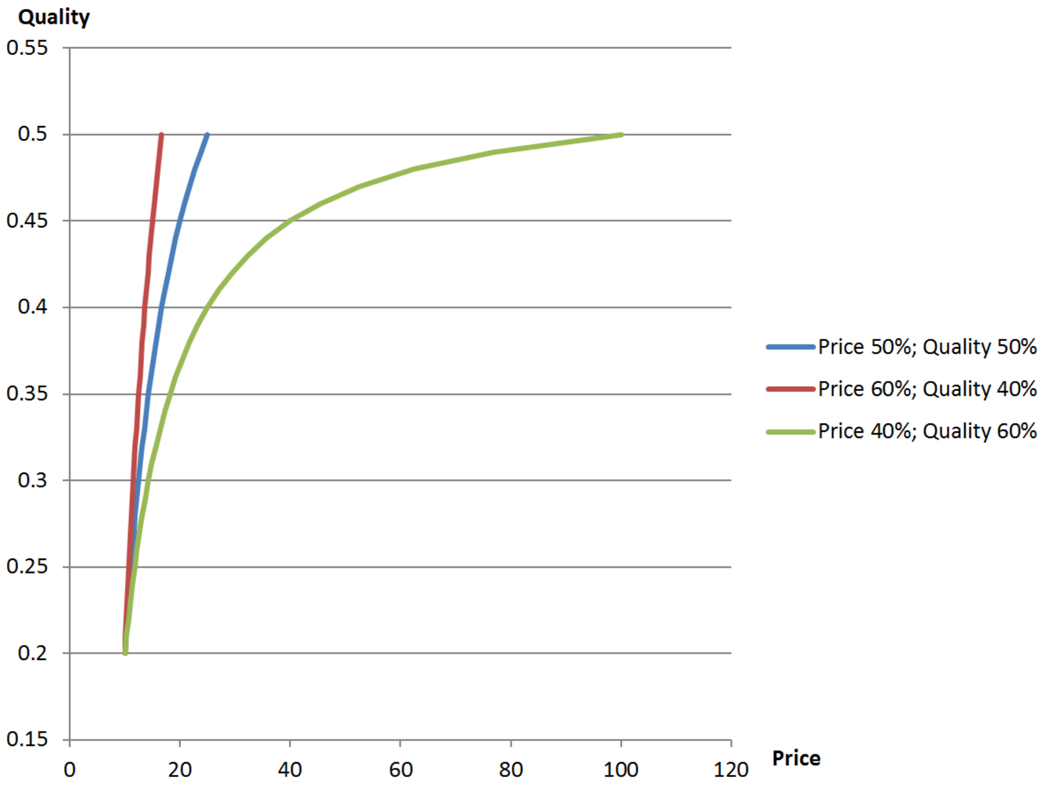
## 6.8 Coventry City Council

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| R | | | | | | | | |
| | | | 0.00% | 0.00% | 0.00% | 0.71% | 0.00% | 0.00% |

It is a formula used by the Coventry City Council. The bid ranked number 1 is the one with the highest score. This formula is given by:

$$Score = W_{Quality} \cdot \frac{Q_i}{Q_{Best}} + W_{Price} \cdot \frac{P_{Best}}{P_i}.$$

This is a relative formula since it depends on the highest quality and the lowest price of all submitted bids. Bidders, by bidding low prices, can increase the likelihood of getting the highest price score and reduce the price score obtained by other bids. On the other hand, bidders by offering high quality, can increase the likelihood of getting the highest quality score and reduce the quality score obtained by other bids. Another issue is that this formula doesn't provide protection against an extremely high price. As one can see, the price of each bid, $P_i$ is in the denominator. Thus, $\frac{P_{Best}}{P_i}$ can become negligible if $P_i$ is much higher than $P_{Best}$. As in the example in Section 6.1, if the weight of quality is very high, a bidder who can offer a very high quality while having a sufficient quality advantage over other bidders can charge whatever they wish and still be ranked number 1. It is impossible to calculate the score of a bid with a price equal to 0 since it would involve division by 0, which is undefined. For the same reason it is impossible to calculate the scores if the highest quality is 0.




The indifference curves on the above graph are concave, they are bowed out. The marginal rate of substitution of quality for price is positive and increasing which means that consecutive units of quality are more and more expensive. Another observation is that as the weight of quality increases, the indifference curve becomes more concave.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 358, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 280, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 173, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox occurred 2 out of 280 times, which is 0.71% and an intrusive ranking paradox didn't occur. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

Since this is a relative formula, price and quality related ranking paradoxes are possible, including also possibilities of intrusive ranking paradoxes. The necessary condition for a ranking paradox to occur is the change in the lowest price or the highest quality of all submitted bids.

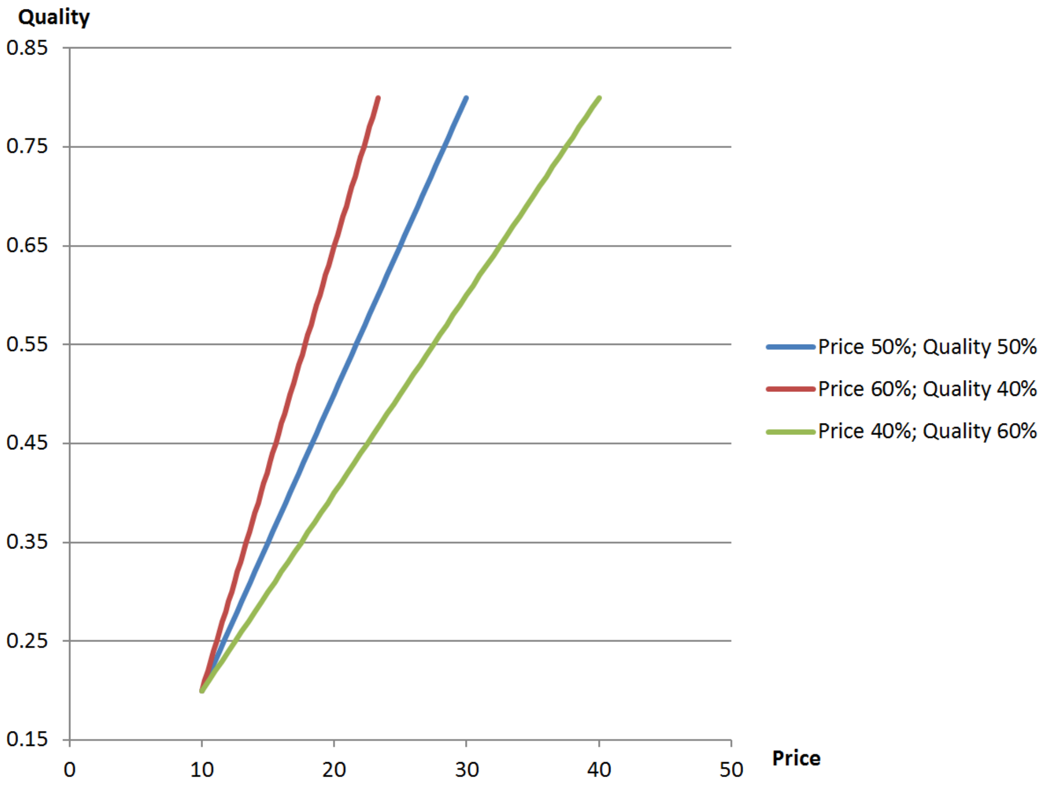
## 6.9 European Organization for Nuclear Research (CERN)

| R | / | ∞ | 2  | 3  | 4  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| | | | 0.84% | 0.71% | 1.16% | 0.71% | 0.00% | 0.00% |

The following is a formula used by the European Organization for Nuclear Research (CERN). The bid ranked number 1 is the one with the highest score. This formula calculates as follows:

$$Score = W_{Quality} \cdot \frac{Q_i}{Q_{Best}} - 0.5 \cdot W_{Price} \cdot \left( \frac{P_i}{P_{Best}} - 1 \right).$$

It is a relative formula since it is based on the highest quality and the lowest price of all submitted bids. Bidders, by bidding low, can increase the likelihood of getting the highest price score and reduce the price score obtained by other bids. On the other hand, a bidding price higher than the lowest price of all submitted bids results in the price score being subtracted from the quality score. Also, bidders providing a high quality can increase the likelihood of getting the highest quality score and reduce the quality score obtained by other bids. This formula provides protection against an extremely high price. This is because the price of each bid is compared against the lowest price, and if it is higher than the lowest price of all submitted bids, the price score is subtracted from the quality score. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they cannot charge whatever amount they see fit and still retain the number 1 ranking. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. Finally, it is impossible to calculate the scores if the lowest price is 0 or the highest quality is 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 3 out of 358 times, which is 0.84%. When at least 3 bids have been submitted, it was 2 out of 280 times, which is 0.71% and when at least 4 bids have been submitted, it was 2 out of 173 times, which is 1.16%. In the 4th experiment, a non-intrusive ranking paradox occurred 2 out of 280 times, which is 0.71% and an intrusive ranking paradox didn't occur. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it

would involve division by 0, which is undefined.

Since this is a relative formula, price and quality related ranking paradoxes are possible, including also possibilities of intrusive ranking paradoxes. The necessary condition for a ranking paradox to occur is the change in the lowest price or the highest quality of all submitted bids.

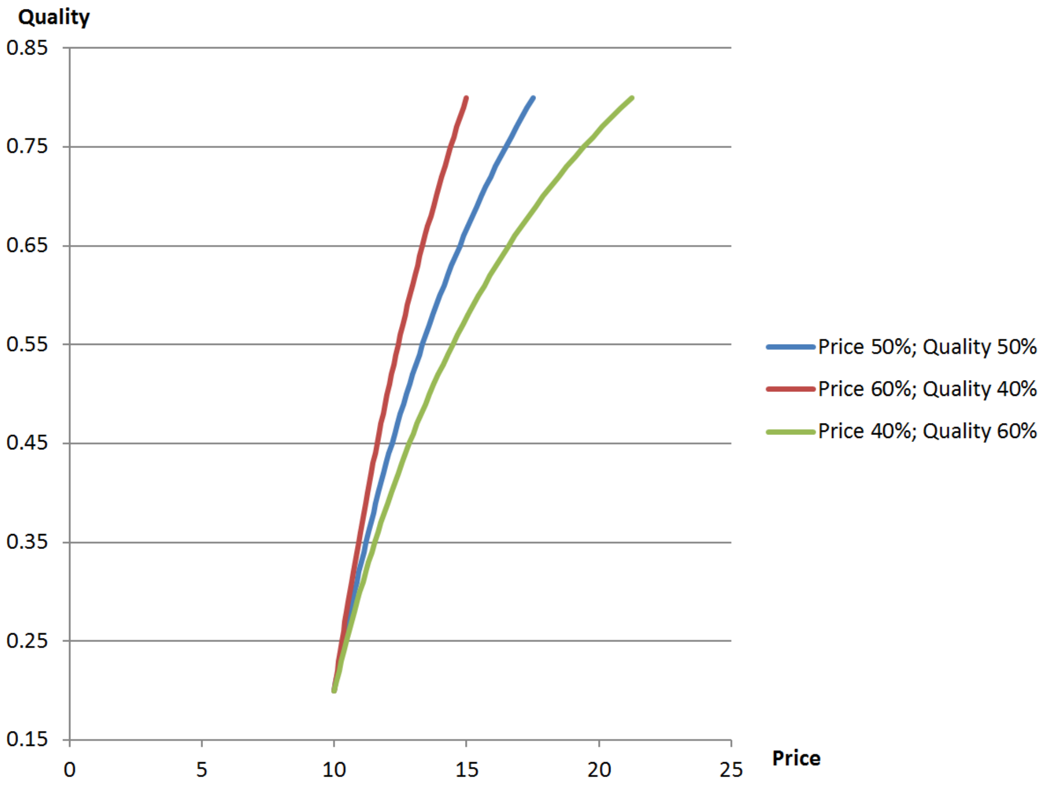
## 6.10 Tennet

| R | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 1.12% | 1.07% | 1.16% | 1.07% | 0.00% | 0.00% |

The following is a formula used by Tennet. The bid ranked number 1 is the one with the lowest score. This formula is:

$$Score = P_i + \left(1 - \frac{Q_i}{Q_{Best}}\right) \cdot \frac{P_i \cdot W_{Quality}}{W_{Price}}.$$

It is a relative formula since it depends on the highest quality of all submitted bids. The score of each bid is equal to its price corrected for quality deficiency. How much is added to the initial price depends on quality deficiency with respect to the highest quality of all submitted bids, the initial price and weights of price and quality. The score of a bid with the highest quality is equal to its price. This formula provides protection against an extremely high price if the weight of price is other than 0. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they can't charge whatever amount they see fit and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. If the weight of price is 0 or the highest quality is 0, it is impossible to calculate the scores because it would involve division by 0, which is undefined. If the weight of price is 0, there should be an additional step describing what to do, for example, rank bids by quality in a descending order.




The indifference curves on the above graph are concave, they are bowed out. The marginal rate of substitution of quality for price is positive and increasing which means that consecutive units of quality are more and more expensive. Another observation is that as the weight of quality increases, the indifference curve becomes more concave.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 4 out of 358 times, which is 1.12%. When at least 3 bids have been submitted, it was 3 out of 280 times, which is 1.07% and when at least 4 bids have been submitted, it was 2 out of 173 times, which is 1.16%. In the 4th experiment, a non-intrusive ranking paradox occurred 3 out of 280 times, which is 1.07% and an intrusive ranking paradox didn't occur. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies both the symmetry of the score and the zero score property.

As I have already mentioned, the formula presented in this section is relative and therefore a quality related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the highest quality of all submitted bids changes.

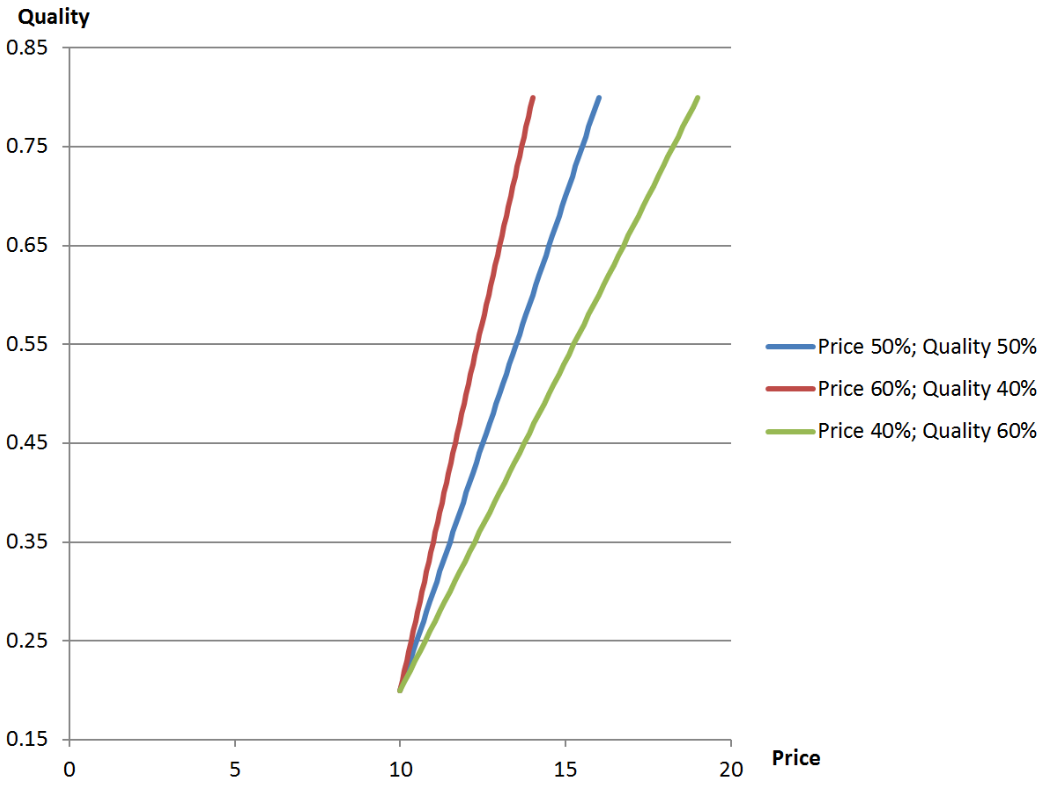
## 6.11 Mercer

| R | / | |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| | | | 0.00% | 0.00% | 0.00% | 1.06% | 0.00% | 0.00% |

The following is a formula used by Mercer. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = \begin{cases} Q_i \cdot W_{Quality} + \left(1 - \frac{P_i - P_{Best}}{P_{Best}}\right) \cdot W_{Price}, & \text{if } \frac{P_i - P_{Best}}{P_{Best}} \leq 1 \\ Q_i \cdot W_{Quality}, & \text{otherwise.} \end{cases}$$

It is a relative formula since it is based on the lowest price of all submitted bids. A bidding price more than twice the lowest price of all submitted bids makes the price score equal to 0. Unfortunately, this formula doesn't provide protection against an extremely high price. To illustrate, suppose that the weight of quality is very high and there is one bid with an extremely high price and very high quality. According to this formula, the final score of this bid is equal to its quality score. If the quality difference between this bid and all other bids is large enough, it can be ranked number 1, regardless of its price. Finally, it is impossible to calculate the scores if the lowest price is 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 360, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox occurred 3 out of 282 times, which is 1.06% and an intrusive ranking paradox didn't occur. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it

would involve division by 0, which is undefined.

Since this is a relative formula, price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. The necessary condition for a ranking paradox to occur is the change in the lowest price of all submitted bids.

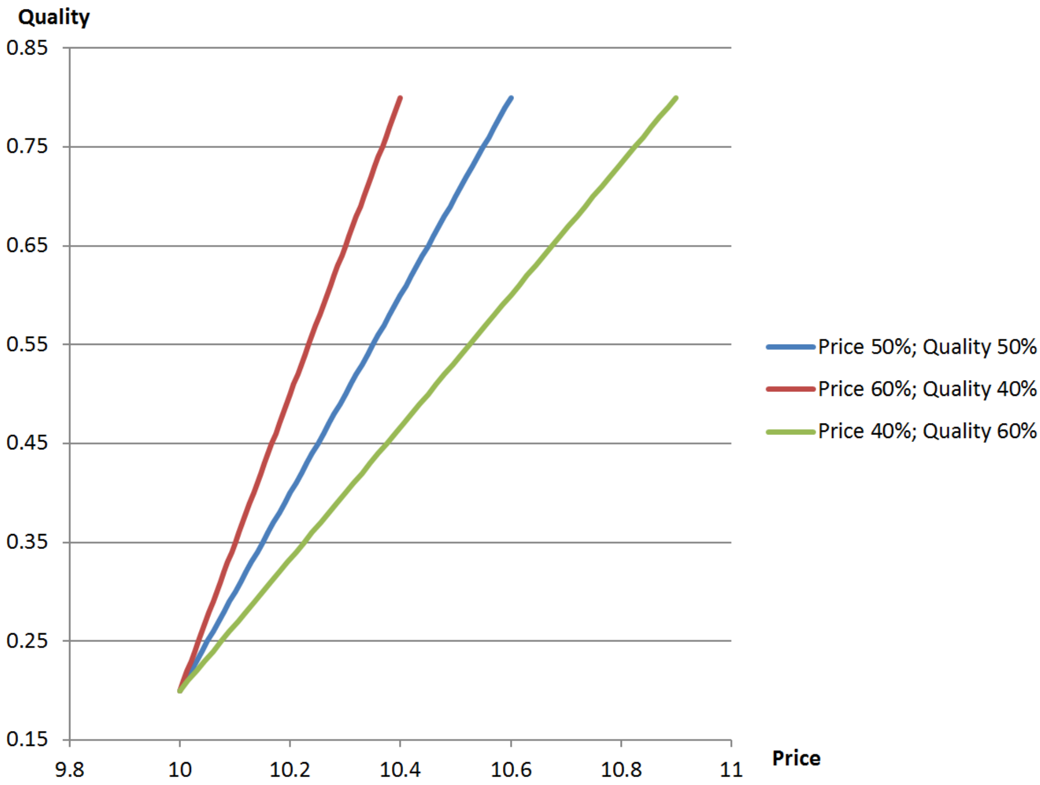
## 6.12 Scottish Government

| R | / | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 0.00% | 0.00% | 0.00% | 0.00% | 0.57% | 0.00% |

The following is a formula used by the Scottish Government. The bid ranked number 1 is the one with the highest score. This formula is:

$$Score = \left[0.5 - \left(\frac{P_i - P_{Avg}}{P_{Avg}}\right)\right] \cdot W_{Price} + W_{Quality} \cdot Q_i.$$

This is a relative formula since it depends on the average price. A bidding price that is more than 50% higher than the average price of all submitted bids causes the price score to be subtracted from the quality score. This formula provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, then they can't charge whatever amount they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. Finally, it is impossible to calculate the scores if all submitted prices are 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 360, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox didn't occur and an intrusive ranking paradox occurred 7 out of 1221 times, which is 0.57%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $1.5 \cdot W_{Price}$.

Since this is a relative formula, price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. The necessary condition for a ranking paradox to occur is the change in the average price. The dependence on the average price makes this formula extremely vulnerable to ranking paradoxes because the average price is the sum of all submitted prices over the number of submitted bids.

## 6.13 Waterschap Brabantse Delta

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| R | / |  |  |  |  |  |  |  |
| | | | 0.28% | 0.35% | 0.57% | 10.28% | 1.23% | 0.28% |

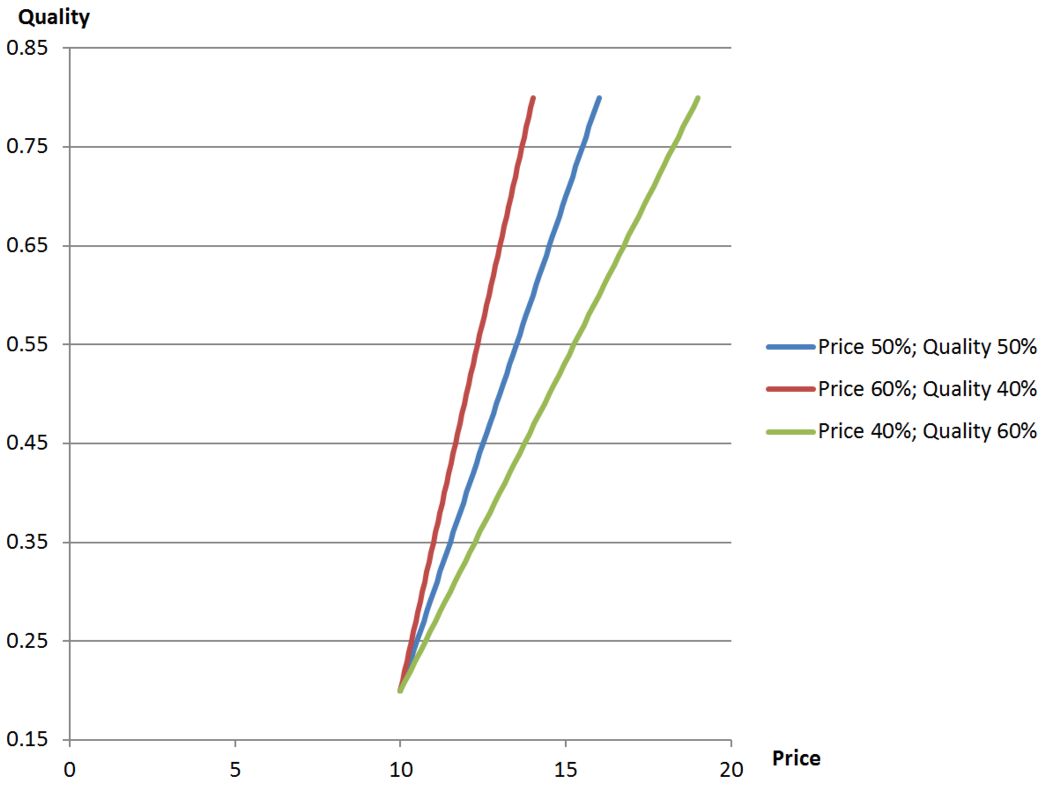
What proceeds is a formula used by the Waterschap Brabantse Delta. The bid ranked number 1 is the one with the highest score. This formula is:

$$Score = W_{Price} \cdot \left(1 - \frac{P_i - P_{Best}}{P_{Best}}\right) + W_{Quality} \cdot Q_i.$$

If the price difference between the lowest bid and the 2nd lowest bid is greater than 20%, then the 2nd lowest bid gets 80% of price points of the lowest bid and the scores of consecutive bids is calculated according to the following formula:

$$Score = W_{Price} \cdot \left(1 - \frac{P_i - P_{2nd\ Best}}{P_{2nd\ Best}}\right) + W_{Quality} \cdot Q_i.$$

This formula is relative since it is based on the lowest price of all submitted bids. This formula provides protection against an extremely high price if at least 3 bids have been submitted. I can safely assume that an extremely high priced bid is neither 1st nor 2nd price-wise. Since the price is extremely high, its final score will be negative. On the other hand, the final score of the lowest priced bid will at least $W_{Price}$. Thus, the lowest priced bid will be ranked higher than an extremely high priced bid. If there have been only 2 bids submitted, then the formula doesn't provide protection against an extremely high price. Suppose that the price of one of the bids is extremely high. Assuming that the price difference between the lowest bid and the 2nd lowest bid is greater than 20% and therefore the special case applies. Hence, the score of this bid is equal to $0.8 \cdot W_{Price} + W_{Quality} \cdot Q_A$. The score of the lowest priced bid is equal to $W_{Price} + W_{Quality} \cdot Q_B$. Thus, if the difference between $Q_A$ and $Q_B$ is large enough, $0.8 \cdot W_{Price} + W_{Quality} \cdot Q_A > W_{Price} + W_{Quality} \cdot Q_B$. Finally, it is impossible to calculate the scores if the lowest price is 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 1 time out of 360, which is 0.28%. When at least 3 bids have been submitted, it was 1 time out of 282, which is 0.35% and when at least 4 bids have been submitted, it was 1 time out of 175, which is 0.57%. In the 4th experiment, a non-intrusive ranking paradox occurred 29 out of 282 times, which is 10.28% and an intrusive ranking paradox occurred 15 out of 1221 times, which is 1.23%. According to the results of the last experiment which are presented in the table on page 94 the formula of Waterschap Brabantse Delta produced non Pareto optimal outcome 1 time out of 360, which is 0.28%. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

Since this is a relative formula, price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. The necessary condition for a ranking paradox to occur is the change in the lowest price of all submitted bids.

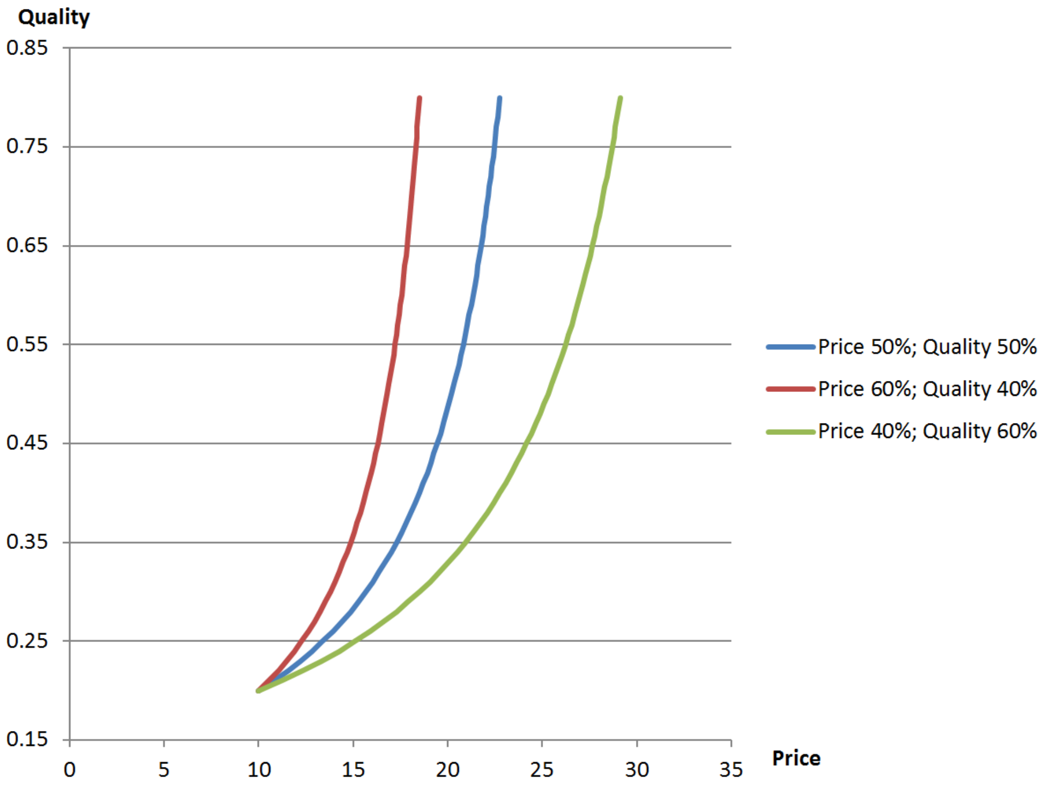
## 6.14 Chen 1

| A | | $\infty$ | 2 | 3 | 4 | ! | !! | |
|---|---|---|---|---|---|---|---|---|
| | | | N/A | N/A | N/A | 0 | 0 | N/A |

This formula comes from[6] and has been modified to include the quality score as well. The bid ranked number 1 is the one with the lowest score. The formula is:

$$Score = W_{Price} \cdot \frac{P_i}{P_{Set\ Max}} + W_{Quality} \cdot \frac{Q_{Set\ Min}}{Q_i}, \text{ where } P_{Set\ Max} \neq 0.$$

This is an absolute formula. The tendering entity sets the maximum price it is willing to pay and the minimum quality it is willing to accept. This formula provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. Finally, it is impossible to calculate the score of a bid with quality equal to 0 since it would involve division by 0, which is undefined.




To plot the indifference curves, $P_{Set\ Max}$ has been set to 17 Euros. The indifference curves on the above graph are convex, they are bowed in. The marginal rate of substitution of quality for price is positive and decreasing which means that consecutive units of quality are less and less expensive. Another observation is that as the weight of quality increases, the indifference curve becomes more convex.

It was impossible to do experiments 3, 4 and 5 because this formula requires an appropriate choice for $P_{Set\ Max}$ and $Q_{Set\ Min}$. Since the results of these experiments depend on the choice of these values, an inappropriate choice would introduce bias. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

Since it is an absolute formula, neither price related nor quality related ranking paradoxes are possible.

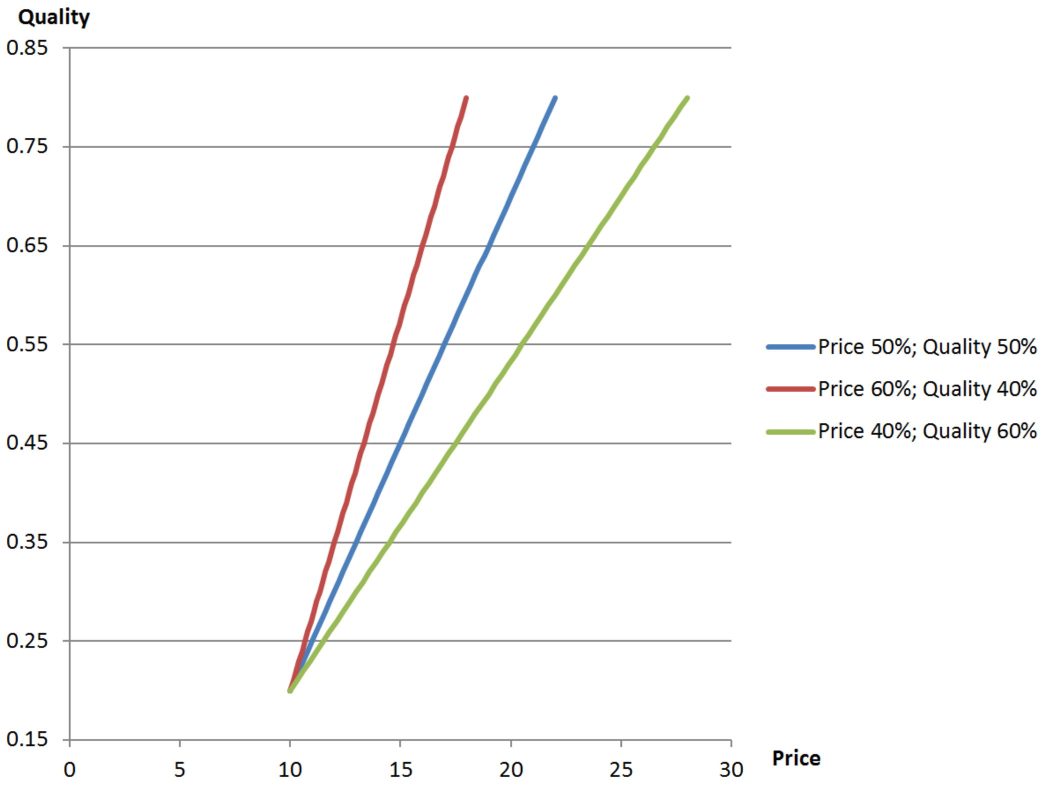
## 6.15 Chen 2

| R | / | ∞ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 0.83% | 0.71% | 1.14% | 0.35% | 0.08% | 0.00% |

This formula comes from[7] and has been modified to include the quality score. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = \left[1 - 0.5 \cdot \frac{P_i}{P_{Best}}\right] \cdot W_{Price} + W_{Quality} \cdot Q_i.$$

This is an example of a relative formula since it depends on the lowest price of all submitted bids. Bidders, by bidding low prices, can increase the likelihood of getting the highest price score and reduce the price score obtained by other bids. On the other hand, a bidding price more than twice the lowest price of all submitted bids results in the price score being subtracted from the quality score. This formula provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. Finally, it is impossible to calculate the scores if the lowest price is 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 3 out of 360 times, which is 0.83%. When at least 3 bids have been submitted, it was 2 out of 282 times, which is 0.71% and when at least 4 bids have been submitted, it was 2 out of 175 times, which is 1.14%. In the 4th experiment, a non-intrusive ranking paradox occurred 1 time out of 282, which is 0.35% and an intrusive ranking paradox occurred 1 time out of 1221, which is 0.08%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

As I have already mentioned, the formula presented in this section is relative and therefore a price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. The necessary condition for ranking paradox to occur is the change in the lowest price of all submitted bids.

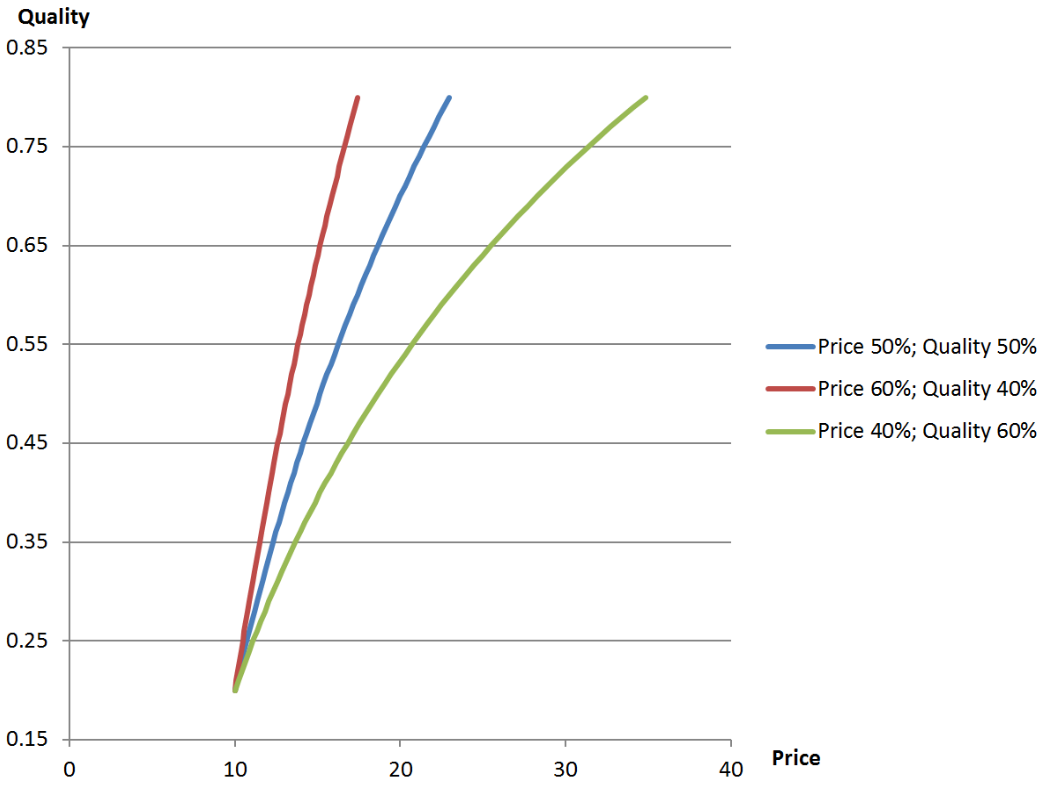
## 6.16 Chen 3

| R | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 0.00% | 0.00% | 0.00% | 0 | 0 | 0.00% |

This formula also comes from[7] and has been also modified to include the quality score. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = \left[1 - 0.5 \cdot \frac{\log\left(\frac{P_i}{P_{Best}}\right)}{\log(2)}\right] \cdot W_{Price} + W_{Quality} \cdot Q_i.$$

This is another an example of a relative formula since it depends on the lowest price of all submitted bids. Bidders, by bidding low prices, can increase the likelihood of getting the highest price score and reduce the price score obtained by other bids. On the other hand, a bidding price more than 4 times the lowest price of all submitted bids results in the price score being subtracted from the quality score. This formula provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. Finally, it is impossible to calculate the scores if the lowest price is 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are concave, they are bowed out. The marginal rate of substitution of quality for price is positive and increasing which means that consecutive units of quality are more and more expensive. Another observation is that as the weight of quality increases, the indifference curve becomes more concave.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 360, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, neither non-intrusive ranking paradoxes nor intrusive ranking paradoxes occurred. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

Although this is a relative formula, price related ranking paradoxes are not possible. The proof of this statement can be found in Appendix A.4.

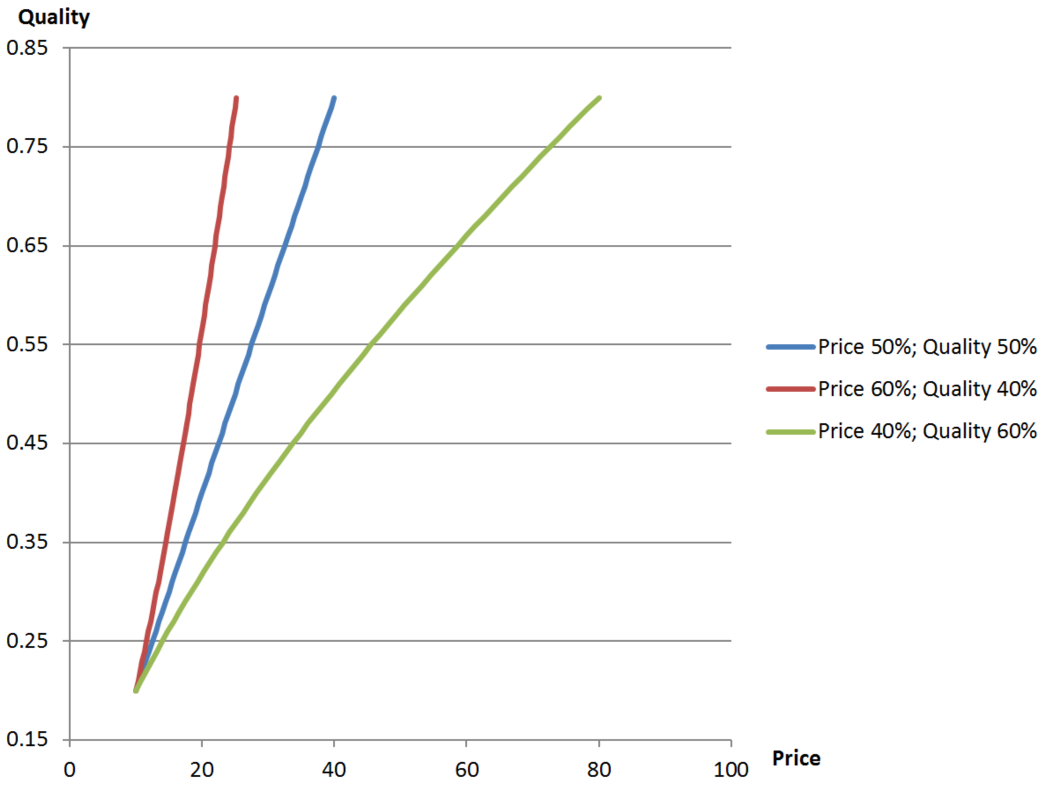
## 6.17 Argitek

| A | / | ∞ | 2 P Q | 3 P Q | 4 P Q | ! | !! |  |
|---|---|---|---|---|---|---|---|---|
| | | | 0.28% | 0.00% | 0.00% | 0 | 0 | 0.00% |

This is a formula used by Argitek. The bid ranked number 1 is the one with the lowest score. This formula is:

$$Score = \frac{P_i}{Q_i^{\frac{w_{Quality}}{w_{Price}}}}.$$

This is an absolute formula. It provides protection against an extremely high price if the weight of price is other than 0. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. It is impossible to calculate the score of a bid with quality equal to 0 since it would involve division by 0, which is undefined. For the same reason it is impossible to calculate the scores if the weight of price is 0. If the weight of price is 0, there should be an additional step describing what to do, for example, rank bids by quality in a descending order.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 1 time out of 360, which is 0.28%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, neither non-intrusive ranking paradoxes nor intrusive ranking paradoxes occurred. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

Since it is an absolute formula, neither price related nor quality related ranking paradoxes are impossible.

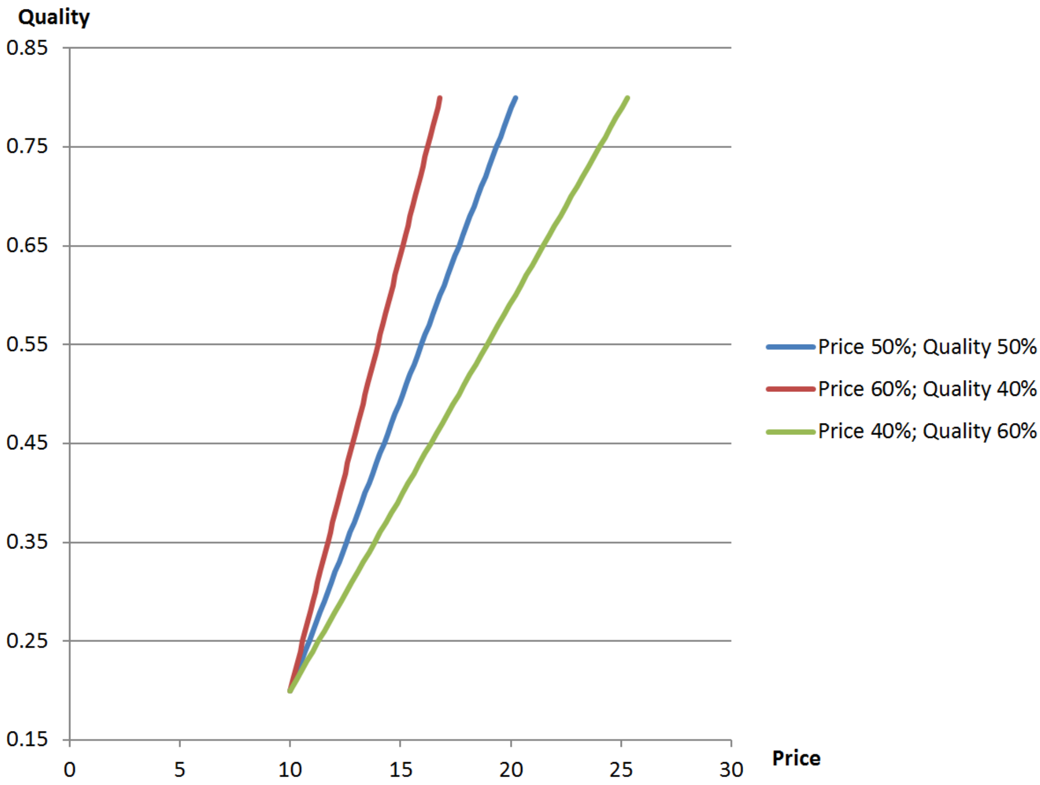
## 6.18 Telgen

| A | / | ∞ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | N/A | N/A | N/A | 0 | 0 | N/A |

The bid ranked number 1 is the one with the highest score. This formula is given by:

$$Score = Q_i \cdot W_{Quality} + W_{Price} \cdot \left( \frac{P_{Set\ Max} - P_i}{P_{Set\ Max} - P_{Set\ Min}} \right), \text{ where } P_{Set\ Max} \neq P_{Set\ Min}.$$

This is yet another example of an absolute formula. A bidding price higher than the upper end of the price range results in the price score being subtracted from the quality score. This is a formula that provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid.




To plot the indifference curves, $P_{Set\ Max}$ has been set to 17 Euros and $P_{Set\ Min}$ has been set to 0 Euros. The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

It was impossible to do experiments 3, 4 and 5 because this formula requires an appropriate choice for $P_{Set\ Max}$ and $P_{Set\ Min}$. Since the results of these experiments depend on the choice of these values, an inappropriate choice would introduce bias. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $W_{Price} \cdot \left(\frac{P_{Set\ Max}}{P_{Set\ Max} - P_{Set\ Min}}\right)$.

Since it is an absolute formula, neither price related nor quality related ranking paradoxes are impossible.

## 6.19 Pauw & Wolvaardt

| R | | | 2 | 3 | 4 | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 4.40% | 3.61% | 2.29% | 0.00% | 0.66% | 0.00% |

This formula has been published in[17] and has been modified to also include the quality score. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = W_{Quality} \cdot Q_i + W_{Price} \cdot \left( \frac{P_{Worst} - P_i}{P_{Worst} - P_{Avg}} \right).$$

This is a relative formula since it is based on the average and the highest price of all submitted bids. It doesn't provide protection against an extremely high price. This can be illustrated with the same example as in Section 6.1. To summarize the example, if the weight of quality is very high, a bidder who can offer a very high quality while having a sufficient quality advantage over other bidders can charge whatever they wish and still be ranked number 1. Finally, it is impossible to calculate the scores if all submitted prices are the same since it would involve division by 0, which is undefined.

As for the indifference curves, assuming that the lowest priced bid is below the average price, then it is impossible to plot them according to the method used in this paper. Note that the highest priced bid gets the final score equal to its quality score. On the other hand, the price score of the lowest priced bid is equal to $W_{Price} \cdot \left( \frac{P_{Worst} - P_{Best}}{P_{Worst} - P_{Avg}} \right)$. Assuming that $P_{Best} < P_{Avg}$, $\left( \frac{P_{Worst} - P_{Best}}{P_{Worst} - P_{Avg}} \right) > 1$. Thus, if the weight of price is 60% and the weight of quality is 40%, then the lowest priced bid will always command a higher score than the highest priced bid.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 15 out of 341 times, which is 4.40%. When at least 3 bids have been submitted, it was 10 out of 277 times, which is 3.61% and when at least 4 bids have been submitted, it was

4 out of 175 times, which is 2.29%. In the 4th experiment, a non-intrusive ranking paradox didn't occur and an intrusive ranking paradox occurred 8 out of 1208 times, which is 0.66%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $W_{Price} \cdot \left(\frac{P_{Worst}}{P_{Worst}-P_{Avg}}\right)$.

Since this formula depends on the average and the highest price, price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the highest or the average price of all submitted bids change. The dependence on the average price makes this formula much more vulnerable to ranking paradoxes than a formula that depends only on the lowest price or the lowest and the highest price.

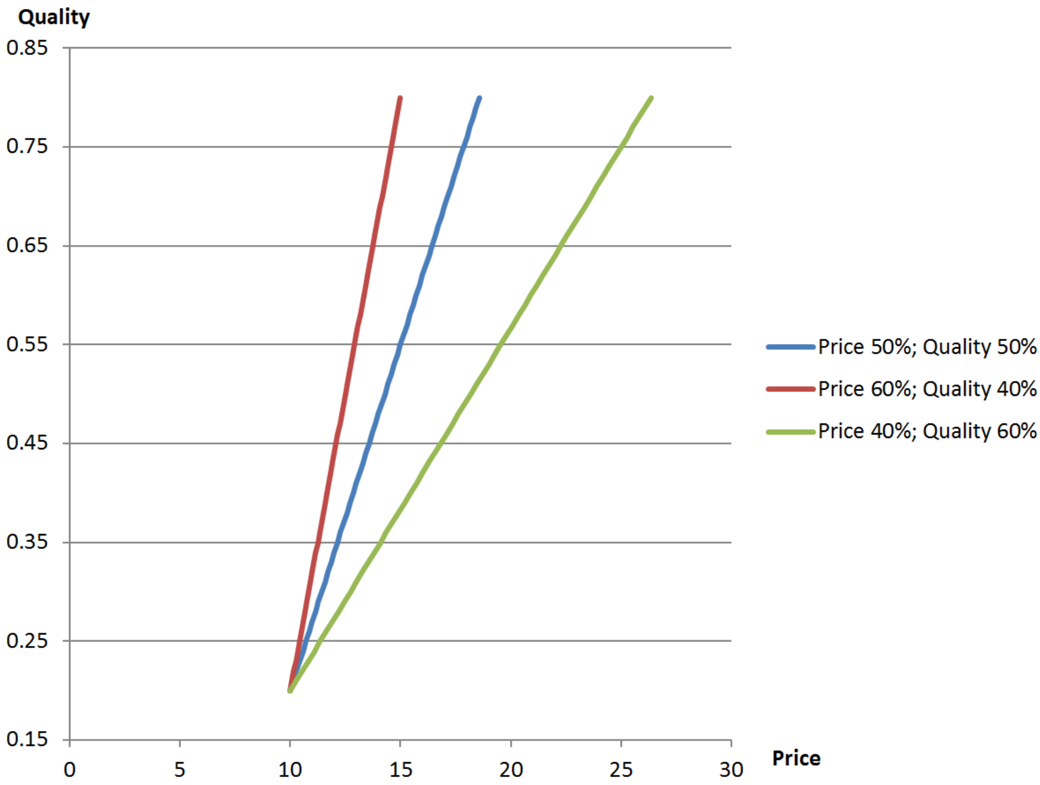
## 6.20 Based on the Average Price

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| R | / | ∞ | | | | | | |
| | | | 0.00% | 0.00% | 0.00% | 0.00% | 0.57% | 0.00% |

This formula comes from[8] and has been also modified to include the quality score. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = W_{Price} \cdot \left(1 - \frac{P_i - P_{Best}}{P_{Avg}}\right) + W_{Quality} \cdot Q_i.$$

This is a relative formula since it is based on the average and the lowest price of all submitted bids. A bidding price more than the sum of the average price and the lowest price of all submitted bids results in the price score being subtracted from the quality score. It provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. Finally, it is impossible to calculate the scores if all submitted prices are 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 0 times out of 360, which is 0.00%. When at least 3 bids have been submitted, it was 0 times out of 282, which is 0.00% and when at least 4 bids have been submitted, it was 0 times out of 175, which is 0.00%. In the 4th experiment, a non-intrusive ranking paradox didn't occur and an intrusive ranking paradox occurred 7 out of 1221 times, which is 0.57%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $W_{Price}$ [7].

The formula based on the average price depends on the lowest and average price and therefore a price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the lowest or the average price of all submitted bids change. The dependence on the average price makes this formula much more vulnerable to ranking paradoxes than a formula that depends only on the lowest price or the lowest and the highest price.

[7] Assuming that $P_{Best} = 0$.

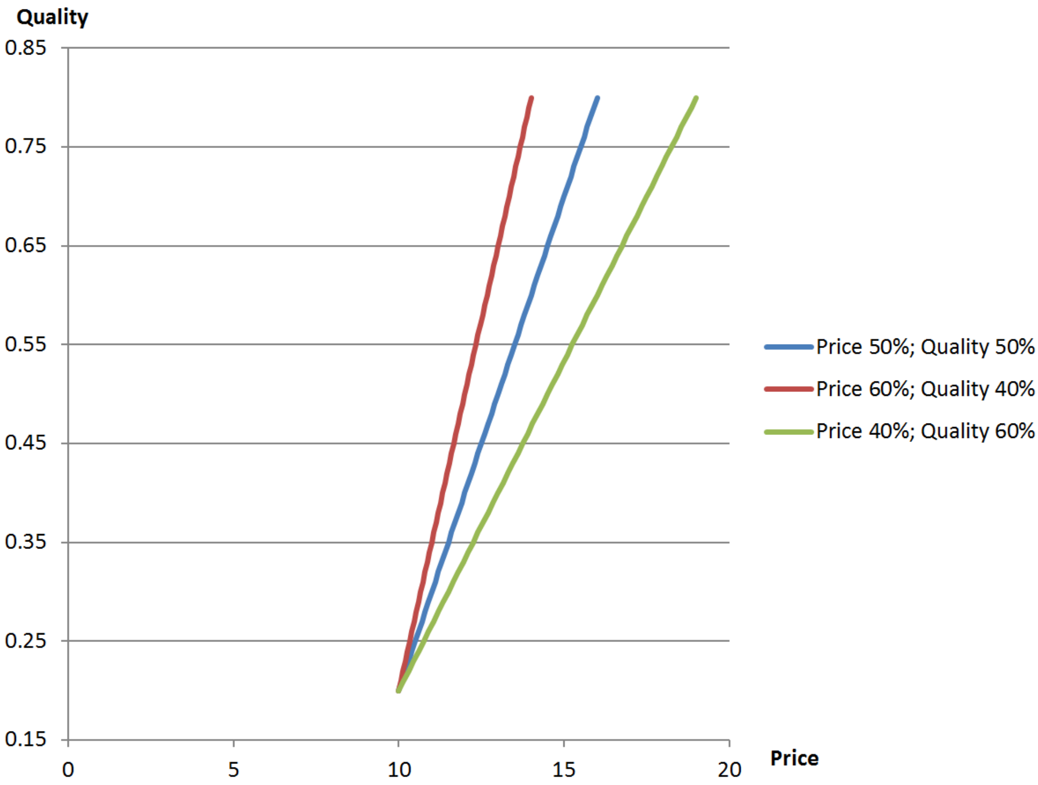
## 6.21 Based on the Lowest Price

| R | / | ∞ | 2 | 3 | 4 | ! | !! | |
|---|---|---|---|---|---|---|---|---|
| | | | 1.11% | 1.06% | 1.14% | 0.35% | 0.00% | 0.00% |

This formula also comes from[8] and has been also modified to include the quality score. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = W_{Price} \cdot \left( \frac{2 \cdot P_{Best} - P_i}{P_{Best}} \right) + W_{Quality} \cdot Q_i.$$

This is a relative formula since it depends on the lowest price of all submitted bids. A bidding price more than twice the lowest price of all submitted bids results in the price score being subtracted from the quality score. It provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. Finally, it is impossible to calculate the scores if the lowest price is 0 since it would involve division by 0, which is undefined.




The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 4 out of 360 times, which is 1.11%. When at least 3 bids have been submitted, it was 3 out of 282 times, which is 1.06% and when at least 4 bids have been submitted, it was 2 out of 175 times, which is 1.14%. In the 4th experiment, a non-intrusive ranking paradox occurred 1 time out of 282, which is 0.35% and an intrusive ranking paradox didn't occur. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it

would involve division by 0, which is undefined.

Since this formula is based on the lowest price, price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the lowest price of all submitted bids changes.

## 6.22 Bisection Method

| R | | |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| | | | 1.76% | 1.81% | 0.57% | 1.10% | 0.91% | 0.00% |

This is the 3rd formula from from[8] which has been also modified to include the quality score. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = W_{Price} \cdot \left( \frac{P_i - P_{Worst}}{P_{Best} - P_{Worst}} \right) + W_{Quality} \cdot Q_i.$$

It is a relative formula since it is based on the lowest and the highest price. It doesn't provide protection against an extremely high price. This can be illustrated with the same example as in Section 6.1. To summarize the example, if the weight of quality is very high, a bidder who can offer a very high quality while having a sufficient quality advantage over other bidders can charge whatever they wish and still be ranked number 1. Finally, it is impossible to calculate the scores if all submitted prices are 0 since it would involve division by 0, which is undefined.

As for the indifference curves, it is impossible to plot them according to the method used in this paper for the same reason as in Section 6.2.

Regarding the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 6 out of 341 times, which is 1.76%. When at least 3 bids have been submitted, it was 5 out of 277 times, which is 1.81% and when at least 4 bids have been submitted, it was 1 time out of 175, which is 0.57%. In the 4th experiment, a non-intrusive ranking paradox occurred 3 out of 273 times, which is 1.10% and an intrusive ranking paradox occurred 11 out of 1208 times, which is 0.91%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0%

receives a score of $-W_{Price}$ [8].

Since it is a relative formula, price related ranking paradox is possible, including also a possibility of an intrusive ranking paradox. A ranking paradox may occur if the lowest or the highest price of all submitted bids change. The dependence on both the lowest and the highest price makes this formula much more vulnerable to ranking paradoxes then a formula that depends only on the lowest price.

[8] Assuming that $P_{Best} = 0$.

## 6.23 Score by Rank

| R | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 8.89% | 8.51% | 5.14% | 1.10% | 0.33% | 0.00% |

This formula can be found in[21] and it has been modified to include the quality score. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = W_{Price} \cdot Score_{Price} + W_{Quality} \cdot Q_i.$$

The highest priced bid earns 0 and the lowest 1 points on the price score. All others are placed at equal increments between. If there are 6 suppliers, they would be placed at 0, 0.2, 0.4, 0.6, 0.8 and 1.

This is a relative formula. Despite the fact that the price score doesn't explicitly depend on the lowest price of all submitted bids, it does depend on the number of submitted bids. Another reason to claim that it is a relative formula is that a ranking paradox is possible because the price score of each bid changes if the number of bidders participating it a tender changes. This will be illustrated with an example. Last but not least, it doesn't provide protection against an extremely high price. This can be illustrated with the same example as in Section 6.1. To summarize the example, if the weight of quality is very high, a bidder who can offer very high quality while having a sufficient quality advantage over other bidders can charge whatever they wish and still be ranked number 1.

As for the indifference curves, it is impossible to plot them according to the method used in this paper for the same reason as in Section 6.2.

Regarding the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 32 out of 360 times, which is 8.89%. When at least 3 bids have been submitted, it was 24 out of 282 times, which is 8.51% and when at least 4 bids have been submitted, it was 9 out of 175 times, which is 5.14%. In the 4th experiment, a non-intrusive ranking paradox occurred 3 out of 273 times, which is 1.10% and an intrusive ranking paradox occurred 4 out of 1208 times, which is 0.33%. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $W_{Price}$ [9].

Here is an example of an intrusive ranking paradox that shows a relative aspect of the score by rank. Suppose that the weight of price has been set to 20% and the weight of quality to 80%. Suppose further that 6 bids have been submitted whose prices and qualities are summarized in the table below.

| Bid | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Price score | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
| Quality score | 0.85 | 0.82 | 0.76 | 0.64 | 0.58 | 0.55 |
| Total score | 0.68 | 0.696 | 0.688 | 0.632 | 0.632 | 0.64 |
| Ranking | 3 | 1 | 2 | 5 | 6 | 4 |

Here is what happens if the bid ranked as number 3 drops out.

| Bid | B | C | D | E | F |
|---|---|---|---|---|---|
| Price score | 0 | 0.25 | 0.5 | 0.75 | 1 |
| Quality score | 0.82 | 0.76 | 0.64 | 0.58 | 0.55 |
| Total score | 0.656 | 0.658 | 0.612 | 0.614 | 0.64 |
| Ranking | 2 | 1 | 5 | 4 | 2 |

As one can see, bid C, which was initially ranked as number 2 is now ranked as number 1 and bid B, which was initially ranked as number 1 is now ranked as number 2. This is a classic example of an intrusive ranking paradox. To conclude, using score by rank raises a possibility of a price related ranking paradox that can either be intrusive or non-intrusive.

[9] Assuming that 0 Euros is the lowest price bid.

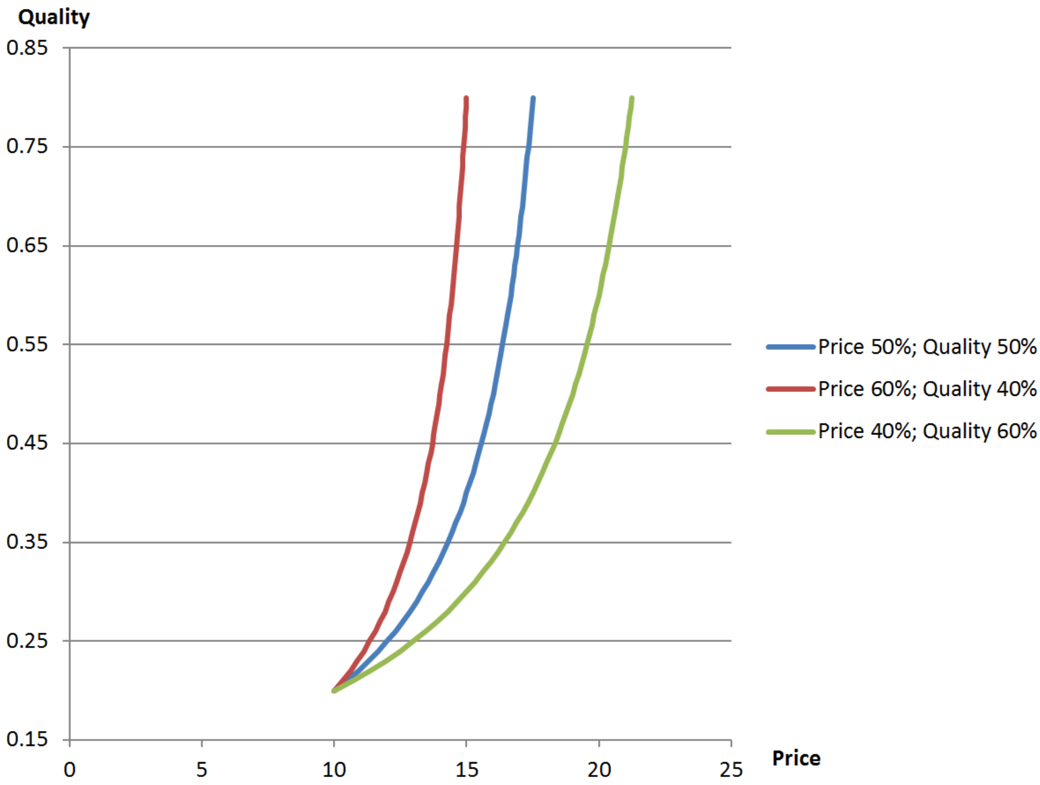
## 6.24 Kuiper 1

| A | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | N/A | N/A | N/A | 0 | 0 | N/A |

This formula comes from[11]. The bid ranked number 1 is the one with the lowest score. This formula is

$$Score = P_i - \frac{W_{Quality}}{W_{Price}} \cdot P_{Ref} \cdot \left(1 - \frac{Q_{Ref}}{Q_i}\right).$$

This is an absolute formula. It provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. It is impossible to calculate the scores if the weight of price is 0. In such situations where it is impossible to calculate the scores because the weight of price is 0, there is needed an additional step - namely the ranking of bids by quality in a descending order. For the same reason it is impossible to calculate the score of a bid with quality equal to 0.




To plot the indifference curves, the reference price has been set to 10 Euros and the reference quality has been set to 20%. The indifference curves on the above graph are convex, they are bowed in. The marginal rate of substitution of quality for price is positive and decreasing which means that consecutive units of quality are less and less expensive. Another observation is that as the weight of quality increases, the indifference curve becomes more convex.

It was impossible to do experiments 3, 4 and 5 because this formula requires an appropriate choice for the reference price and reference quality. Since the results of these experiments depend on the choice of these values, an inappropriate choice would introduce bias. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

Since it is an absolute formula, neither price related nor quality related ranking paradoxes are impossible.

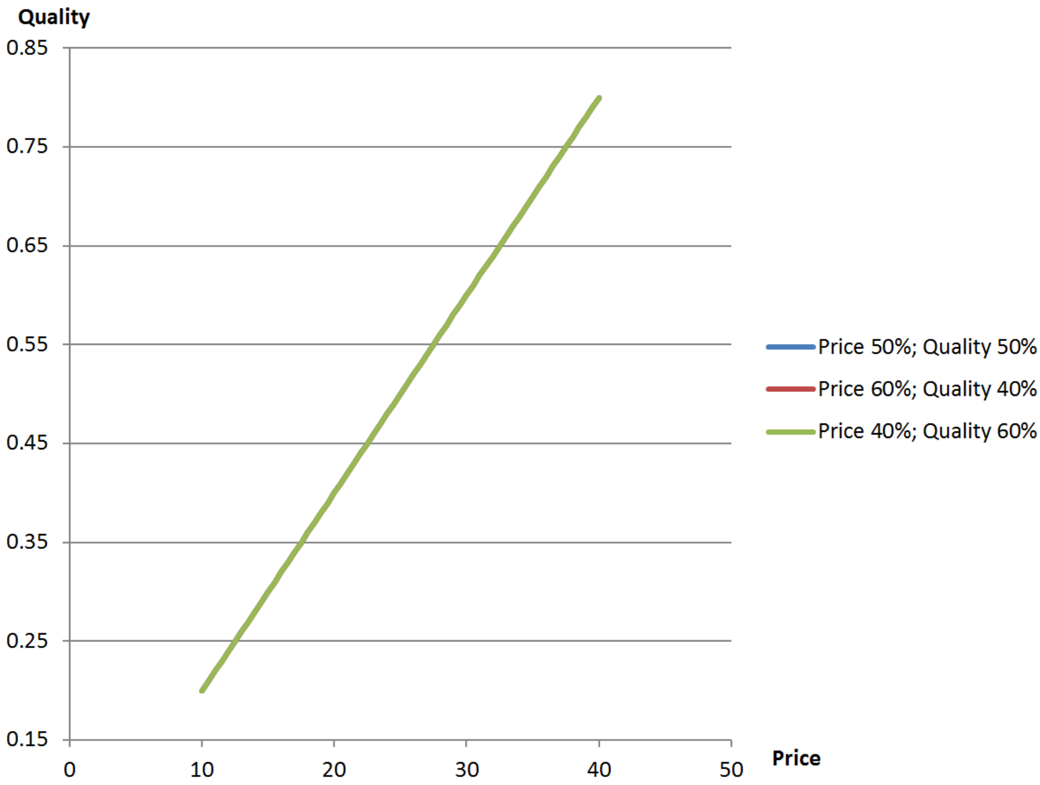
## 6.25 Kuiper 2

| A | / | ∞ |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| | | | 1.76% | 1.81% | 0.57% | 0 | 0 | 0.00% |

This formula also comes from[11]. The bid ranked number 1 is the one with the highest score. This formula is:

$$Score = \frac{Q_i \cdot W_{Quality}}{P_i \cdot W_{Price}}.$$

This is an example of an absolute formula. This formula provides protection against an extremely high price if the weight of price is other than 0. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid. If the weight of price is 0, it is impossible to calculate the scores. In such situations where it is impossible to calculate the scores because the weight of price is 0, there is needed an additional step - namely the ranking of bids by quality in a descending order. For the same reason it is impossible to calculate the score of a bid with a price equal to 0.




The indifference curves are straight lines. The indifference curves on the above graph neither become steeper as the weight of price increases nor do they become more flat as the weight of quality increases. This means that the slope of the indifference curve doesn't depend on the weights of price and quality. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

As regards the results of the 3rd experiment, the formula ranked as the number 1 bid with the lowest price and quality lower than the highest submitted quality, when at least 2 bids have been submitted, 6 out of 341 times, which is 1.76%. When at least 3 bids have been submitted, it was 5 out of 277 times, which is 1.81% and when at least 4 bids have been submitted, it was 1 time out of 175, which is 0.57%. In the 4th experiment, neither non-intrusive ranking paradox nor intrusive ranking paradox occurred. According to the results of the 5th experiment which are presented in the table on page 94, there is no evidence to consider this formula as not Pareto optimal. Finally, this formula satisfies the symmetry of the score. It was impossible to check the zero score property because it would involve division by 0, which is undefined.

Since it is an absolute formula, neither price related nor quality related ranking paradoxes are impossible.

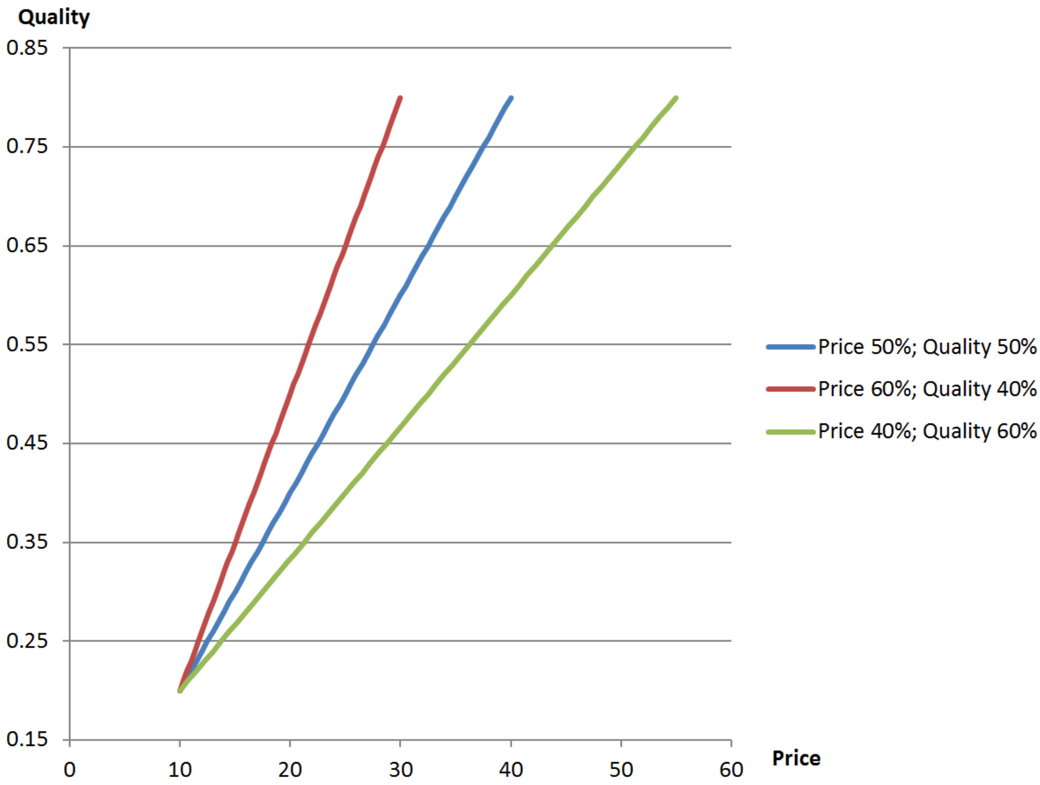
## 6.26 Kuiper 3

| A | / | $\infty$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | N/A | N/A | N/A | 0 | 0 | N/A |

This the 3rd formula from[11]. The bid ranked number 1 is the one with the highest score. The formula is:

$$Score = W_{Quality} \cdot \frac{Q_i}{Q_{Ref}} + W_{Price} \cdot \left(2 - \frac{P_i}{P_{Ref}}\right), \text{ where } Q_{Ref} \neq 0 \text{ and } P_{Ref} \neq 0.$$

This is another example of an absolute formula. A bidding price more than twice the reference price results in the price score being subtracted from the quality score. It provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid.

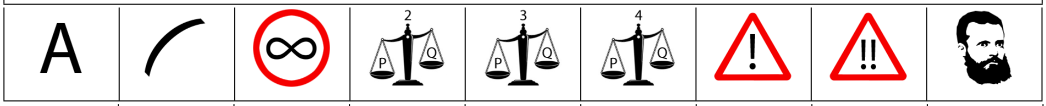


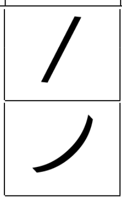

To plot the indifference curves, the reference price has been set to 10 Euros and the reference quality has been set to 20%. The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.

It was impossible to do experiments 3, 4 and 5 because this formula requires an appropriate choice for the reference price and reference quality. Since the results of these experiments depend on the choice of these values, an inappropriate choice would introduce bias. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $2 \cdot W_{Price}$.

Since it is an absolute formula, neither price related nor quality related ranking paradoxes are impossible.

## 6.27 Kuiper's Superformula



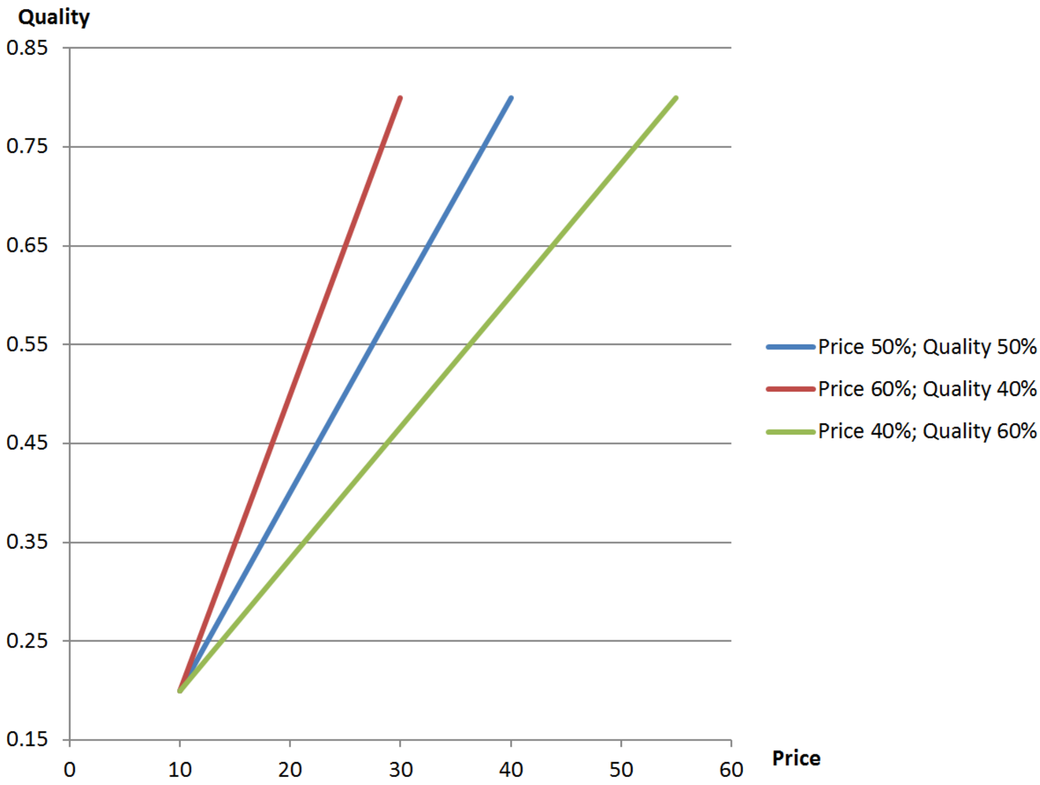



| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | N/A | N/A | N/A | 0 | 0 | N/A |

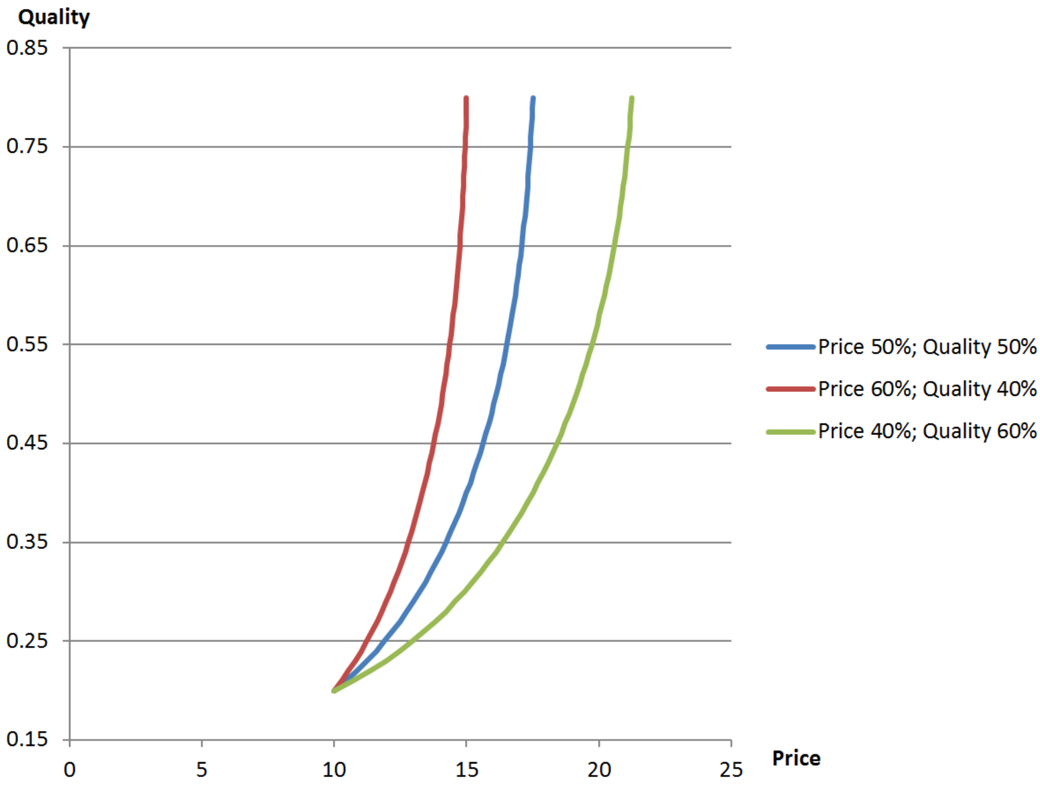
The bid ranked number 1 is the one with the lowest score. The formula is:

$$Score = \sqrt[n]{\left(\frac{P_i}{P_{Q=1}}\right)^n + \left(\frac{1 - Q_i}{1 - Q_{P=0}}\right)^n}, \text{ where } P_{Q=1} \neq 0 \text{ and } Q_{P=0} \neq 1.$$

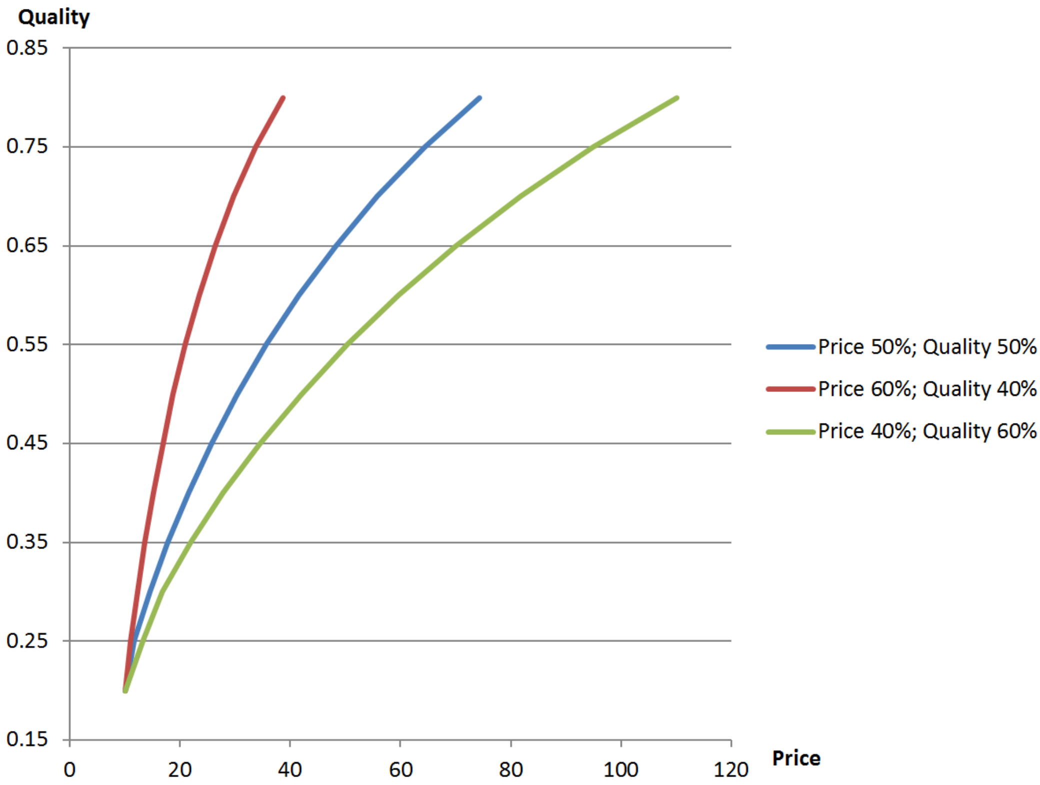
This is the last example of an absolute formula. It depends on 3 parameters: $n$, $P_{Q=1}$ and $Q_{P=0}$. $P_{Q=1}$ denotes the price of a bid with quality 1 and $Q_{P=0}$ denotes the quality of a bid with price 0. In order to determine these parameters, one needs 3 points on the indifference curve. These points define not only the weights of price and quality, but also the shape of the indifference curve. Values of $n$, $P_{Q=1}$ and $Q_{P=0}$ are determined by finding a superellipse with radius 1 that goes through 3 points on the indifference curve using, for example, Solver. This formula provides protection against an extremely high price. Suppose the weight of quality is very high and the weight of quality is very low. Even if one bidder can offer a very high quality, they are unable to charge whatever they wish and still be ranked number 1. In other words, their bid can't dominate the other bids because different prices may have an effect on the ranking of a very high quality bid.




The parameters for the red indifference curve are $P(Q = 10) = 50$, $Q(P = 0) = 0$ and $n = 1$. The parameters for the blue indifference curve are $P(Q = 10) = 33.33$, $Q(P = 0) = 0$ and $n = 1$. The parameters for the green indifference curve are $P(Q = 10) = 70$, $Q(P = 0) = 0$ and $n = 1$. The indifference curves on the above graph are straight lines and become steeper as the weight of price increases and more flat as the weight of quality increases. The marginal rate of substitution of quality for price is positive and constant, meaning every unit of quality is worth the same amount of money. The slope of the indifference curves is positive, meaning higher quality is more expensive than lower quality.




The parameters for the red indifference curve are $P(Q = 10) = 15.06$, $Q(P = 0) = 0.1$ and $n = 2.93$. The parameters for the blue indifference curve are $P(Q = 10) = 17.61$, $Q(P = 0) = 0.13$ and $n = 2.76$. The parameters for the green indifference curve are $P(Q = 10) = 21.45$, $Q(P = 0) = 0.15$ and $n = 2.59$. One should take note that the indifference curves on the above graph are convex, they are bowed in. The marginal rate of substitution of quality for price is positive and decreasing which means that consecutive units of quality are less and less expensive. Another observation is that as the weight of quality increases, the indifference curve becomes more convex.




The parameters for the red indifference curve are $P(Q = 10) = 233.73$, $Q(P = 0) = 0$ and $n = 0.62$. The parameters for the blue indifference curve are $P(Q = 10) = 235.25$, $Q(P = 0) = 0$ and $n = 0.5$. The parameters for the green indifference curve are $P(Q = 10) = 271.2$, $Q(P = 0) = 0$ and $n = 0.39$. The indifference curves on the above graph are concave, they are bowed out. The marginal rate of substitution of quality for price is positive and increasing which means that consecutive units of quality are more and more expensive. Another observation is that as the weight of quality increases, the indifference curve becomes more concave.

It was impossible to do experiments 3, 4 and 5 because this formula requires an appropriate choice for the reference price and reference quality. Since the results of these experiments depend on the choice of these values, an inappropriate choice would introduce bias. Finally, this formula satisfies the symmetry of the score. It doesn't satisfy the zero score property because the bid with a price of 0 Euros and a quality of 0% receives a score of $\sqrt[n]{\left(\frac{1}{1-Q_{P=0}}\right)^n}$.

Since it is an absolute formula, neither price related nor quality related ranking paradoxes are impossible.

# 7 Summary of the Results

| Formula | Type | Shape of the indifference curve |
|---|---|---|
| Lowest Bid Scoring | Relative | Concave |
| Highest Bid-Lowest Bid Scoring | Relative | N/A |
| Average Scoring | Relative | N/A |
| Based on Bid Spread | Relative | N/A |
| Based on Average Bid | Relative | Straight line |
| Maximum Price Deviation Model | Relative | Straight line |
| Negometrix | Relative | Straight line |
| Coventry City Council | Relative | Concave |
| European Organization for Nuclear Research (CERN) | Relative | Straight line |
| Tennet | Relative | Concave |
| Mercer | Relative | Straight line |
| Scottish Government | Relative | Straight line |
| Waterschap Brabantse Delta | Relative | Straight line |
| Chen 1 | Absolute | Convex |
| Chen 2 | Relative | Straight line |
| Chen 3 | Relative | Concave |
| Argitek | Absolute | Straight line |
| Telgen | Absolute | Straight line |
| Pauw & Wolvaardt | Relative | N/A |
| Based on the Average Price | Relative | Straight line |
| Based on the Lowest Price | Relative | Straight line |
| Bisection Method | Relative | N/A |
| Score by Rank | Relative | N/A |
| Kuiper 1 | Absolute | Convex |
| Kuiper 2 | Absolute | Straight line |
| Kuiper 3 | Absolute | Straight line |
| Kuiper's Superformula | Absolute | Any |

| Formula | Mistake of overvaluing the weight of price and under-valuing the weight of quality | | |
|---|---|---|---|
| | when at least 2 bids have been submitted | when at least 3 bids have been submitted | when at least 4 bids have been submitted |
| Lowest Bid Scoring | 0.00% | 0.00% | 0.00% |
| Highest Bid-Lowest Bid Scoring | 1.76% | 1.81% | 0.57% |
| Average Scoring | 1.17% | 1.08% | 0.00% |
| Based on Bid Spread | 1.76% | 1.81% | 0.57% |
| Based on Average Bid | 0.00% | 0.00% | 0.00% |
| Maximum Price Deviation Model | 0.00% | 0.00% | 0.00% |
| Negometrix | 0.00% | 0.00% | 0.00% |
| Coventry City Council | 0.00% | 0.00% | 0.00% |
| European Organization for Nuclear Research (CERN) | 0.84% | 0.71% | 1.16% |
| Tennet | 1.12% | 1.07% | 1.16% |
| Mercer | 0.00% | 0.00% | 0.00% |
| Scottish Government | 0.00% | 0.00% | 0.00% |
| Waterschap Brabantse Delta | 0.28% | 0.35% | 0.57% |
| Chen 1 | N/A | N/A | N/A |
| Chen 2 | 0.83% | 0.71% | 1.14% |
| Chen 3 | 0.00% | 0.00% | 0.00% |
| Argitek | 0.28% | 0.00% | 0.00% |
| Telgen | N/A | N/A | N/A |
| Pauw & Wolvaardt | 4.40% | 3.61% | 2.29% |
| Based on the Average Price | 0.00% | 0.00% | 0.00% |
| Based on the Lowest Price | 1.11% | 1.06% | 1.14% |
| Bisection Method | 1.76% | 1.81% | 0.57% |
| Score by Rank | 8.89% | 8.51% | 5.14% |
| Kuiper 1 | N/A | N/A | N/A |
| Kuiper 2 | 1.76% | 1.81% | 0.57% |
| Kuiper 3 | N/A | N/A | N/A |
| Kuiper's Superformula | N/A | N/A | N/A |

| Formula | Likelihood of a non-intrusive ranking paradox | Likelihood of an intrusive ranking paradox |
|---|---|---|
| Lowest Bid Scoring | 0.00% | 0.08% |
| Highest Bid-Lowest Bid Scoring | 1.10% | 0.91% |
| Average Scoring | 8.79% | 7.53% |
| Based on Bid Spread | 1.10% | 0.91% |
| Based on Average Bid | 0.00% | 0.57% |
| Maximum Price Deviation Model | 0.00% | 0.08% |
| Negometrix | 2.13% | 0.00% |
| Coventry City Council | 0.71% | 0.00% |
| European Organization for Nuclear Research (CERN) | 0.71% | 0.00% |
| Tennet | 1.07% | 0.00% |
| Mercer | 1.06% | 0.00% |
| Scottish Government | 0.00% | 0.57% |
| Waterschap Brabantse Delta | 10.28% | 1.23% |
| Chen 1 | 0 | 0 |
| Chen 2 | 0.35% | 0.08% |
| Chen 3 | 0 | 0 |
| Argitek | 0 | 0 |
| Telgen | 0 | 0 |
| Pauw & Wolvaardt | 0.00% | 0.66% |
| Based on the Average Price | 0.00% | 0.57% |
| Based on the Lowest Price | 0.35% | 0.00% |
| Bisection Method | 1.10% | 0.91% |
| Score by Rank | 1.10% | 0.33% |
| Kuiper 1 | 0 | 0 |
| Kuiper 2 | 0 | 0 |
| Kuiper 3 | 0 | 0 |
| Kuiper's Superformula | 0 | 0 |

| Formula | Protection against an extremely high price | Pareto non-optimality |
|---|---|---|
| Lowest Bid Scoring | No | 0.00% |
| Highest Bid-Lowest Bid Scoring | No | 0.00% |
| Average Scoring | No | 0.00% |
| Based on Bid Spread | No | 0.00% |
| Based on Average Bid | Yes | 0.00% |
| Maximum Price Deviation Model | No | 0.00% |
| Negometrix | Yes | 0.00% |
| Coventry City Council | No | 0.00% |
| European Organization for Nuclear Research (CERN) | Yes | 0.00% |
| Tennet | Yes | 0.00% |
| Mercer | No | 0.00% |
| Scottish Government | Yes | 0.00% |
| Waterschap Brabantse Delta | Yes | 0.28% |
| Chen 1 | Yes | N/A |
| Chen 2 | Yes | 0.00% |
| Chen 3 | Yes | 0.00% |
| Argitek | Yes | 0.00% |
| Telgen | Yes | N/A |
| Pauw & Wolvaardt | No | 0.00% |
| Based on the Average Price | Yes | 0.00% |
| Based on the Lowest Price | Yes | 0.00% |
| Bisection Method | No | 0.00% |
| Score by Rank | No | 0.00% |
| Kuiper 1 | Yes | N/A |
| Kuiper 2 | Yes | 0.00% |
| Kuiper 3 | Yes | N/A |
| Kuiper's Superformula | Yes | N/A |

| Formula | Symmetry of the score | Zero Score |
|---|---|---|
| Lowest Bid Scoring | Yes | N/A |
| Highest Bid-Lowest Bid Scoring | Yes | No |
| Average Scoring | Yes | No |
| Based on Bid Spread | Yes | No |
| Based on Average Bid | Yes | Yes |
| Maximum Price Deviation Model | Yes | Yes |
| Negometrix | Yes | N/A |
| Coventry City Council | Yes | N/A |
| European Organization for Nuclear Research (CERN) | Yes | N/A |
| Tennet | Yes | Yes |
| Mercer | Yes | N/A |
| Scottish Government | Yes | No |
| Waterschap Brabantse Delta | Yes | N/A |
| Chen 1 | Yes | N/A |
| Chen 2 | Yes | N/A |
| Chen 3 | Yes | N/A |
| Argitek | Yes | N/A |
| Telgen | Yes | No |
| Pauw & Wolvaardt | Yes | No |
| Based on the Average Price | Yes | No |
| Based on the Lowest Price | Yes | N/A |
| Bisection Method | Yes | No |
| Score by Rank | Yes | No |
| Kuiper 1 | Yes | N/A |
| Kuiper 2 | Yes | N/A |
| Kuiper 3 | Yes | No |
| Kuiper's Superformula | Yes | No |

| Formula | Price related intrusive ranking paradox | Price related non-intrusive ranking paradox |
|---|---|---|
| Lowest Bid Scoring | Yes | Yes |
| Highest Bid-Lowest Bid Scoring | Yes | Yes |
| Average Scoring | Yes | Yes |
| Based on Bid Spread | Yes | Yes |
| Based on Average Bid | Yes | Yes |
| Maximum Price Deviation Model | Yes | Yes |
| Negometrix | No | Yes |
| Coventry City Council | Yes | Yes |
| European Organization for Nuclear Research (CERN) | Yes | Yes |
| Tennet | No | No |
| Mercer | Yes | Yes |
| Scottish Government | Yes | Yes |
| Waterschap Brabantse Delta | Yes | Yes |
| Chen 1 | No | No |
| Chen 2 | Yes | Yes |
| Chen 3 | No | No |
| Argitek | No | No |
| Telgen | No | No |
| Pauw & Wolvaardt | Yes | Yes |
| Based on the Average Price | Yes | Yes |
| Based on the Lowest Price | Yes | Yes |
| Bisection Method | Yes | Yes |
| Score by Rank | Yes | Yes |
| Kuiper 1 | No | No |
| Kuiper 2 | No | No |
| Kuiper 3 | No | No |
| Kuiper's Superformula | No | No |

| Formula | Quality related intrusive ranking paradox | Quality related non-intrusive ranking paradox |
|---|---|---|
| Lowest Bid Scoring | No | No |
| Highest Bid-Lowest Bid Scoring | No | No |
| Average Scoring | No | No |
| Based on Bid Spread | No | No |
| Based on Average Bid | No | No |
| Maximum Price Deviation Model | No | No |
| Negometrix | Yes | Yes |
| Coventry City Council | Yes | Yes |
| European Organization for Nuclear Research (CERN) | Yes | Yes |
| Tennet | Yes | Yes |
| Mercer | No | No |
| Scottish Government | No | No |
| Waterschap Brabantse Delta | No | No |
| Chen 1 | No | No |
| Chen 2 | No | No |
| Chen 3 | No | No |
| Argitek | No | No |
| Telgen | No | No |
| Pauw & Wolvaardt | No | No |
| Based on the Average Price | No | No |
| Based on the Lowest Price | No | No |
| Bisection Method | No | No |
| Score by Rank | No | No |
| Kuiper 1 | No | No |
| Kuiper 2 | No | No |
| Kuiper 3 | No | No |
| Kuiper's Superformula | No | No |

# 8 Conclusion & Advice

As a tendering entity:

1. Choose a formula that reflects your preferences regarding price and quality.

What is meant here is that if the weight of price is higher than the weight of quality, then the formula should favour cheaper bids. Conversely, if the weight of quality is higher than the weight of price, then the formula should favour better quality bids. Most importantly, the outcome of a tender should meet the preferences of the tendering entity regarding price and quality. For example, if the tendering entity desires a high quality, it should set a high weight of quality and a good formula should reflect this preference. If a formula doesn't reflect this preference, there may be a mismatch between the preference of the tendering entity and the outcome of the tender causing the failure of the whole procurement process. Another important observation is that supplier selection must be understood as a system where quality criteria, price, and their respective weights interact.

2. Do you accept ranking paradox?

I have covered both the relative and absolute approaches in Chapter 3 and explained in detail ranking paradox in Chapter 4. If you accept ranking paradox, a relative formula will suit your purposes. If you don't accept ranking paradox, then you would be better served by an absolute formula.

3. How much are you willing to pay for consecutive units of quality?

This question refers to the marginal rate of substitution of quality for price. As we have seen, different formulas imply increasing, decreasing or constant marginal rates of substitution. Your choice of formula should also reflect whether you are willing to pay more, less or the same for consecutive units of quality. If you are willing to pay more, then a formula with a concave indifference curve will reflect this willingness. If you are willing to pay less, then a formula with convex indifference curve will satisfy this willingness. Finally, if you are willing to pay the same, then a formula with straight indifference curve will better suit this end.

4. Use a formula that provides protection against an extremely high price.

5. Specify your needs in as much detail as possible and reveal this information.

There are at least 3 reasons why you should specify your needs in as much detail as possible and reveal this information. First, it will allow potential suppliers to better respond to these needs. Second, this information may reduce the uncertainty about the price that potential suppliers face. Third, this information enhances transparency.

6. It is optimal to disclose all the details of the award mechanism to potential suppliers.

Potential suppliers face choices when preparing their bids. For example, they may choose whether to give an extended warranty for a higher price. Hence, they have a bid selection problem before they submit their bids. If you disclose all the details of the award mechanism to the potential suppliers, they will use this knowledge to meet your needs and maximize their chances of winning the tender.

7. Keep it simple and smart.

Consider 2 examples. The 1st example is about procuring cleaning services. Suppose that potential contractors have to bid 2 prices: one for cleaning office space and the other for cleaning common areas. The following table summarizes prices that were submitted.

| bid | Office space | Common areas |
|---|---|---|
| A | 40 EUR | 0 EUR |
| B | 20 EUR | 20 EUR |

Suppose further that the contract award is based only on the total price score and that the score is calculated according to the following formula:

$$Score = \frac{P_{Best} + 20}{P_i + 20} \cdot 50.$$

The above formula is applied separately to the submitted prices for cleaning office space and common areas. Scores of bidders A and B are displayed in the table below.

| bid | Office space | Common areas | Total score |
|---|---|---|---|
| A | $33\frac{1}{3}$ | 50 | $83\frac{1}{3}$ |
| B | 50 | 20 | 70 |

According to the scoring rule, the contract will be awarded to bidder A. If the area of the office space is larger than the common areas, the result of a tender is not cost efficient for the tendering entity because bidder B would charge lower total price. The tendering entity would be better off if it asked the bidders to bid the total price.

The 2nd example is about procuring pens. Suppose that potential contractors have to bid 3 prices according to the quantity of the order. The following table summarizes submitted prices.

| Quantity | A | B |
|---|---|---|
| $0 - 1000$ | 2.5 EUR | 2 EUR |
| $1000 - 5000$ | 2 EUR | 2 EUR |
| $> 5001$ | 1.5 EUR | 2 EUR |

Suppose further that the contract award is based only on the total price score and that the score is calculated according to the following formula:

$$Score = \frac{P_{Best}}{P_i}.$$

The above formula is applied separately to submitted prices for each quantity range. Scores are summarized in the table below.

| Quantity | A | B |
|---|---|---|
| $0 - 1000$ | 0.8 | 1 |
| $1000 - 5000$ | 1 | 1 |
| $> 5001$ | 1 | 0.75 |
| Total score | 2.8 | 2.75 |

Although sometimes it could be difficult to estimate the exact quantity of procured goods, it is worthwhile to spend some time to come up with a nearest approximation because this exercise may pay off. In the above example, the contract will be awarded to bidder A, but if the tendering entity requires more than 5001 pens, it would be better off if the contract was awarded to bidder B.

The conclusion from both examples is that it is more efficient to ask the bidders to bid the total price because otherwise bidders may have an incentive to abuse the scoring rule and the outcome of the tender may be unfavourable for the tendering entity.

8. Use a decision support system.

The procurement process is complex. If done manually, there could be a significant amount of mistakes in the procurement process caused by human errors. According to Padumadasa and Rehan[16], award decision tends to lack transparency and fairness due to bias inherent in human nature. Finally, decision support systems can also help to save time and money.

9. Visualize

It is worth creating a price-quality graph of all bids after they have been submitted by placing individual prices on the horizontal axis and individual qualities on the vertical axis. Visualization provides insight not only into the importance of price and quality, but also which bid should be ranked as number 1.

10. No variants

Each bidder should only be permitted to submit 1 bid. In case a relative formula is used, this should decrease the likelihood of a ranking paradox.

11. The choice of formula may significantly influence the results. Different formulas may lead to different rankings.

12. Contractual penalty when relative formula is used and 'winner' drops out.

If a relative formula is used, the likelihood of a ranking paradox should be decreased.

13. Stimulate competition - get as many bidders as possible.

# A Appendix

## A.1 Preference Relations

Let $\mathcal{X}$ be a non-empty set representing possible choices an economic agent. If presented with 2 choices $x, y \in \mathcal{X}$, the agent may prefer one over the other.

**Definition 1.** *A* (strict) preference relation *or* preference order *on $\mathcal{X}$ is a binary relation $\succ$ with the following properties:*
*(a) Asymmetry: If $x \succ y$, then $y \not\succ x$.*
*(b) Negative transitivity: If $x \succ y$ and $z \in \mathcal{X}$, then $x \succ z$ or $z \succ y$.*

Asymmetry implies that for no $y, x \in \mathcal{X}$ it is true that both $x \succ y$ and $y \succ x$. Negative transitivity states that if there is a clear preference between 2 choices $x$ and $y$ and if a 3rd choice $z$ is added, then there is a choice, which is least preferable, that is $y$ if $z \succ y$ or most preferable, that is $x$ if $x \succ z$.

**Definition 2.** *A* weak preference relation *or* preference order *on $\mathcal{X}$ is a binary relation $\succeq$ with the following properties:*
*(a) Completeness: For all $x, y \in \mathcal{X}$ one has $x \succeq y$ or $y \succeq x$.*
*(b) Transitivity: If $x \succeq y$ and $y \succeq z$, then $x \succeq z$.*

Completeness means that every 2 alternatives are comparable. Transitivity states that if $x$ is at least as good as $y$ and $y$ is at least as good as $z$ then $x$ is at least as good as $z$.

Once a weak preference relation is defined, $\succeq$, an indifference relation $\sim$ is given by:

$$x \sim y :\Leftrightarrow x \succeq y \text{ and } y \succeq x.$$

Thus $x \sim y$ means that either $x$ is preferred over $y$ or there is no clear preference between $x$ and $y$. The indifference relation $\sim$ is an equivalence relation. It is reflective, symmetric, and transitive.

## A.2

2 bids have the same score if their utilities are equal. Take bid A with price $P_A$ and quality $Q_A$ and bid B with price $P_B$ and quality $Q_B$. Assume that $P_A > P_B$ and $Q_A > Q_B$.

$$
\begin{aligned}
U_A &= U_B \\
\frac{1-(Q_A-Q_A)\cdot\frac{W_{Quality}}{W_{Price}}}{\frac{P_A}{P_B}} &= \frac{1-(Q_A-Q_B)\cdot\frac{W_{Quality}}{W_{Price}}}{\frac{P_B}{P_B}} \\
\frac{1}{\frac{P_A}{P_B}} &= 1-(Q_A-Q_B)\cdot\frac{W_{Quality}}{W_{Price}} \\
\frac{P_B}{P_A} &= 1-(Q_A-Q_B)\cdot\frac{W_{Quality}}{W_{Price}} \\
1-\frac{P_B}{P_A} &= (Q_A-Q_B)\cdot\frac{W_{Quality}}{W_{Price}} \\
1-\frac{P_B}{P_A} &= \Delta Q\cdot\frac{W_{Quality}}{W_{Price}} \\
\Delta P\cdot W_{Price} &= \Delta Q\cdot W_{Quality}
\end{aligned}
$$

## A.3

It is assumed that 1 bid which won't be ranked number 1 enters the bid competition or 1 bid which wasn't ranked number 1 leaves the bid competition and that the lowest price changes. $P^*$ and $Q^*$ denote price and quality of the highest utility bid. As on can see, the scores don't depend on the lowest price and therefore intrusive price related ranking paradox is impossible.

$$
\begin{aligned}
& P_i\cdot\left(\frac{\max(U_1,\cdots,U_N)-U_i}{\max(U_1,\cdots,U_N)}\right)= \\
= {} & P_i\cdot\left(1-\frac{\left(1-(Q_{Best}-Q_i)\cdot\frac{W_{Quality}}{W_{Price}}\right)\cdot P_{Best}}{P_i}\cdot\frac{P^*}{\left(1-(Q_{Best}-Q^*)\cdot\frac{W_{Quality}}{W_{Price}}\right)\cdot P_{Best}}\right)= \\
= {} & P_i\cdot\left(1-\frac{P^*\cdot\left(1-(Q_{Best}-Q_i)\frac{W_{Quality}}{W_{Price}}\right)}{P_i\cdot\left(1-(Q_{Best}-Q^*)\frac{W_{Quality}}{W_{Price}}\right)}\right).
\end{aligned}
$$

## A.4

Take bid A with price $P_A$ and quality $Q_A$ and bid B with price $P_B$ and quality $Q_B$. Calculate the scores difference between bids A and B. As on can see, the scores difference doesn't depend on the lowest price and therefore a price related ranking paradox is impossible.

$$
\begin{aligned}
& \left(1-0.5\cdot\frac{\log\left(\frac{P_A}{P_{Best}}\right)}{\log(2)}\right)\cdot W_{Price}+W_{Quality}\cdot Q_A+\\
-\ & \left(1-0.5\cdot\frac{\log\left(\frac{P_B}{P_{Best}}\right)}{\log(2)}\right)\cdot W_{Price}-W_{Quality}\cdot Q_B=\\
=\ & W_{Price}-\frac{0.5\cdot W_{Price}}{\log(2)}\cdot\log\left(\frac{P_A}{P_{Best}}\right)+W_{Quality}\cdot Q_A+\\
-\ & W_{Price}+\frac{0.5\cdot W_{Price}}{\log(2)}\cdot\log\left(\frac{P_B}{P_{Best}}\right)-W_{Quality}\cdot Q_B=\\
=\ & \frac{0.5\cdot W_{Price}}{\log(2)}\cdot\left(\log\left(\frac{P_B}{P_{Best}}\right)-\log\left(\frac{P_A}{P_{Best}}\right)\right)+W_{Quality}\cdot(Q_A-Q_B)=\\
=\ & \frac{0.5\cdot W_{Price}}{\log(2)}\cdot\left(\log(P_B)-\log(P_{Best})-\log(P_A)+\log(P_{Best})\right)+W_{Quality}\cdot(Q_A-Q_B)=\\
=\ & \frac{0.5\cdot W_{Price}}{\log(2)}\cdot\left(\log(P_B)-\log(P_A)\right)+W_{Quality}\cdot(Q_A-Q_B)=\\
=\ & \frac{0.5\cdot W_{Price}}{\log(2)}\cdot\log\left(\frac{P_B}{P_A}\right)+W_{Quality}\cdot(Q_A-Q_B).
\end{aligned}
$$

# References

[1] Adotévi S. (2004) Public procurement performance. International Public Procurement Conference Proceedings, 3:253-261.

[2] Albano G.L., F. Dini, and R. Zampino (2009) Bidding for complex projects: Evidence from Italian government's acquisitions of IT services. Lecture Notes in Computer Science, 5693:353-363.

[3] de Boer L., E. Labro, and P. Morlacchi (2001), A review of methods supporting supplier selection. European Journal of Purchasing & Supply Management, 7:75-89.

[4] de Boer L., M.M. Linthorst, F. Schotanus, and J. Telgen (2006) An analysis of some mistakes, miracles and myths in supplier selection. 15th IPSERA Annual Conference, 6-8 April 2006, San Diego, California, USA.

[5] Chen T.H. (2005) Mathematical properties of award systems and their legal consequences. Tijdschrift Aanbestedingsrecht, 2:51-61.

[6] Chen T.H. (2006) De economische dimensie van het criterium economisch voordeligste aanbieding. Tijdschrift Aanbestedingsrecht, 3:101-110.

[7] Chen T.H. (2008) An economic approach to public procurement. Journal of Public Procurement, 8(3):407-430.

[8] CROW (2007) Gunnen op waarde: Hoe doe je dat? Praktische handreiking voor bouwopdrachten. CROW.

[9] Dimitri N., G. Piga, and G. Spagnolo (2006) Handbook of Procurement. CUP.

[10] de Gouveia M.R. (2002) The price factor in EC public tenders. Public Contract Law Journal, 31(4):679-693.

[11] Kuiper H. (2009) De Economisch meest voordelige inschrijving gevisualiseerd. Nederlandse Vereniging Bouw Kostendeskundigen, 2:4-11.

[12] Mateusa R., J.A. Ferreira, and J. Carreira (2009) Full disclosure of tender evaluation models: Background and application in Portuguese public procurement. Journal of Purchasing and Supply Management, 16(3):206-215.

[13] Morand, P.H. and L. Thomas (2006) Efficient procurement with quality concerns. Recherches Economiques de Louvain 72, 129-155.

[14] Onesime O.C.T., X.F. Xu, and D.C. Zhan (2004) A decision support system for supplier selection process. International Journal of Information Technology & Decision Making, 3(3):453-470.

[15] Palaneeswaran E., M. Kumaraswamy, and T. Ng (2003) Targeting optimum value in public sector projects through 'best value' - focused contractor selection. Engineering, Construction and Architectural Management, 10(6):418-431.

[16] Padumadasa E.U. and S. Rehan (2009) Investigation in to decision support systems and multiple criteria decision making to develop a web based tender management system. 10th International Symposium on the Analytic Hierarchy Process, July 29-August 1, 2009, Pittsburgh, Pennsylvania, USA.

[17] Pauw, J.C. and J.S. Wolvaardt (2009) Multi-criteria decision analysis in public procurement - a plan from the south. Politeia, 28(1):66-88.

[18] Quayle M. (2006) Purchasing and supply chain management: Strategies and realities. IGI Global.

[19] Soudry O. (2004) Promoting economy: Electronic reverse auctions under the EC directives on public procurement. Journal of Public Procurement, 4(3):340-374.

[20] Saussier S., C. Staropoli, and A. Yvrande-Billon (2009) Public-private agreements, institutions and competition: When economic theory meets facts. Review of Industrial Organization, 35:1-18.

[21] Smith P. (2010) Evaluating bids and tenders - the scoring trap. Danish Purchasing and Logistics Forum, 47(4):34-39.

[22] Telgen J., F. Schotanus (2010) Effects of full transparency in supplier selection on subjectivity and bid quality. 4th International Public Procurement Conference, August 26-28, 2010, Seoul, South Korea.

[23] Thai K. (ed) (2009) International Handbook of Public Procurement. CRC Press.

[24] Waara F. and J. Brochner (2006) Price and nonprice criteria for contractor selection. Journal of Construction Engineering and Management, 132(8):797-804.

[25] van Weele A.J. (2002) Purchasing and supply chain management: Analysis, planning and practice. Thomson Learning.